Neutrino-electron scattering in hot magnetic fields in the Magellanic System with allowance for polarizations of electrons

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Abstract. The energy-momentum loss by neutrinos in the neutrino-electron scattering in hot stellar magnetic fields in the Magellanic System with allowance for the longitudinal polarizations of electrons is calculated. Several numerical estimations on the asymmetric energy-momentum loss by neutrinos in the neutrino-electron scattering in hot stellar magnetic fields in the Magellanic System are presented. Some reasons of asymmetric recoil and asymmetric heating of strongly magnetized astrophysical objects in the Magellanic System are shown.

Keywords. Energy-momentum loss, neutrino-electron scattering, Magellanic System, stellar magnetic fields, polarization of electrons

1. Introduction

The polarization effects, field effects and some other effects arising in the scattering of neutrinos (antineutrinos) at electrons in a magnetic field (MF) have been studied by numerous authors (see, e.g. [1-6]). One of the main purposes of this work is to present analytic formulae for the energy-momentum loss (EML) by neutrinos in the neutrino-electron scattering (NES) $\nu_i + e^- \to \nu_i + e^-$ in hot stellar magnetic fields in the Magellanic System with allowance for the longitudinal polarizations of the initial and final electrons, where $\nu_i = \nu_e, \nu_\mu, \nu_\tau$. We also want to show that the EML by neutrinos in the NES in hot stellar magnetic fields is sensitive to the spin variable of the initial and final electrons, the direction of the incident and scattered neutrinos momenta and the neutrino flavour. Our purpose is also to explain the reasons of asymmetric recoil and asymmetric heating of strongly magnetized astrophysical objects in the Magellanic System.

We use the standard Weinberg-Salam-Glashow electroweak interaction theory. When the momentum transferred is relatively small, $\left|q^2\right| \ll m_W^2, m_Z^2$ (m_W is the $W^\pm$-boson mass, m_Z is the Z-boson mass), the 4-fermion approximation of the Weinberg-Salam-Glashow standard model can be used. The gauge of a 4-potential is $A^\mu = (0,0,xH,0)$ and an external MF vector $\vec{H}$ is directed along the axis Oz. We deal with a massless neutrino. We use the pseudo-Euclidean metric with signature $(+---)$ and the system of units $k_B = \hbar = c = 1$, where k_B is the Boltzman constant.

2. Energy-momentum loss by neutrinos in the NES in a magnetic field

The EML by neutrinos in the NES in a magnetic field per unit of volume of the medium per unit time is defined as

$$\frac{dP_\beta}{dt}=\left(\frac{dW}{dt},\vec{F}\right)=(2\pi)^{-7}G_F^2 eH\int\sum_{n,n'=0}^{\infty}\sum_i \frac{E_i E_i'}{\left|E_i' p_{zi}-E_i p_{zi}'\right|}Q\omega^2\omega'^2\times \tag{1}$$
$$\times q_\beta f_\nu\left(1-f_\nu'\right)f_e\left(1-f_e'\right)d\omega d\omega' d\Omega d\Omega'$$

where Q [2] is the function of a MF strength H, spin variables ζ,ζ' and energies E, E' of electrons in initial and final states, the polar angle of incident (scattered) neutrino momentum ϑ (ϑ'), the difference between the azimuthal angles of incident neutrino momentum and scattered neutrino momentum $\alpha-\alpha'$, the angle φ ($\tan\varphi = q_y/q_x$, $q_\beta = k_\beta - k_\beta'$, $k_\beta\left(k_\beta'\right)$ is the incident (scattered) neutrino 4-momentum, $\beta = 0,1,2,3$) and the parameter

$$x=\left(1/2eH\right)\left[\omega^2\sin^2\vartheta+\omega'^2\sin^2\vartheta'-2\omega\omega'\sin\vartheta\sin\vartheta'\cos\left(\alpha-\alpha'\right)\right], \tag{2}$$

$p_{zi}\left(p_{zi}'\right)$ is the third component of the initial (final) electron momentum which satisfies the equation $k_z+p_z=k_z'+p_z'$, $\omega\left(\omega'\right)$ is the incident (scattered) neutrino energy, G_F is the Fermi constant, e is the elementary electric charge, $d\Omega\left(d\Omega'\right)$ is the solid angle element along the initial (final) neutrino momentum, $f = f_e\left(E,T_e\right)$ is the Fermi-Dirac distribution (FDD) of electrons in the initial state, E is the energy of an electron in the initial state, T_e is the temperature of the matter (electron gas) before scattering, $f' = f_e'\left(E',T_e'\right)$ is the FDD of electrons in the final state, E' is the energy of an electron in the final state, T_e' is the temperature of the matter (electron gas) after scattering, $f_\nu = f_\nu\left(\omega,T_\nu\right)$ is the FDD of incident neutrinos, T_ν is the initial temperature of the incident neutrino gas, $f_\nu' = f_\nu'\left(\omega',T_\nu'\right)$ is the FDD of scattered neutrinos, T_ν' is the final temperature of the scattered neutrino gas.

3. Asymmetric momentum loss by neutrinos

The momentum loss by neutrinos in the NES in a MF happens asymmetrically depending on the polarization states of electrons. In general, the gas consisting of electrons having a left-hand circular polarization and the gas consisting of electrons having a right-hand circular polarization are recoiled by neutrinos asymmetrically. The asymmetry of momentum loss by neutrinos (or the asymmetry of recoil of electrons depending on their polarization states) in the NES in a MF is determined by the following formula:

$$A_M=\frac{\vec{F}_R-\vec{F}_L}{\vec{F}_R+\vec{F}_L} \tag{3}$$

where

$$\vec{F}_R=\vec{F}\left(\zeta=1,\zeta'=1\right),\quad \vec{F}_L=\vec{F}\left(\zeta=-1,\zeta'=-1\right) \tag{4}$$

and the value $\zeta,\zeta'=+1(-1)$ corresponds to right-hand (left-hand) helicity of an electron. ζ belongs to the electrons in the initial state, ζ' belongs to the electrons in the final state. For the strongly magnetized stars we can take $H\simeq 4.41\times10^{15}G$ and for relativistic electrons $\left(E,E'\gg m_e\right)$ we suppose that $p_{zi}/E\ll1$, $p_{zi}'/E'\ll1$. On the other hand at the temperatures $T\simeq10^{11}K$ the characteristic energy for electrons is $E\simeq8.5MeV$. It is obtained from the formula $E=\sqrt{m_e^2+2eHn+p_z^2}$ that the energy $E\simeq8.5MeV$ corresponds to the Landau quantum number $n=1$. Let us consider the transition of $n=1\rightarrow n'=2$. At $n'=2$ and $H\simeq4.41\times10^{15}G$ the energy of the final electrons is $E'\simeq12MeV$. On the other hand it is known that the chemical potential of the electron gas in a very strong magnetic field is determined as (see, e.g., [6])

$$\mu = \frac{2\pi^2 n_0}{eH} \simeq 25.68 MeV \left(\frac{n_0}{10^{33} cm^{-3}}\right)\left(\frac{10^{15} G}{H}\right) \qquad (5)$$

At the electron density $n_0 \sim 10^{33} cm^{-3}$ and $H \sim 10^{15} G$ we have $\mu \simeq E' \simeq 12 MeV$. It means that $(E' - \mu)/T'_e \ll 1$. In this case the asymmetry of heating is determined as

$$A_M = \frac{Q_R - Q_L}{Q_R + Q_L}. \qquad (6)$$

The analyses enable us to come to the conclusion that, in general, the asymmetry of momentum loss by neutrinos is sensitive to the neutrino flavour, the magnetic field strength, the energies (or the Landau quantum numbers and the third components of the momentum) of the initial and final electrons, the polar angle of the incident (scattered) neutrino momentum, the difference between the azimuthal angles of the incident and scattered neutrino momenta, the angle φ, the incident (scattered) neutrino energy.

4. Numerical estimations for asymmetric momentum loss by neutrinos

Let us consider the case of $\vartheta = 0$, $\vartheta' = \pi/2$, $\alpha' = \varphi$ for numerical estimations for the asymmetric momentum loss by neutrinos. In this case $x = \omega'^2/(2eH)$ and the following expression is obtained for the asymmetry of momentum loss by neutrinos

$$A_M = \frac{g_R^2 I_3^2 - g_L^2 I_4^2 - \left(g_L^2 - g_R^2\right) I_2^2 - 2\left(g_R^2 I_2 I_4 - g_L^2 I_2 I_3\right)}{g_R^2 I_3^2 + g_L^2 I_4^2 + \left(g_L^2 + g_R^2\right) I_2^2 - 2\left(g_R^2 I_2 I_4 + g_L^2 I_2 I_3\right)} \qquad (7)$$

where $I_2 = I_{n-1,n'}$, $I_3 = I_{n-1,n'-1}$, $I_4 = I_{n,n'}$ are the Laguerre functions [2], $g_L = 0.5 + \sin^2\theta_w$, $g_R = \sin^2\theta_w$ for $\nu_e e^-$-scattering and $g_L = -0.5 + \sin^2\theta_w$, $g_R = \sin^2\theta_w$ for $\nu_\mu e^-$- and $\nu_\tau e^-$-scatterings. Here θ_w is the Weinberg angle. We see that in the expression (7) A_M does not contain the Laguerre function $I_1 = I_{n,n'-1}$.

If we consider the transition $n = 1 \to n' = 2$, the strongly magnetized stars of the strength $\sim 10^{15} G$ (e.g., $H \simeq 4.41 \times 10^{15} G$) and the neutrinos of energy $\omega' \simeq 1 MeV$, we obtain $x \simeq 0.019$ and

$$A_{M\nu_e e^-} \simeq -0.89 \qquad (8)$$

for the $\nu_e e^- \to \nu_e e^-$ process. Numerical estimations show that for the considered case of the magnetic field strength and for the same neutrino energy

$$A_{M\nu_\mu e^-} = A_{M\nu_\tau e^-} \simeq -0.4, \qquad (9)$$

i.e.

$$A_{M\nu_e e^-} \simeq 2.23 A_{M\nu_\mu e^-} = 2.23 A_{M\nu_\tau e^-}. \qquad (10)$$

These estimations show that the momentum loss by neutrinos in the NES in a MF happens asymmetrically depending on the polarization state of electrons. It enables us to come to the conclusion that within the considered kinematics the electrons (electron gas) having a left-hand circular polarization and the electrons (electron gas) having a right-hand circular polarization are recoiled by neutrinos asymmetrically and the asymmetry of recoil of electrons in different polarization states by neutrinos is sensitive to the neutrino flavour, the MF strength and the scattered neutrino energy.

5. Asymmetric energy loss by neutrinos and numerical estimations

The asymmetry of energy loss by neutrinos (or the asymmetry of heating of electrons depending on their polarization states) in the NES in a MF is determined by the following formula:

$$A_E = \frac{\left(dW/dt\right)\big|_R - \left(dW/dt\right)\big|_L}{\left(dW/dt\right)\big|_R + \left(dW/dt\right)\big|_L} \qquad (11)$$

where

$$\left(dW/dt\right)\big|_R = \left(dW/dt\right)\left(\zeta = 1, \zeta' = 1\right), \ \left(dW/dt\right)\big|_L = \left(dW/dt\right)\left(\zeta = -1, \zeta' = -1\right). \qquad (12)$$

The analyses show that the asymmetry of energy loss by neutrinos (or the asymmetry of heating of electrons depending on their polarization states) is sensitive to the neutrino flavour, the MF strength, the energies (or the Landau quantum numbers and the third components of the momentum) of the initial and final electrons, the polar angle of the incident (scattered) neutrino momentum, the difference between the azimuthal angles of the incident and scattered neutrino momenta, the angle φ, the incident (scattered) neutrino energy.

In a protoneutron star the gas consisting of the electrons having a left-hand circular polarization and the gas consisting of the electrons having a right-hand circular polarization will be in thermodynamic equilibrium after the scattering of neutrinos at electrons. Because in a protoneutron star the gas consisting of the electrons having a left-hand circular polarization and the gas consisting of the electrons having a right-hand circular polarization are mixed. It means that for the final temperature of the electron gas (after scattering) we have $T_L = T_R = T'$. Let us consider the case of $\vartheta = 0$, $\vartheta' = \pi/2$, $\alpha' = \varphi$ for numerical estimations. For the strongly magnetized stars ($H \simeq 4.41 \times 10^{15} G$) and the neutrinos of energy $\omega' \simeq 1 MeV$ we obtain $A_{E\nu_e e^-} \simeq -0.89$ for the $\nu_e e^- \to \nu_e e^-$ process. Numerical estimations show that for the considered case of the MF strength and the neutrino energy $A_{E\nu_\mu e^-} = A_{E\nu_\tau e^-} \simeq -0.4$, i.e. $A_{E\nu_e e^-} \simeq 2.23 A_{E\nu_\mu e^-} = 2.23 A_{E\nu_\tau e^-}$.

6. Conclusions

The momentum loss by neutrinos in the NES in a MF happens asymmetrically depending on the polarization states of electrons. Within the considered kinematics the electrons (electron gas) having a left-hand circular polarization and the electrons (electron gas) having a right-hand circular polarization are recoiled by neutrinos asymmetrically and the asymmetry of recoil of electrons in different polarization states by neutrinos is sensitive to the neutrino flavour, the MF strength and the scattered neutrino energy. The asymmetric recoil of electrons in different polarization states by neutrinos enables us to explain the reasons of asymmetric recoil of strongly magnetized astrophysical objects in the Magellanic System. In the heating process of the electrons by the neutrinos the dominant role belongs to the electron neutrinos compared with the contribution of the muon neutrinos or the tauon neutrinos. For the strongly magnetized stars ($H \simeq 4.41 \times 10^{15} G$) and the neutrinos of $1 MeV$ energy, within the considered kinematics, the electrons (electron gas) having a left-hand circular polarization and the electrons (electron gas) having a right-hand circular polarization are heated by the neutrinos asymmetrically and the asymmetry of heating is sensitive to the neutrino flavour, the magnetic field strength and the scattered neutrino energy. The effect of asymmetrical heating of electrons by neutrinos could contribute to asymmetry and anisotropy of the subsequent explosion of the outer layers of the collapsing stellar core in the Magellanic System.

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