

Eclipsing binary stars in open clusters

J. K. Taylor

Department of Chemistry and Physics, Keele University, Staffordshire, ST5 5BG, UK

Submitted for postgraduate research degree qualification on 3rd February 2005

This is NOT the version which was accepted on 1st March 2006

ABSTRACT

The study of detached eclipsing binary stars allows accurate absolute masses, radii and luminosities to be measured for two stars of the same chemical composition, distance and age. These data can provide a good test of theoretical stellar evolutionary models, aid the investigation of the properties of peculiar stars, and allow the distance to the eclipsing system to be found using empirical methods. Detached eclipsing binaries which are members of open clusters provide a more powerful test of theoretical models, which must match the properties of the eclipsing system whilst simultaneously predicting the morphology of the cluster in photometric diagrams. They also allow the distance and the metal abundance of the cluster to be found, avoiding problems with fitting empirical or theoretical isochrones in colour-magnitude diagrams.

Absolute dimensions have been found for V615 Per and V618 Per, which are eclipsing members of the χ Persei open cluster. This has allowed the fractional metal abundance of the cluster to be measured to be $Z \approx 0.01$, in disagreement with the solar chemical composition often assumed in the literature.

Accurate absolute dimensions (masses to 1.4%, radii to 1.1% and effective temperatures to within 800 K) have been measured for V453 Cygni, a member of NGC 6871. The current generation of theoretical stellar models can successfully match these properties, as well as the central concentration of mass of the primary star as derived from a study of the apsidal motion of the system. A Monte Carlo analysis technique has been implemented to determine robust uncertainties in the results of the photometric analysis of detached eclipsing binaries.

The B-type subgiant eclipsing system V621 Per, a member of the open cluster χ Persei, which is related to χ Persei, has been studied. The absolute dimensions of the system have not been measured as the secondary star is not detectable in our spectroscopic observations, but have been inferred from a comparison with theoretical models. The secondary star should be detectable in very high-quality spectra, in which case further study of this system will be very rewarding.

Absolute dimensions have been determined for HD 23642, an eclipsing member of the Pleiades open cluster. This has allowed an investigation into the usefulness of different methods to find the distances to eclipsing binaries. A new method has been introduced, based on calibrations between surface brightness and effective temperature, and used to find an accurate distance to the Pleiades of 139 ± 4 pc. This value is in good agreement with other distance measurements but does not agree with the controversial distance measurement derived from parallaxes obtained by the Hipparcos satellite.

The metallic-lined eclipsing binary WW Aur has been studied using extensive new spectroscopy and published light curves. The masses and radii have been found, to accuracies of 0.4% and 0.6% respectively, using entirely empirical methods. The effective temperatures of both stars have been found using a method which is almost fundamental. The predictions of theoretical models can only match the properties of WW Aur by adopting a large metal abundance of $Z = 0.060 \pm 0.005$.

1	Stellar properties	3
1.1	Spectral classification	3
1.2	Brightness and distance	3
1.2.1	Interstellar extinction	4
1.3	Stellar characteristics	5
1.3.1	Stellar interferometry	5
1.3.2	The effective temperature scale	5
1.3.3	T_{eff} s and angular diameters from the IRFM	6
1.3.4	Stellar chemical compositions	6
1.3.5	Bolometric corrections	7
1.3.6	Surface brightness relations	7
1.4	Limb darkening	9
1.4.1	Limb darkening laws	10
1.4.2	Limb darkening and eclipsing binaries	11
1.5	Gravity darkening	11
2	Stellar evolution	12
2.1	The evolution of single stars	12
2.1.1	The formation of stars	12
2.1.2	Main sequence evolution	13
2.1.3	The evolution of low-mass stars	13
2.1.4	The evolution of intermediate-mass stars	13
2.1.5	The evolution of massive stars	14
3	Modelling of stars	14
3.1	Physical phenomena in models	14
3.1.1	Equation of state	14
3.1.2	Opacity	14
3.1.3	Energy transport	14
3.1.4	Convective core overshooting	15
3.1.5	Convective efficiency	16
3.1.6	The effect of stellar rotation	16
3.1.7	The effect of mass loss	16
3.1.8	The effect of diffusion	17
3.1.9	The effect of magnetic fields	17
3.2	Available theoretical stellar models	17
3.2.1	Hejlesen theoretical models	17
3.2.2	Granada theoretical models	17
3.2.3	Geneva theoretical models	17
3.2.4	Padova theoretical models	17
3.2.5	Cambridge theoretical models	17
3.2.6	Other theoretical models	18
3.3	Comments on stellar models	18
4	Spectral characteristics of stars	18
4.1	Spectral lines	18
4.1.1	Spectral line broadening	19
4.2	Spectral features in stars	20
4.3	Stellar model atmospheres	20
4.3.1	The current status of model atmospheres	20
4.3.2	Convection in model atmospheres	21
4.3.3	The future of stellar model atmospheres	21
4.4	Calculation of theoretical stellar spectra	21
4.4.1	Microturbulence velocity	21
4.4.2	The UCLSYN spectral synthesis code	22
4.4.3	Abundance analysis of stellar spectra	22
4.5	Spectral peculiarity in stars	22
4.5.1	Metallic-lined stars	22
4.5.2	Chemically peculiar stars	23
5	Multiple stars	23
5.1	Dynamical characteristics of multiple stars	23
5.2	Binary star systems	24
5.3	Eclipsing binary star systems	25
6	Detached eclipsing binary star systems	26
6.1	Comparison with theoretical stellar models	28
6.1.1	The methods of comparison	28
6.1.2	Further work	29
6.1.3	The difference between binary and single stars	30
6.2	Metal and helium abundances of nearby stars	30
6.3	Detached eclipsing binaries as standard candles	31
6.3.1	Distances from bolometric corrections	31
6.3.2	Distances from surface brightness relations	32
6.3.3	Distances from modelling of stellar SEDs	32
6.3.4	Recent results for the distance to eclipsing binaries	32
6.4	Detached eclipsing binaries in stellar systems	33
6.4.1	Literature results on dEBs in open clusters	33
7	Tidal effects	34
7.1	Orbital circularization and rotational synchronism	34
7.1.1	The theory of Zahn	34
7.1.2	The theory of Tassoul & Tassoul	35
7.1.3	The theory of Press, Wiita & Smarr	35
7.1.4	The theory of Hut	35
7.1.5	Comparison with observations	35
7.1.6	Summary	36
7.2	Apsidal motion	37
7.2.1	Relativistic apsidal motion	37
7.2.2	Comparison with theoretical models	38
7.2.3	Comparison between observations and theory	38
8	Open clusters	38
8.1	Photometric characteristics of open clusters	39
8.2	Colour-magnitude diagrams of open clusters	40

8.3	Dynamical characteristics of open clusters	42
9	The galactic and extragalactic distance scale	42
9.1	Parallax-based distances to stars	42
9.1.1	Trigonometrical parallax	42
9.1.2	Spectroscopic and photometric parallax	43
9.2	Distances to binary stars	43
9.2.1	Visual binaries	43
9.2.2	Eclipsing binaries	43
9.3	Variable stars as standard candles	43
9.3.1	δ Cepheid variables	43
9.3.2	RR Lyrae variables	43
9.3.3	Type Ia supernovae	44
9.4	Distances to stellar clusters	44
9.5	The Galactic and extragalactic distance scale	44
10	Obtaining and reducing astronomical data	45
10.1	Telescopes	45
10.1.1	Optical aberration	45
10.2	Charge-coupled devices	45
10.2.1	Advantages and disadvantages of CCDs	46
10.2.2	Reduction of CCD data	46
10.2.3	Debiasing CCD images	46
10.2.4	Flat-fielding CCD images	47
10.2.5	Photometry from CCD images	47
10.2.6	Aperture photometry	47
10.2.7	Point spread function photometry	48
10.2.8	Optimal photometry	48
10.3	Grating spectrographs	48
10.3.1	Reduction of CCD grating spectra	48
10.4	Échelle spectrographs	49
10.5	Observational procedures for the study of dEBs	49
10.5.1	CCD photometry	49
10.5.2	Grating spectroscopy	49
11	Determination of spectroscopic orbits	50
11.1	The equations of spectroscopic orbits	50
11.2	The fundamental concept of radial velocity	50
11.3	Radial velocities from observed spectra	50
11.3.1	Radial velocities from spectral lines	51
11.3.2	Radial velocities using 1D cross-correlation	52
11.3.3	Directly observing cross-correlation functions	53
11.3.4	Radial velocities using 2D cross-correlation	53
11.3.5	Radial velocities using spectral disentangling	54
11.3.6	Radial velocities using Doppler tomography	54
11.4	Determination of spectroscopic orbits	55
11.4.1	SBOP – Spectroscopic Binary Orbit Program	56
11.5	Determination of rotational velocities	56
12	Photometry	56
12.1	Photometric systems	56
12.1.1	Broad-band photometric systems	57
12.1.2	Broad-band photometric calibrations	58
12.1.3	Strömgren photometry	58
12.1.4	Strömgren photometric calibrations	59
12.1.5	Other photometric systems	61
13	Light curve analysis of dEBs	61
13.1	Models for the simulation of dEB light curves	62
13.1.1	Rectification	62
13.1.2	EBOP – Eclipsing Binary Orbit Program	62
13.1.3	WINK by D. B. Wood	63
13.1.4	The Wilson-Devinney (WD) code	63
13.1.5	Comparison between light curve codes	64
13.1.6	Other light curve fitting codes	65
13.1.7	Least-squares fitting algorithms	65
13.2	Solving light curves	65
13.2.1	Calculation of the orbital ephemeris	66
13.2.2	Initial conditions	66
13.2.3	Parameter determinacy and correlations	67
13.2.4	Final parameter values	68
13.3	Uncertainties in the parameters	69
13.3.1	The problem	69
13.3.2	The solutions	69
14	V615 Persei and V618 Persei in h Persei	70
14	V453 Cygni in NGC 6871	70
14	V621 Persei in χ Persei	70
14	HD 23642 in the Pleiades	70
14	The metallic-lined system WW Aurigae	70
15	Conclusion	71
15.1	What this work can tell us	71
15.1.1	The observation and analysis of dEBs	71
15.1.2	Studying stellar clusters using dEBs	71
15.1.3	Theoretical evolutionary models and dEBs	71
15.2	Further work	72
15.2.1	Further study of the dEBs in this work	72
15.2.2	Other dEBs in open clusters	72
15.2.3	dEBs in globular clusters	72
15.2.4	dEBs in other galaxies	72
15.2.5	dEBs in clusters containing δ Cepheids	72
15.2.6	dEBs which are otherwise interesting	73
15.2.7	dEBs found by large-scale variability studies	73

1 STARS

A star is a sphere of matter held together by its own gravity and generating energy by means of nuclear fusion in its interior (Wordsworth Dictionary of Science and Technology, 1995).

Stars form from large clouds of gas and dust which attain a sufficient density to gravitationally collapse and form a protostar. The gravitational energy of the cloud is converted to thermal energy, which is transported by convection to the surface and then lost in the form of radiation. This gravitational collapse continues until the centre of the protostar is sufficiently hot and dense for thermonuclear fusion of hydrogen to begin. The minimum mass for this to occur is approximately $0.08 M_{\odot}$. The maximum initial mass of a star is strongly dependent on the chemical composition of the material from which it formed, but is of the order of $100 M_{\odot}$ for a solar chemical composition. Once thermonuclear fusion becomes the main source of energy for the protostar, it ceases to contract and settles down into a steady state.

The fundamental original properties of a star are its initial mass (M), chemical composition, rotational velocity and age. Given these quantities, stellar evolutionary theories can predict the radius (R), effective temperature (T_{eff}), luminosity (L), and structure of any star. The luminosity of the star is defined to be the total amount of radiative energy emitted, summed over all wavelengths, per unit time in all directions (Hilditch 2001). The radius of a star is actually not a precisely defined quantity, because stars do not have exact radii but merely a progressive loss of density (Scholz 1998), but is usually taken as the radius of the photosphere at an optical depth of $\frac{2}{3}$ (e.g., Siess, Dufour & Forestini 2000). T_{eff} is defined to be the temperature of a black body emitting the same flux per surface area as the star. Use of geometry and the Stefan-Boltzmann law gives, by definition,

$$L = 4\pi R^2 \sigma_{\text{SB}} T_{\text{eff}}^4 \quad (1)$$

where the Stefan-Boltzmann constant $\sigma_{\text{SB}} = 5.670400(40) \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ (Institute of Physics, UK). The uncertainty in the final digit of this value is given in parentheses; this convention will be used below. The properties of a star are often given in units of the equivalent value for the Sun. The fundamental properties of the Sun are given in Table 1.

1.1 The spectral classification of stars

The first classification system for stellar spectra was introduced by the Italian Jesuit astronomer A. Secchi in the 1860s. Towards the end of the 19th century a scheme was developed by astronomers in which the spectra of stars were assigned letters between A and P depending on the strength of the hydrogen Balmer lines, where A had the strongest lines and P had no detectable lines (Zeilik & Gregory 1998).

In the early 20th century, a team led by E. C. Pickering (Harvard College Observatory) developed a spectral classification scheme where the strengths of spectral features change smoothly between classes. The team applied this to 225 300 stars to produce the Henry Draper Catalogue, which was named after the wealthy amateur astronomer who financed their work. The team dropped some of the previously used spectral classes and rearranged the remaining ones into the order of decreasing temperature: OBAFGKM. One member of the Harvard team, A. Cannon, further divided each class into ten numerically-designated subclasses, where number 0 refers to the hottest and number 9 to the coolest stars in each spectral class (Kaufmann 1990).

Stars can also be divided into luminosity classes which are represented, in increasing order of luminosity, by VI (subdwarfs), V (MS stars), IV (subgiants), III (giant stars), II (bright giants) and I (supergiants). The original scheme, by W. S. Adams and A. Kohlschutter, was refined by W. W. Morgan and P. C. Keenan (see Morgan, Keenan & Kellman 1943). These researchers used the letters a, ab, b (in order of decreasing luminosity) to indicate luminosity subclasses (Zeilik & Gregory 1998, p. 255). The MK luminosity classification scheme is still in use today, but the subclasses are rarely used by researchers except for supergiants.

Spectral classification is an indicator of the ionisation and

Table 1. The fundamental properties of the Sun. Note that $M_{\text{bol}\odot}$ is a defined quantity and not a measured value.

References: (1) Zombeck (1990); (2) Bessell, Castelli & Plez (1998); (3) Anders & Grevesse (1989).

Quantity	Value	Units	Ref
Mass	1.9891×10^{30}	kg	1
Radius	6.9599×10^8	m	1
$\log g$	4.4377	cm s^{-2}	1
Spectral type	G2 V		1
Luminosity	$3.855(6) \times 10^{26}$	W	2
T_{eff}	5781	K	2
M_{bol}	+4.74	mag	2
M_V	+4.81	mag	2
Bolometric correction	−0.07	mag	2
Hydrogen abundance	0.70683		3
Helium abundance	0.27431		3
Metal abundance	0.01886		3

excitation state of a stellar photosphere, which depends mainly on T_{eff} . Surface gravity also has some effect through the process of collisional excitation; supergiants have T_{eff} s up to 8000 K lower than MS stars with the same spectral class (Böhm-Vitense 1981). Luminosity classification is made using the ratios of widths of strong spectral lines, which depend mainly on surface gravity through the effect of pressure broadening. A two-dimensional spectral type (consisting of a spectral and a luminosity class) is therefore an indicator of the T_{eff} and surface gravity of a star.

Spectral types are unimportant when studying relatively well understood stars; for example dwarfs and giants of spectral types B, A, F, G and K; for which T_{eff} s and surface gravities can be estimated with relative ease. As spectral types are discrete and atmospheric parameters are continuous, the only reason to continue quoting spectral types is to provide a convenient and straightforward indicator of the properties of a star. For classes of star which are less well understood, for example O stars, M dwarfs and cooler objects, and supergiants, the procedure of spectral typing has continued to be important. This is because it depends entirely on observed spectral features, so provides a means of classifying stars before their properties are understood in detail.

The spectral classification sequence for cool stars has been extended from M to L and T. The L classification was formalised by Kirkpatrick et al. (1999) and the T class by Burgasser et al. (2002); the transition between them seems to be caused by cloud characteristics rather than a change in T_{eff} (Leggett et al. 2004). The next spectral class has been suggested to be Y, which will refer to objects as small as Jupiter and Saturn (H. R. A. Jones, in Leggett et al. 2004).

Fig. 1 shows two T_{eff} scales for O5 to M8 stars.

1.2 Brightness and distance

The brightness of stars to observers at the Earth is usually given in magnitudes, for historical reasons and for convenience. In the second century BC the Greek astronomer Hipparchus divided the stars visible to the naked eye into six groups, where group one contained the brightest stars and group six the dimmest. In the late 18th century W. Herschel found that the stars in group one were about one hundred times brighter than the stars in group six. It was subsequently discovered that the human eye detects light in a logarithmic manner, and in 1856 N. R. Pogson proposed a formal definition of the magnitude scale. The difference between the magnitudes m_1 and m_2 corresponding to a ratio of received flux densities, f_2/f_1 , is

$$m_1 - m_2 = -2.5 \log_{10} \left(\frac{f_1}{f_2} \right) \quad (2)$$

This does not define a zeropoint for the magnitude scale. Photometric systems usually define the magnitude of the star Vega to be zero if observed from the top of Earth's atmosphere. Alterna-

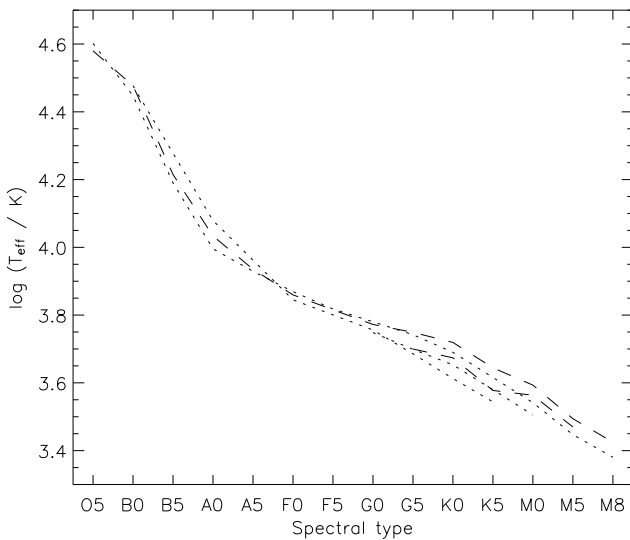


Figure 1. Two T_{eff} scales plotted against spectral type. The T_{eff} scales from Allen (1973) are plotted for (in decreasing T_{eff}) MS, giant and supergiant stars (dotted lines). The T_{eff} scales from Zombeck (1990) are plotted for (in decreasing T_{eff}) MS and giant stars (dashed lines).

tive systems exist based on a more useful physical definition, e.g., the AB_V system used by the Sloan Digital Sky Survey (Oke & Gunn 1983; Fukugita et al. 1996).

As stars generally have a different spectral energy distribution to Vega, the magnitudes of stars relative to Vega depend on the wavelength at which observations are undertaken. The usual convention is to use the visual magnitudes of stars, which is taken to be the magnitude as viewed through the Johnson V passband (see Sec. 12), and denoted as m_V . The absolute magnitude of a star is a measure of its intrinsic brightness and is defined to be the apparent magnitude of the stars as viewed from a distance of ten parsecs. Using eq. 2 gives the equation

$$m_V - M_V = 5 \log_{10}(d) - 5 \quad (3)$$

where d is the distance in parsecs and the quantity $(m_V - M_V)$ is the apparent distance modulus.

The absolute magnitude of a star when considering the radiation it emits summed over all wavelengths is the absolute bolometric magnitude (M_{bol}). This is usually given relative to the Sun using the equation

$$M_{\text{bol}} - M_{\text{bol}\odot} = -2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right) \quad (4)$$

The relation between the absolute visual magnitude, M_V , and absolute bolometric magnitude, M_{bol} , of a star is

$$M_V = M_{\text{bol}} - BC_V \quad (5)$$

where BC_V is the V -band bolometric correction (see Sec. 1.3.5)

Colour indices for stars are the ratio of flux densities at two different wavelengths (or viewed through two different passbands) relative to Vega, for example the colour index for a star between the B and V passbands is

$$m_B - m_V = -2.5 \log_{10} \left[\left(\frac{f_B}{f_V} \right)_{\text{star}} \left(\frac{f_V}{f_B} \right)_{\text{Vega}} \right] \quad (6)$$

The colour indices for Vega are all zero by definition.

1.2.1 Interstellar extinction

The matter between stars attenuates the light which passes through it. The amount of light which is attenuated is a function of wavelength, so interstellar material affects the colours of stars as well as their apparent brightnesses. The main attenuation is due to scattering, but some light is also absorbed. As blue

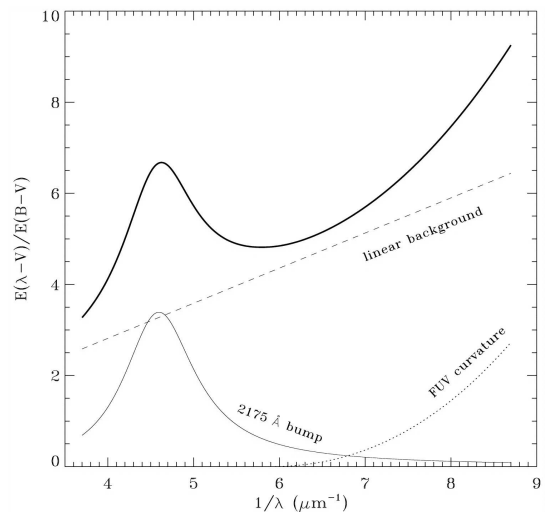


Figure 2. Decomposition of the analytical fitting function for extinction curves introduced by Fitzpatrick & Massa (1986, 1988, 1990). Taken from Fitzpatrick (1998).

light is attenuated more than red light, this causes stars to appear to be redder than they actually are, a phenomenon which is termed ‘reddening’. As blue light is scattered more than red light, this term should technically be replaced by ‘de-bluening’ (Zeilik & Gregory 1999, p. 285) but this is a minor philosophical point.

The total extinction as a function of wavelength λ is represented by A_{λ} , which is in units of magnitudes. The amount of extinction depends on the amount and composition and grain size of the interstellar matter which is causing it (Fitzpatrick 1999). The main reason for studying the interstellar absorption in this work is to quantify and remove its effects, although knowledge of the properties of the interstellar medium is important for constructing models of galactic chemical evolution.

The total extinction at wavelength λ depends on the reddening (also called colour excess) between the B and V passbands:

$$A_{\lambda} = R_{\lambda} E_{B-V} \quad (7)$$

where the constant of proportionality, R , is traditionally applied only to finding A_V but is actually applied at many wavelengths. The value of R_V is generally found – and assumed – to be between 3.0 and 3.2 (e.g., Allen 1973; Zombeck 1990), but values between 2.2 and 5.8 have been reported. It is important to remember that R_{λ} is also weakly dependent on spectral type (Crawford & Mandwewala 1976; Bessell, Castelli & Plez 1998) because the effective wavelengths of photometric passbands are redder for late-type stars as they produce relatively more flux in the redder part of the passband response function. As R_{λ} is smaller at longer wavelengths, it is smaller for the late-type stars, so E_{B-V} must be larger for these stars to give the same A_{λ} .

R_{λ} depends on the physical properties of the material causing the interstellar extinction (Ducati, Ribeiro & Rembold 2003), which for optical light is mainly dust grains. Smaller dust grains are more important in the ultraviolet (UV) and blue wavelength regions whilst larger grains are more important at red and infrared (IR) wavelengths. This means that the spectral dependence of reddening varies slightly throughout the Milky Way Galaxy (Ducati, Ribeiro & Rembold 2003); there is considerable structure between 4000 and 7000 Å (Jacoby, Hunter & Christian 1984).

Fitzpatrick (1998) has made a detailed investigation of the effects of interstellar extinction and how these may be removed from astronomical observations. That investigation was based on an analytical fitting function for extinction curves introduced by Fitzpatrick & Massa (1990), consisting of a linear background, a steep rise in extinction at shorter wavelengths, and a ‘bump’ increase in extinction centred at 2176 Å (Fig. 2). Whilst the centre of the ‘bump’ is very stable, its width depends on the type of material causing the extinction (Fitzpatrick & Massa 1986). The

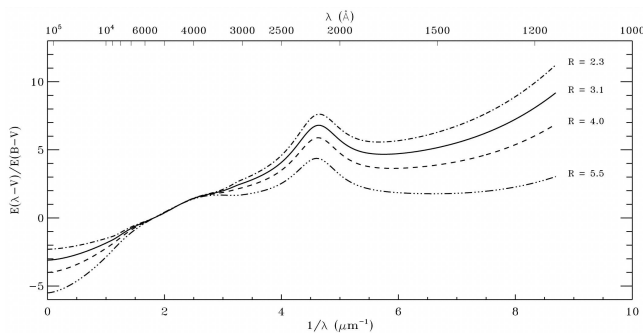


Figure 3. The form of extinction curves for different values of R . Taken from Fitzpatrick (1998).

shape of the far-UV curvature appears to be invariant in the Milky Way Galaxy (Fitzpatrick & Massa 1988).

Fitzpatrick (1998) recommends that the constant of proportionality should be taken to be $R_V = 3.1$ (see Fig. 3 for the effect of different values on an extinction curve). An illustration of the total absorption, A_λ , for the Johnson *UBVR* and Strömgren *uvby* passbands is given in Fig. 4.

Using the equation recommended by Fitzpatrick (1998),

$$A_V = R_V E_{B-V} = 3.1 E_{B-V} \quad (8)$$

we can adjust eq. 3 to allow for interstellar absorption:

$$(m_V - M_V)_0 = 5 \log_{10}(d) - 5 - A_V = 5 \log_{10}(d) - 5 - 3.1 E_{B-V} \quad (9)$$

where a subscripted zero denotes a quantity from which the effects of reddening have been removed. Likewise, $B-V$ becomes

$$(m_B - m_V)_0 = (m_B - m_V) - E_{B-V} \quad (10)$$

A useful equation for detached eclipsing binaries (dEBs) can be derived from eq. 3 and the definitions of luminosity and M_{bol} :

$$(m_V - M_V)_0 = (m_V - A_V) - (M_{\text{bol}} + BC) \quad (11)$$

$$= 5 \log \frac{R}{R_\odot} + (m_V - A_V) - M_{\text{bol}\odot} + 10 \log \frac{T_{\text{eff}}}{T_{\text{eff}\odot}} + BC \quad (12)$$

(e.g., Clausen 2004).

1.3 Stellar characteristics

1.3.1 Stellar interferometry

Interferometric measurements of the radii of nearby stars are of fundamental importance to astrophysics. When combined with good parallax measurements they allow accurate linear radii of stars to be determined. Knowledge of the distance (from parallax) and apparent brightness of a star allows its absolute brightness to be found. If the linear radius of the star is known, its T_{eff} can be calculated directly. This allows calibration of the stellar T_{eff} and BC scales. The application of interferometry to visual binary stars also allows the masses of such stars to be found, allowing investigation of the stellar mass-luminosity relation.

There are several problems associated with interferometric measurements of stellar radii:-

- Only nearby stars can be studied and most of these have chemical compositions similar to the Sun, so stars with other chemical compositions are not accessible.
- Only nearby and bright stars can be studied and these are all of spectral types approximately later than A0, so stars with T_{eff} s greater than about 10 000 K are not easily accessible.
- Only very nearby MS stars can be studied as the radii of these stars are relatively small.
- The interferometrically observed radius of a star is in general not the actual linear radius but the equivalent ‘uniform disc’ of a star which displays no limb darkening (Hanbury Brown et al. 1974; Davis, Tango & Booth 2000). This problem is easily solved

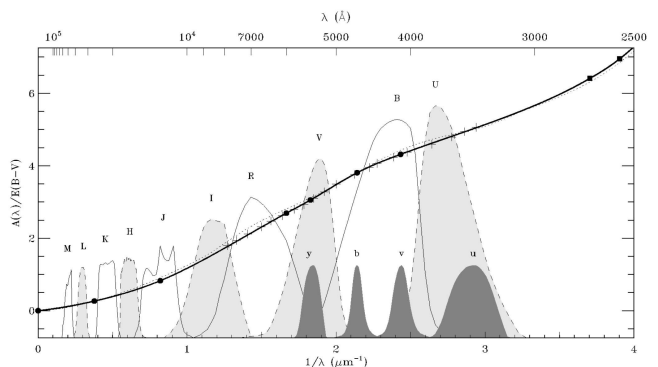


Figure 4. Illustration of the wavelength-dependent variation in A_λ and how this affects the Johnson *UBVR* and Strömgren *uvby* passbands. From Fitzpatrick (1998).

by applying a correction, but such corrections are normally derived from model atmospheres so have a theoretical dependence.

The first modern stellar interferometer was constructed and operated at Narrabri (Australia) by Hanbury Brown and his collaborators (Hanbury Brown, Davis & Allen 1967, 1974; Hanbury Brown et al. 1967) and consisted of two 6.7 m reflecting telescopes mounted on a circular railway track 188 m in diameter. Observations from this instrument were used to establish empirical T_{eff} and BC scales (Code et al. 1976). Barnes, Evans & Moffett (1978) had access to radius measurements of 76 stars with accuracies better than 25% in order to investigate stellar surface brightness.

There was little immediate progress in the field of stellar interferometry once the Narrabri research was discontinued, but several instruments are now in use and producing important results. The Mark III Optical Interferometer (Pasadena, California) has produced angular diameters of over 100 stars (Mozurkewich et al. 1991, 2003) and is now retired. The Navy Prototype Optical Interferometer (NPOI) has superseded this instrument and is currently operational at Flagstaff, Arizona (Nordgren et al. 1999; Nordgren, Sudol & Mozurkewich 2001). The Palomar Testbed Interferometer (PTI) is also operational (Lane & Colavita 2003), as is the Sydney University Stellar Interferometer (SUSI; Davis et al. 1999a, 1999b). The twin Keck telescopes (Hawaii) can also be linked to form a stellar interferometer, and the first results are now being published (Colavita et al. 2003).

The most interesting and productive stellar interferometer currently in operation is the Very Large Telescope Interferometer (VLTI) at ESO Paranal, Chile. Observations from this instrument have been used to calibrate the Cepheid period-luminosity relation (Kervella et al. 2004b, 2004c), observe the limb darkening of giant stars (Wittkowski, Aufferberg & Kervella 2004), severely constrain theoretical models of stellar structure (Kervella et al. 2003, 2004a), provide the first interferometric measurements of the diameters of M-type dwarfs (Ségransan et al. 2003) and calibrate many stellar surface brightness relations (Kervella et al. 2004d). The VLTI is capable of deriving linear diameters of the closest stars, using Hipparcos parallax observations, to accuracies of better than 1% (Di Folco et al. 2004).

1.3.2 The effective temperature scale

The T_{eff} of a star is defined to be the temperature of a black body emitting the same flux per surface area as the star. This means that the T_{eff} of a star is a precisely defined concept, but as stars are quite different from black bodies, the physical interpretation of T_{eff} is not straightforward. Therefore a scale of T_{eff} s has been established by several researchers. The recent T_{eff} scale for MS stars (Cox 2000) is shown in Fig. 5. The empirical T_{eff} scale constructed by Flower (1996) is shown in Fig. 6 for MS, subgiant, giant and supergiant stars.

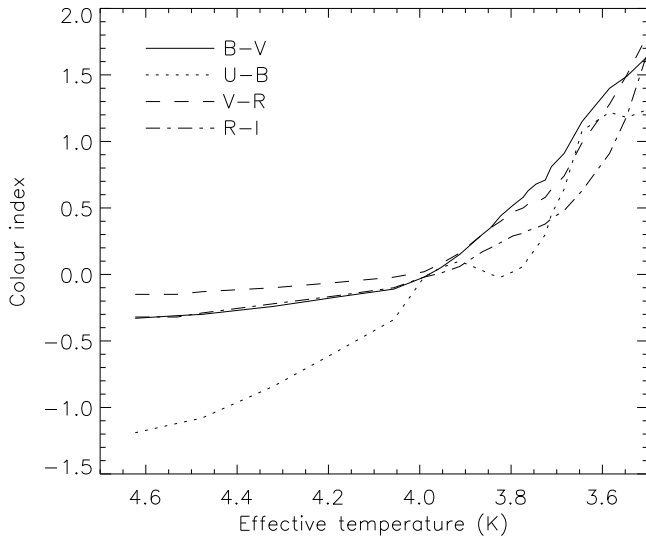


Figure 5. The T_{eff} scale as a function of Johnson photometric colour indices given by Cox (2000).

1.3.3 Stellar effective temperatures and angular diameters from the Infra-Red Flux Method

The Infra-Red Flux Method (IRFM) was introduced by Blackwell & Shallis (1977; Blackwell, Shallis & Selby 1979) as a way of deriving T_{eff} s and angular diameters for cool stars, to accuracies of potentially 1% and 2% respectively. The angular diameter of a star, θ_* , can be calculated from the monochromatic fluxes of the star at its surface, $F_{*,\lambda}$, and at the Earth, $F_{E,\lambda}$ according to

$$\theta_* = 2 \sqrt{\frac{F_{E,\lambda}}{F_{*,\lambda}}} \quad (13)$$

and using the small-angle approximation. At optical wavelengths the monochromatic flux of a star can be strongly dependent on T_{eff} , but at IR wavelengths it is generally more weakly dependent on T_{eff} , allowing T_{eff} to be determined by iteration from an initial guess.

The method was replaced in 1980 by a more simple and direct procedure (Blackwell, Petford & Shallis 1980) in which the total integrated stellar flux, J_E , is found from

$$J_E = \int_0^\infty F_{E,\lambda} d\lambda \quad (14)$$

The T_{eff} is then found from

$$\frac{J_E}{F_{E,\lambda}} = \frac{\sigma_{\text{SB}} T_{\text{eff}}^4}{\Phi(T_{\text{eff}}, \log g, \lambda, A)} \quad (15)$$

where σ_{SB} is the Stefan-Boltzmann constant and $\Phi(T_{\text{eff}}, \log g, \lambda, A)$ represents the monochromatic flux from the star as a function of T_{eff} , surface gravity, wavelength and abundances. The angular diameter is then calculated using

$$J_E = \frac{\theta_*^2}{4} \sigma_{\text{SB}} T_{\text{eff}}^4 \quad (16)$$

(Blackwell et al. 1990). $\Phi(T_{\text{eff}}, \log g, \lambda, A)$ must be calculated using model atmospheres, so this method is not entirely empirical.

Mégessier (1994) studied the effects of using different model atmospheres in the IRFM and found that different atmospheres gave results different by up to 1%, and that abundances had to be taken into account as they could have an effect of similar size. The presence of a cool stellar companion or circumstellar dust ring can have a significant effect on the T_{eff} of a star derived from the IRFM (Smalley 1993, 1996).

1.3.4 Stellar chemical compositions

Shortly after the Big Bang, the Universe contained mostly hydrogen, with some helium and a trace of lithium. Since this point, the

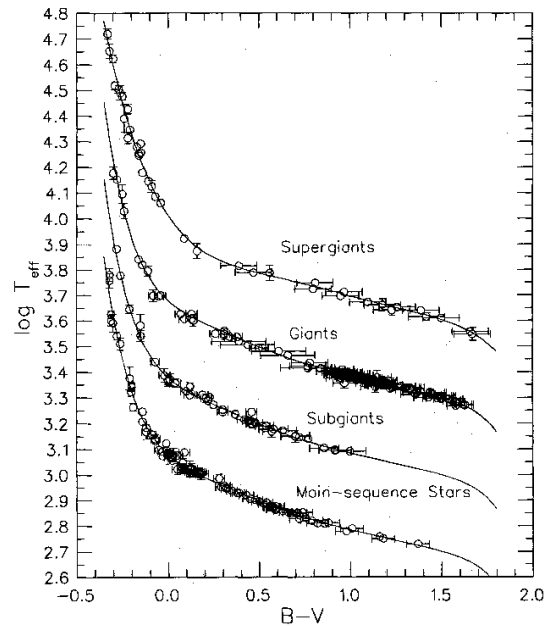


Figure 6. The T_{eff} scale as a function of the Johnson photometric colour index $B-V$ for stars in different evolutionary stages (indicated). Giant, subgiant and MS scales are shifted by $-0.3, -0.6, -0.9$ dex in $\log T_{\text{eff}}$. Taken from Flower (1996).

thermonuclear processes inside stars have been converting these light elements into heavier elements, which are ejected back into the interstellar environment when the star dies.

The abundances of individual elements are generally expressed logarithmically with respect to the abundance of that element in the Sun, using the formula

$$\left[\frac{E}{H} \right] = \log \left(\frac{N_E}{N_H} \right)_* - \log \left(\frac{N_E}{N_H} \right)_\odot \quad (17)$$

where the fractional abundance of element E is N_E , the fractional abundance of hydrogen is N_H , and the subscripts $*$ and $\odot$ refer to the star and to the Sun, respectively. Abundance ratios, for example $[C/Fe]$, are defined in a similar way.

The fractional abundances by mass of hydrogen, helium and ‘metals’ (all other elements) are denoted by X , Y and Z , respectively. The values of these quantities for the Sun are generally taken to be $X_\odot = 0.70683$, $Y_\odot = 0.27431$ and $Z_\odot = 0.01886$ (Anders & Grevesse 1989). $Z_\odot$ is found from laboratory studies of pristine meteorites (the ‘C1 chondrite’ class) and from spectroscopic studies of the solar photosphere and corona, and is dominated by the important volatile elements carbon, oxygen and nitrogen (Grevesse, Noels & Sauval 1996).

Most theoretical studies of stellar evolution adopt metal abundances which are scaled up or down from the solar values, but some studies also adjust the abundances of the ‘ α -elements’. These are the products of α -capture and are ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ca and ^{48}Ti . They are primarily made by thermonuclear fusion of carbon, oxygen and neon in the later stages of stellar evolution (Cowley 1995).

More recent values for the solar abundances have been given by Asplund, Grevesse & Sauval (2004) and are $X_\odot = 0.7392$, $Y_\odot = 0.2486$ and $Z_\odot = 0.0122$. These values are quite different from those of Anders & Grevesse (1989), and have important implications for stellar astrophysics if they are correct, but are unlikely to be adopted until published in a refereed journal (A. Claret, 2004, private communication). They are in very poor agreement with the results of helioseismological investigations (Bahcall et al. 2005).

Kurucz (2002) has stated that “One of the curiosities of astronomy is the quantity $\left[\frac{\text{Fe}}{\text{H}} \right]$ because the solar iron abundance is not well known and many different answers exist.”

The abundances of helium and metals are expected to increase over time as stars manufacture them from hydrogen and

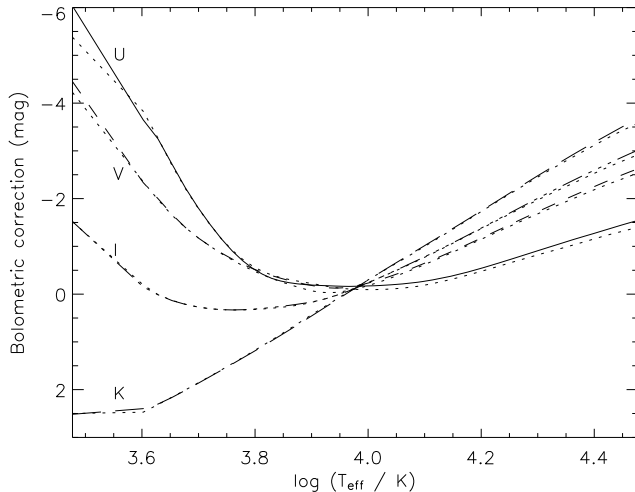


Figure 7. Plot of the BCs of Girardi et al. (2002) for the U , V , I and K passbands. BCs for $\log g = 4.5$ are shown with several styles of line. BCs for $\log g = 3.5$ are shown with dotted lines.

then eject them into the interstellar medium via winds, binary mass loss and supernovae. Whilst the early Universe contained some helium, negligible amounts of metals were made in the Big Bang. The abundances of helium and metals are therefore expected to be related according to the equation

$$Y = Y_{\text{prim}} + \frac{\Delta Y}{\Delta Z} Z \quad (18)$$

where Y_{prim} is the primordial helium abundance and $\frac{\Delta Y}{\Delta Z}$ is the enrichment slope. Ribas et al. (2000) found $Y_{\text{prim}} = 0.225 \pm 0.013$ and $\frac{\Delta Y}{\Delta Z} = 2.2 \pm 0.8$ from fitting theoretical evolutionary models to the properties of several dEBs. This is in good agreement with other determinations of both quantities.

1.3.5 Bolometric corrections

The bolometric flux produced by a star is the total electromagnetic flux summed over all wavelengths. Thus luminosity is a bolometric quantity but the magnitude of a star observed through a photometric passband is not. Transformation between the bolometric magnitude and a passband-specific magnitude of a star requires bolometric corrections (BCs), which are defined using

$$M_{\lambda} = M_{\text{bol}} - BC_{\lambda} \quad (19)$$

where M_{λ} is the absolute magnitude of a star in passband λ .

The zeropoint of the BC scale is thus set by the physical properties adopted for the Sun, and different sources of BCs may adopt different zeropoints. BCs are used in the study of dEBs to aid in measuring the distance to a dEB from the luminosities of the stars and the overall apparent magnitudes of the dEB. For this method there are two types of sources for BCs.

Empirical BCs can be found using two methods. The first method is to obtain spectrophotometric observations of stars over as wide a wavelength range as possible. This is difficult for hot stars as they emit a significant fraction of their light at UV wavelengths, and light at wavelengths below $912 \text{ \AA}$ is not observable as it is strongly absorbed by the interstellar medium. The second method is to resolve the surfaces of stars using interferometry, and find their distances by parallax. This provides a fundamental measurement of their T_{eff} s, and their absolute magnitudes can be found from their known distances and apparent magnitudes.

Empirical BCs have been tabulated by several researchers, including Code et al. (1976), Habets & Heintze (1981), Malagnini et al. (1986) and Flower (1996). The study of dEBs can provide empirically-determined BCs (Habets & Heintze 1981) as the surfaces of the stars are resolved by the analysis of light curves. The disadvantages of empirical BCs is that their values have observational uncertainty and are only relevant to stars of a similar

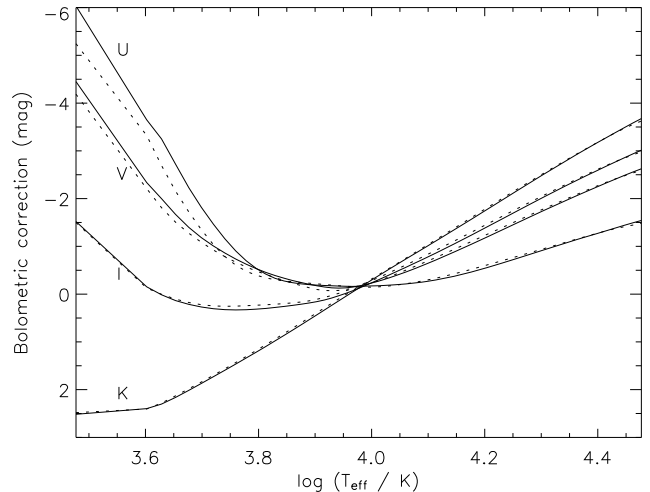


Figure 8. Plot of the BCs of Girardi et al. (2002), for the U , V , I and K passbands, showing the variation with metal abundance, $\left[\frac{M}{H}\right]$. The solid lines show BCs for a solar metal abundance and the dotted lines show BCs for $\left[\frac{M}{H}\right] = -1.0$.

chemical composition to the stars used to find the BCs. As most empirical BCs are determined using interferometry, this limits the chemical composition to approximately solar, as this is the chemical composition of the nearby stars which are resolvable with current interferometric instruments.

Theoretical BCs can be derived using theoretical model atmospheres, meaning they are exact and that they can be derived for any realistic set of atmospheric parameters, including chemical composition. Although they have no random errors, the use of theoretical calculations in the derivation of BCs means that they are subject to systematic errors. Whilst these systematic errors can be difficult to investigate, the comparison between several different theoretical BCs and empirical BCs can be useful. Theoretical BCs for the V and K passbands have been tabulated by Bessell, Castelli & Plez (1998) for a solar chemical composition. Girardi et al. (2002) have provided BCs for several wide-band photometric systems, including $UBVR_{\text{IJKL}}$, for metal abundances, $\left[\frac{M}{H}\right]$, of -2.5 to $+0.5$ in steps of 0.5 . Girardi et al. (2004) have extended this to the SDSS $u'g'r'i'z'$ passbands (Sec. 12.1.5).

Figs. 7 and 8 show the form of the BC function in the U , V , I and K passbands, and the effects of changes in surface gravity and metal abundance. The BCs for very hot and cool stars are larger and more uncertain than for intermediate-temperature stars because hot and cool stars emit a large fraction of their light at non-optical wavelengths (Harries, Hilditch & Howarth 2003). Fig. 9 shows the empirical BC scale of Flower (1996).

1.3.6 Surface brightness relations

The concept of surface brightness was first used in the analysis of eclipsing binaries (EBs) almost one century ago (Kruszewski & Semeniuk 1999), when Stebbins (1910) used the known trigonometrical parallax and inferred linear radii of the components of Algol (HD 19356) to estimate the surface brightnesses of both stars relative to the Sun. Stebbins (1911) applied this analysis to β Aurigae, which was the first EB with a double-lined spectroscopic orbit (Baker 1910). Kopal (1939) was able to provide a calibration of surface brightness (expressed as an equivalent T_{eff}) in terms of spectral type from the analysis of EBs.

The first analysis to use surface brightness relations to find the distance to an EB, rather than the other way round, was by Gaposchkin (1962), who determined the distance to M 31 from the study of an EB inside this galaxy. Further work was directed towards finding the distance of the Large and Small Magellanic Clouds (Gaposchkin 1970). Compared to modern distance values, the results were quite reasonable (although the quoted uncertain-

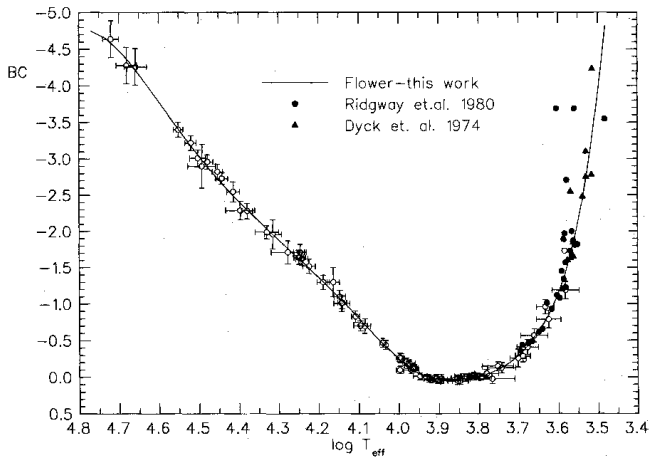


Figure 9. The BC scale as a function of T_{eff} (Flower 1996).

ties were much too small) but a little large, probably due to the inclusion of more complicated semidetached binaries (Kruszewski & Semeniuk 1999).

Wesselink (1969) calibrated the visual surface brightness of stars, s_V , in terms of $(B-V)_0$. The definition of s_V is

$$s_V = V_0 + 5 \log \phi_{\text{as}} \quad (20)$$

where V_0 is the dereddened apparent V magnitude and ϕ_{as} is the angular diameter of the star in arcseconds. By this definition, a zeroth magnitude star with an angular diameter of one arcsecond would have $s_V = 0$ mag. By consideration of equations involving parallax and absolute visual magnitude, M_V , we get

$$M_V - s_V + 5 \log R = +15.15 \quad (21)$$

where R is the radius of the star in solar units. If the Sun is used as a calibration point, then

$$s_V = -10 \log T_{\text{eff}} - BC_V + 27.28 \quad (22)$$

The calibration of Wesselink is valid for $-0.30 < (B-V)_0 < 1.80$, although there is a large gap at $0.64 < (B-V)_0 < 1.45$ where no calibration points lie. The calibration is shown in Fig. 10 and was used by Wesselink to determine the absolute magnitude of Cepheid variables. An updated calibration was provided by Marocco & Mazzitelli (1976).

Barnes & Evans (1976; Barnes, Evans & Parson 1976; Barnes, Evans & Moffett 1978) used the angular diameters of 52 stars, most of which had been studied using interferometry, to investigate the relations between surface brightness and colour indices involving the Johnson $BVRI$ broad-band passbands. They discovered that the best relation, in terms of having the smallest scatter, used the $V-R$ colour index. As surface brightness relations in terms of colour index were not originally their idea, it is best to refer to only the surface brightness – $(V-R)$ calibration as being the Barnes-Evans relation (Kruszewski & Semeniuk 1999). Barnes, Evans & Moffett (1978) improved the definition of the relation by adding data for another 40 stars. The relations in $B-V$ and $R-I$ have more scatter due to a dependence on surface gravity and increased “cosmic scatter” (intrinsic variation between similar stars). The relation for $U-B$ is of no use as it is strongly affected by surface gravity, “cosmic scatter”, line blanketing and Balmer line emission. These effects mean that the $U-B$ relation is not monotonic. The $B-V$ relation has a similar problem for stars cooler than mid K-type. The calibrations are shown in Fig. 11.

The Barnes-Evans relation is defined using the equation

$$F_V = \log T_{\text{eff}} + 0.1 BC_V = 4.2207 - 0.1 V_0 - 0.5 \log \phi_{\text{mas}} \quad (23)$$

where F_V is the surface brightness parameter and ϕ_{mas} is the angular diameter of the star in milliarcseconds. The constant 4.2207 is found using the Sun as a calibration point as it has a known angular diameter and absolute bolometric flux (Di Benedetto 1993). This means that BCs are not required to find F_V ; in fact they can be determined using the above equation. The calibrations

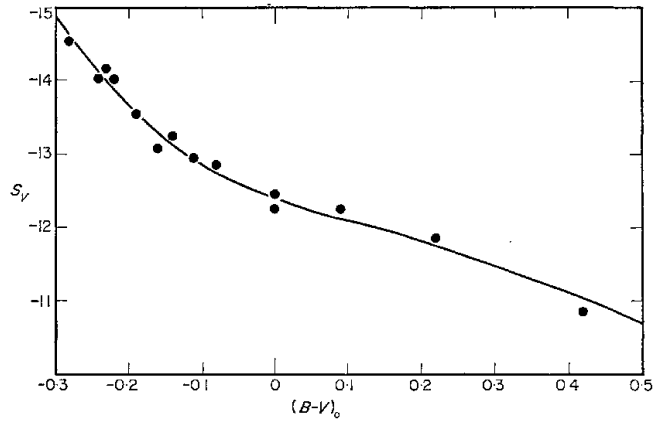


Figure 10. Calibration of surface brightness against dereddened $B-V$ for early-type stars. Taken from Wesselink (1969).

are valid for spectral types between approximately O4 and M8. Barnes & Evans state that there is no dependence on luminosity class. The relation between the s_V parameter of Wesselink (1969) and the F_V parameter of Barnes & Evans is

$$F_V = -0.1 s_V + 2.728 \quad (24)$$

where the constant depends on the solar properties adopted by Wesselink (1969).

An important aspect of the Barnes-Evans relation is that it is stated to be applicable to all types of stars, including pulsating variables and carbon stars. This means that it can be used to find the distance to, and linear radii of, δ Cepheids, so can be used to calibrate an important distance indicator. However, there is some evidence that the measured angular diameters of late-type stars have a dependence on wavelength as a result of circumstellar matter and their spectral characteristics (Barnes & Evans 1976).

Popper (1968) suggested that a calibration between surface brightness and $B-V$ would be useful, and Popper (1980) provided tabular relations between the Barnes-Evans F_V parameter and the colour indices $B-V$, $V-R$ and $b-y$, which have been used by many researchers. A calibration between F_V and $b-y$ was also given by Moon (1984), allowing stellar radii to be predicted using $wby\beta$ photometry (Moon 1985b). Eaton & Poe (1984) recalibrated the Barnes-Evans relation, and also introduced a relation based on the $B-I$ index. The suggestion behind the latter calibration is that the B passband is affected by the Balmer jump in a very similar way to the effect on the I passband of the Paschen jump, so the $B-I$ index is reliable for hot stars. It should be remembered that the B passband is sensitive to metallicity through the effect of line blanketing (Sec. 12.1.1).

The Barnes-Evans relation was applied by Lacy (1977a) in the determination of the distance moduli to nine dEBs, with accuracies of about 0.2 mag. It was also applied by Lacy (1978) to three dEBs which are members of nearby open clusters or associations. The resulting distances were in reasonable agreement with the distances found by MS fitting methods, although there were suggestions of a systematic discrepancy of 0.1 mag. Lacy (1977c) used the Barnes-Evans relation to find the radii of a large number of nearby single stars. O'Dell, Hendry & Collier Cameron (1994) recalibrated the $F_V - (B-V)$ relation and presented a method to determine the distance to a sample of stars, for example the members of an open cluster, using their recalibration.

The concept of a zeroth-magnitude angular diameter was introduced by Mozurkewich et al. (1991) and is the angular diameter of a star with an apparent magnitude of zero. Adopting consistent notation from this point, the surface brightness in passband λ is defined to be

$$S_{m_\lambda} = m_\lambda + 5 \log \phi \quad (25)$$

where m_λ is the apparent magnitude in passband λ and ϕ is the angular diameter of the star in milliarcseconds (Di Benedetto 1998). Note that S_λ is not the same quantity as the previously

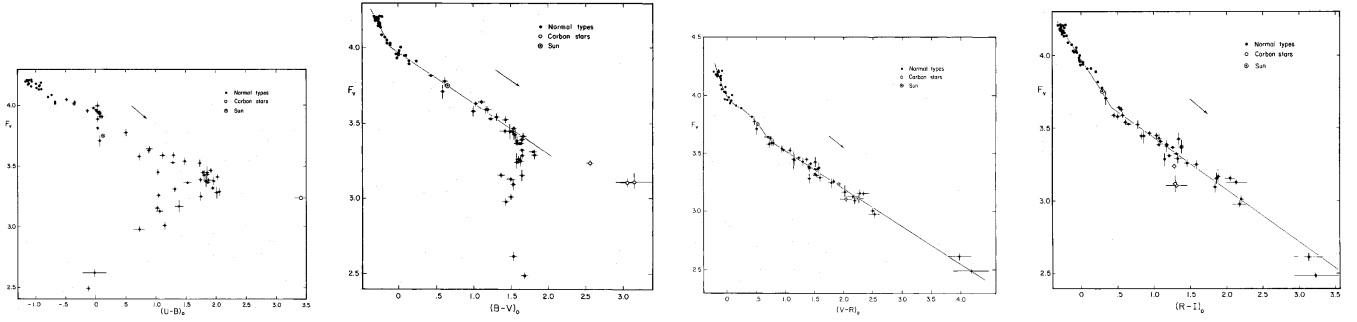


Figure 11. Relation between stellar surface brightness and several colour indices. The arrows show the effect of one magnitude of interstellar extinction. Taken from Barnes, Evans & Moffett (1978).

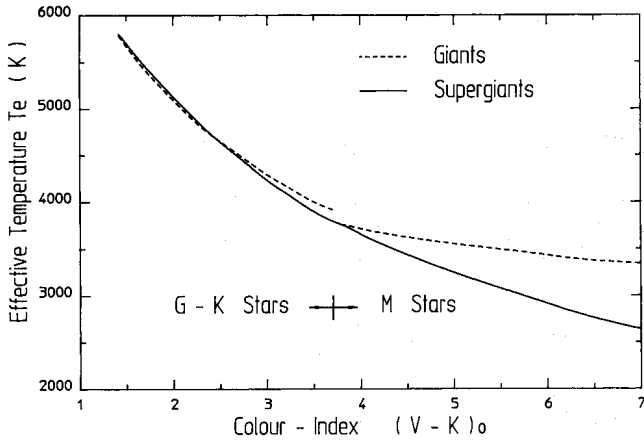


Figure 12. Relation between visual surface brightness and $(V-K)_0$ for late-type stars. Note the discontinuity between the calibrations for G and K giants and M giants (Di Benedetto 1993).

mentioned s_V or F_V . The zeroth-magnitude angular diameter is defined to be

$$\phi(m_{\lambda=0}) = \phi \times 10^{\frac{m_{\lambda}}{5}} \quad (26)$$

This means that $\phi(m_{\lambda=0})$ is a measure of surface brightness:

$$\phi(m_{\lambda=0}) = 10^{\frac{S_{m_{\lambda}}}{5}} \quad (27)$$

Calibrations for $\phi(m_{\lambda=0})$ were given for the $B-K$ and $V-K$ indices by van Belle (1999). Calibrations for S_V were constructed by Thompson et al. (2001) for the $V-I$, $V-J$, $V-H$ and $V-K$ indices and used to find the distance to the dEB OGLE GC 17, a member of the globular cluster ω Centauri.

Di Benedetto (1993) investigated the $S_V - (V-K)$ relation and found significant differences between the calibrations for M-type giants and supergiants (Fig. 12), contrasting with the claim that the Barnes-Evans relation is applicable to almost all types of star. Di Benedetto (1998) calibrated several other broad-band colour indices and found that the $V-K$ index remains the best indicator of surface brightness; although $V-K$ has a dependence on metal abundance through the effects of line blanketing, the effect is only about 1%.

Salaris & Groenewegen (2002) noted that the zeroth-magnitude angular diameter is strongly correlated with the Strömgren c_1 index in B-type stars. They calibrated the relationship using stars in nearby dEBs (Fig. 14) and found

$$\phi^{V=0} = 1.824(180)c_1 + 1.294(78) \quad (28)$$

Salaris & Groenewegen state that this relationship may need a more detailed investigation but that it may be useful in determining the distance to the LMC using B-stars in EBs.

Kervella et al. (2004d) used interferometric data for nearby stars to provide calibrations for surface brightness based on

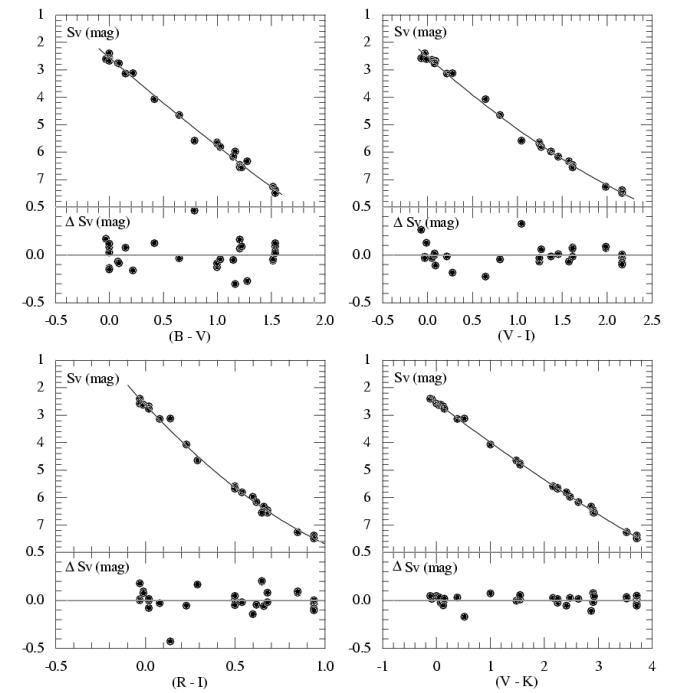


Figure 13. Relation between visual surface brightness and several photometric indices. Taken from Di Benedetto (1998).

every photometric index which uses two passbands out of $UBVRIJHKL$ (Fig. 15). The calibrations are linear, although some are indicated to be a bad representation of nonlinear data. Estimates of “cosmic scatter” are also made; this is below 1% for calibrations based on the $U-L$, $B-K$, $B-L$, $V-K$, $V-L$ and $R-I$ indices. Calibrations for $\phi(m_{\lambda=0})$ in terms of T_{eff} are also given for all the passbands mentioned above (Fig. 16). Further investigation by Groenewegen (2004) has revealed a dependence of $V-K$ on $\left[\frac{\text{Fe}}{\text{H}}\right]$; this has been quantified. Groenewegen calibrated S_V against $V-R$ and $V-K$, and S_K against $J-K$; the latter relation has a statistically insignificant dependence on $\left[\frac{\text{Fe}}{\text{H}}\right]$.

1.4 Limb darkening

When stars are viewed from a particular direction they do not appear to be uniform discs. Although stars are normally approximately spherically symmetric, towards the edge of the star they appear to get dimmer. This limb darkening (LD) occurs because when we look obliquely into the surface of a star we are seeing a cooler gas overall than when we look from normal to the surface. As cooler gases are less bright, the limb of a star appears dimmer.

LD is important in several areas of stellar astrophysics:–

- Determination of stellar angular diameters from interferometry requires a correction from the observed uniform disc size to

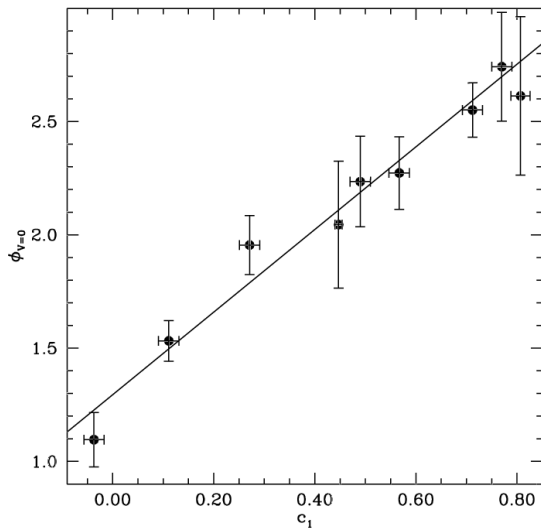


Figure 14. Relation between the V-band zeroth-magnitude angular diameter and the Strömgren photometric index c_1 . Taken from Salaris & Groenewegen (2002).

the actual LD disc size. This correction is small but usually theoretically derived (e.g., Davis, Tango & Booth 2000).

- Line profiles of rotating stars (e.g., Hutsemekers 1993).
- Transits of extrasolar planets across their parent stars (e.g., Brown et al. 2001). The light variation can be analysed to determine the relative radii of the star and planet, but such an analysis needs to include the effect of LD.
- Gravitational microlensing (Heyrovský 2003).
- Mode identification in the study of pulsating variable stars (Barban et al. 2003).
- Analysing light curves of EBs to determine their properties.

LD is a fundamental effect which must be allowed for when analysing the light curves of EBs. The neglect, or inadequate representation, of LD can create systematic uncertainties in the stellar radii derived from light curve analysis. For the purposes of modelling light curves, the variation in brightness over a stellar disc is represented by various parameterisations called LD laws.

LD coefficients can be determined observationally by:-

- Analysis of the light curves of EBs.
- Interferometry of nearby stars, for example the M4III star ψ Phoenicis (Wittkowski, Aufdenberg & Kervella 2004).
- Analysis of the light curves of gravitational microlensing events (Heyrovský 2003).

These methods generally require very high quality data and the results can be imprecise, particularly when attempting to derive coefficients of the more complex LD laws. Popper (1984) states that only single-parameter LD law coefficients can be derived from the light curves of EBs, and only a few investigations have been able to determine reliable LD characteristics of a star.

Many tabulations exist of LD coefficients determined theoretically using model atmospheres. Whilst this can introduce a dependence on theoretical models into the analysis of the light curves of EBs, there is no alternative when the observations are not good enough to allow the derivation of LD coefficients from the light curves themselves. The general theoretical method is to derive the emergent flux at different angles from a plane-parallel model atmosphere and fit this with the relevant LD law.

1.4.1 Limb darkening laws

The simplest LD law is the linear law. This is formulated using $\mu = \cos \theta$ where θ is the angle of incidence of a sight line to the stellar surface. The linear LD law is given by

$$\frac{I(\mu)}{I(1)} = 1 - u(1 - \mu) \quad (29)$$

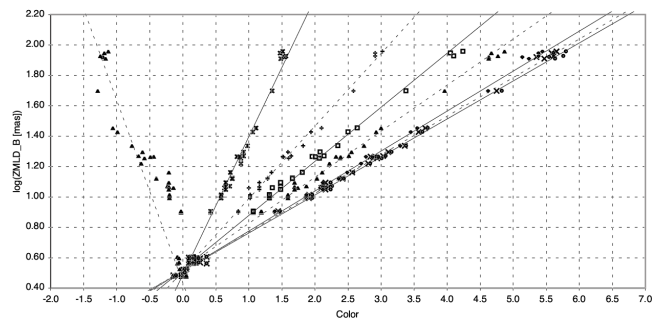


Figure 15. Relation between zeroth-magnitude angular diameter and (from left to right on the diagram) $B-U$, $B-V$, $B-R$, $B-I$, $B-J$, $B-H$, $B-K$ and $B-L$. Note the strong nonlinearity in the $B-U$ data. Taken from Kervella et al. (2004d).

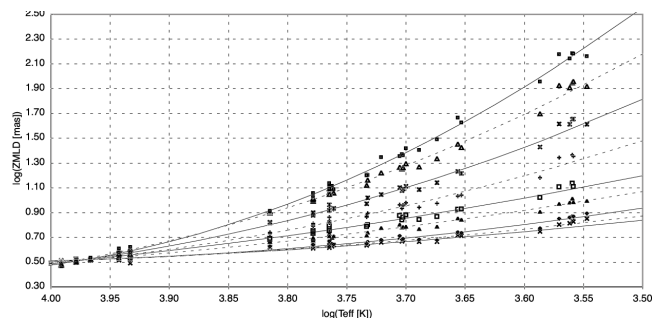


Figure 16. Relation between zeroth-magnitude angular diameter and T_{eff} . From top to bottom of the diagram, the lines are for the $UBVR IJHKL$ passbands. Taken from Kervella et al. (2004d).

where $I(\mu)$ is the flux per unit area received at angle θ , $I(1)$ is the flux per unit area from the centre of the stellar disc. The coefficient u depends on the wavelength of observation, the T_{eff} , the surface gravity and the chemical composition of the star.

Two-coefficient laws have been introduced to provide a better representation to the (theoretically derived) LD characteristics of stars. The quadratic law has often been used:

$$\frac{I(\mu)}{I(1)} = 1 - a(1 - \mu) - b(1 - \mu)^2 \quad (30)$$

which contains the coefficients a and b . Klingsmith & Sobieski (1970) introduced the logarithmic LD law

$$\frac{I(\mu)}{I(1)} = 1 - c(1 - \mu) - d\mu \ln \mu \quad (31)$$

which contains the coefficients c and d . Díaz-Cordovés & Giménez (1992) introduced the square-root law

$$\frac{I(\mu)}{I(1)} = 1 - e(1 - \mu) - f(1 - \sqrt{\mu}) \quad (32)$$

which contains the coefficients e and f . Barban et al. (2003) generalised the cubic law to

$$\frac{I(\mu)}{I(1)} = 1 - p(1 - \mu) - q(1 - \mu)^2 - r(1 - \mu)^3 \quad (33)$$

where the fitted coefficients are p , q and r .

Claret (2000b, 2003) investigated a four-coefficient law, with coefficients a_k , which can be represented as

$$\frac{I(\mu)}{I(1)} = 1 - \sum_{k=1}^4 a_k (1 - \mu^{k/2}) \quad (34)$$

Claret (2000b) claims that this law is more successful at fitting all types of star than the two-coefficient laws. Claret & Hauschildt (2003) introduced a new biparametric approximation given by

$$\frac{I(\mu)}{I(1)} = 1 - g(1 - \mu) - \frac{h}{(1 - e^\mu)} \quad (35)$$

Table 2. Tabulations of LD coefficients in the literature for different LD laws.

Reference	Linear	Log	Quad	Cubic	Sqrt	Exp	4coeff	Additional remarks
Grygar (1965)	*							
Klinglesmith & Sobieski (1970)	*	*						$T_{\text{eff}} \geq 10000 \text{ K}$.
Al-Naimiy (1978)	*							
Muthsam (1979)			*					
Wade & Rucinski (1985)	*		*					
Claret & Giménez (1990a)	*		*					$T_{\text{eff}} \leq 6730 \text{ K}$.
Claret & Giménez (1990b)	*		*					$T_{\text{eff}} \leq 6730 \text{ K}$. Not tabulated.
Díaz-Cordovés & Giménez (1992)	*		*		*			Not tabulated.
van Hamme (1993)	*	*			*			
Díaz-Cordovés, Claret & Giménez (1995)	*		*		*			<i>uvby</i> and <i>UBV</i> passbands
Claret, Díaz-Cordovés & Giménez (1995)	*		*		*			<i>RIJHK</i> passbands.
Claret (1998)	*		*		*			
Barban et al. (2003)	*		*	*	*			<i>uvby</i> passbands, A and F stars.
Claret (2000b)	*	*	*		*		*	<i>uvby</i> and <i>UBVRIJHK</i> passbands
Claret (2003)	*	*	*		*		*	Geneva and Walraven passbands
Claret & Hauschildt (2003)	*	*	*		*	*	*	$5000 \geq T_{\text{eff}} \geq 10000 \text{ K}$
Claret (2004b)	*	*	*		*		*	Sloan <i>u'g'r'i'z</i> passbands

in an attempt to better fit the theoretical LD predicted by recent spherical model atmospheres. The last two laws are notably more successful at short and long wavelengths, where success is measured by the agreement between the predicted LD and the LD law used to fit the predictions. In particular, spherical model atmospheres predict a severe drop in flux significantly before the observed edge of the disc (Claret & Hauschildt 2003), and the last two laws are the most successful at representing this.

1.4.2 Limb darkening and eclipsing binaries

Many tabulations of LD coefficients are collected in Table 2. When analysing a light curve, the choice of LD law is restricted to those implemented in the light curve code one is using. It is important to produce results for several different coefficients to determine the effect they have on the solution.

The atmospheres of close binaries are modified by flux incident from the other star in the system, changing the LD characteristics. Theoretical coefficients usually refer to isolated stars but the LD of irradiated atmospheres have been investigated by Claret & Giménez (1990b) and by Alencar & Vaz (1999). These authors also compared theoretical results with linear LD coefficients derived from photometric observations and found reasonable agreement within the (quite large) errors. Other comparisons between theory and observation exist (for example Al-Naimiy 1978) and agreement is generally good. However, the linear LD law does not represent well the flux characteristics of model atmospheres. It is also important to remember that theoretical LD coefficients are known to depend on atmospheric metal abundance (Wade & Rucinski 1985; Claret 1998) and the treatment of convection (Barban et al. 2003). Theoretical and observed linear LD coefficients disagree at UV wavelengths, which is important to remember when fitting light curves observed through the passbands such as Strömgren *u* and Johnson *U* (Wade & Rucinski 1985).

The EBOP light curve analysis code (see Sec. 13.1.2) is restricted to the linear LD law, although attempts have been made by Dr. A. Giménez and Dr. J. Díaz-Cordovés to include nonlinear LD (Etzel 1993). The Wilson-Devinney code (see Sec. 13.1.4) can perform calculations using the linear, logarithmic and the square-root laws (equations 29, 31 and 32). Van Hamme (1993) has provided extensive tabulations of the relevant coefficients, and their goodness of fit, to aid the decision as to which law is better in a particular case. In general, the square-root law is better at UV wavelengths and the logarithmic law is better in the IR. In the optical, the square-root law is better for hotter stars and the logarithmic law is better for cooler stars, the transition region being between T_{eff} of 8000 K and 10 000 K.

The incorporation of model atmosphere results into light

curve analysis codes allows the direct use of theoretical LD characteristics without parameterisation and approximation into an LD law. This procedure has been implemented by Bayne et al. (2004) using tabulations of Kurucz (1993b) model atmosphere predictions inside a version of the 1993 Wilson-Devinney code.

1.5 Gravity darkening

The flux emergent from different parts of a stellar surface is dependent on the local value of surface gravity. This dependence takes the form of the gravity darkening exponent designated β_1 (following the notation of Claret 1998), defined by the relation

$$F \propto T_{\text{eff}}^4 \propto g^{\beta_1} \quad (36)$$

where F is the bolometric flux and g is the local surface gravity. An alternative definition, which has often been used, is $T_{\text{eff}} \propto g^{\beta}$ (Hilditch 2001, p. 243). Thus the emergent flux from a star which is distorted by surface inhomogeneities or rotation, or the presence of an orbiting companion, is dependent on the position of emergence. Gravity darkening is an important effect in the analysis of the light curves of EBs and also in the study of rotational effects on single stars (Claret 2000a). It also affects the FWHM of the spectral lines of rapidly rotating stars (Shan 2000).

von Zeipel (1924) was the first to investigate gravity darkening analytically, and found that for a stellar atmosphere in radiative and hydrostatic equilibrium, $\beta_1^{\text{rad}} = 1.0$. Lucy (1967) investigated the properties of convective envelopes, and from numerical methods found an average value of $\beta_1^{\text{conv}} = 0.32$. These values are generally assumed to be correct and were confirmed observationally by Rafert & Twigg (1980), who found mean values of $\beta_1^{\text{rad}} = 0.96$ and $\beta_1^{\text{conv}} = 0.31$ from light curve analyses of a wide sample of dEBs. Hydrodynamical simulations by Ludwig, Freytag & Steffen (1999) found that β_1^{conv} is between about 0.28 and 0.40. The radiative-convective boundary is at $T_{\text{eff}} \approx 7250 \text{ K}$ (Claret 2000a).

The canonical assumption of $\beta_1^{\text{rad}} = 1.0$ and $\beta_1^{\text{conv}} = 0.32$ is unsatisfactory because there is a discontinuity in the value at the boundary between convective and radiative envelopes. This is unphysical because in such situations both types of energy transport can exist simultaneously in the envelope of a star (Claret 1998), suggesting that β_1 varies smoothly over all conditions.

Claret (1998, 2000a) presented tabulations of β_1 calculated using the Granada theoretical stellar evolutionary models (see Sec. 3.2.2). These works have shown that β_1 is a parameter which depends on surface gravity, T_{eff} , surface metal abundance, the type of convection theory, and evolutionary phase. A plot of β_1 versus stellar mass is given in Fig. 17; note that the transition between radiative and convective values is very sharp, but it is

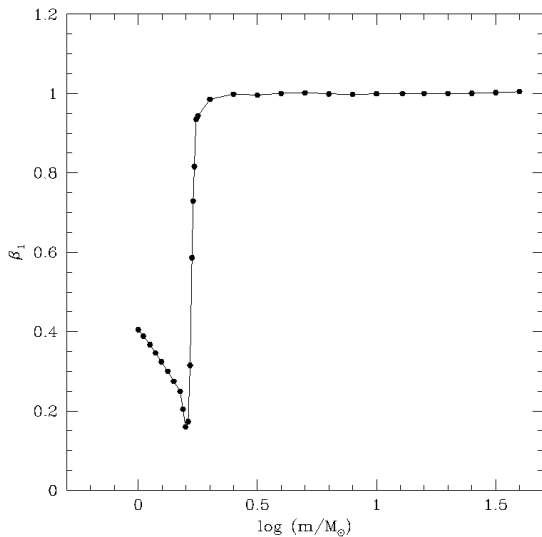


Figure 17. The dependence of the gravity darkening exponent β_1 on mass for homogeneous models calculated using the Granada stellar evolution code. Taken from Claret (1998).

continuous. In general β_1^{conv} is between 0.2 and 0.4 for low-mass stars, whereas for stars with masses above about $1.7 M_\odot$, $\beta_1^{\text{rad}} \approx 1.0$. The evolutionary effects on β_1 are highlighted by Fig. 18, where the change in β_1 is shown for the evolution of a $2 M_\odot$ star from the zero-age MS (ZAMS) to the base of the red giant branch. Note that the envelope of the $2 M_\odot$ star is radiative at the ZAMS but becomes convective during its MS evolution.

2 STELLAR EVOLUTION

2.1 The evolution of single stars

Stellar evolution is generally illustrated using Hertzsprung-Russell (HR) diagrams, on which stars are placed according to their T_{eff} and luminosity. HR diagrams for two different chemical compositions are shown in Fig. 19 and Fig. 20.

2.1.1 The formation of stars

Stars form from giant interstellar clouds of gas and dust. For an interstellar cloud to contract, its gravitational energy must be greater than its thermal energy. If we equate the gravitational and kinetic energies, we can derive the critical mass and density required for an interstellar cloud to collapse. These are the Jeans mass, M_J , and Jeans density, ρ_J and are:

$$M_J = \frac{3kT}{2Gm} R \quad \rho_J = \frac{3}{4\pi M^2} \left(\frac{3kT}{2Gm} \right)^3 \quad (37)$$

where $k = 1.38065 \times 10^{-23} \text{ J K}^{-1}$ is the Boltzmann constant, T , R and M are the temperature, radius and mass of the cloud, $G = 6.673(10) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is the gravitational constant and m is the mean particle mass (Phillips 1999, p. 14).

The Jeans conditions are more easily satisfied for larger interstellar clouds. Once a large cloud has contracted significantly, smaller parts of the cloud individually satisfy the Jeans conditions and so begin to contract themselves. The cloud therefore fragments into many smaller clouds, which collapse on free-fall timescales to form protostars. This means that most stars are born in clusters (Phillips 1999, p. 15).

The cores of protostars collapse more quickly than the outer regions, and begin to radiate a lot of energy outwards. This energy tends to slow the collapse of the outer regions of a protostar, and the radiation pressure will eject some matter from the protostar. Once the outer envelope has been accreted or ejected, the protostar becomes visible. The locus in the HR diagram where stellar objects of different masses become observable is called the

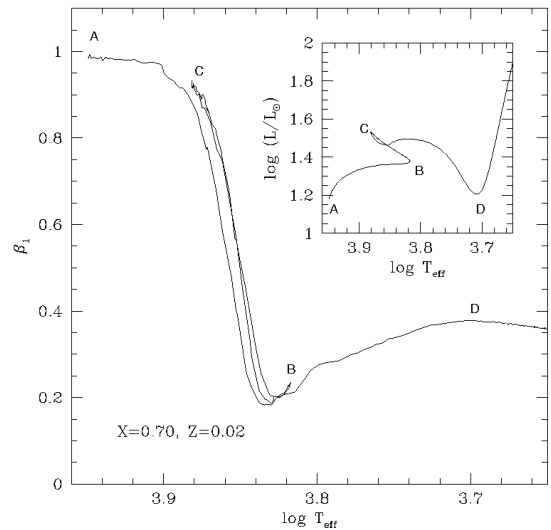


Figure 18. The evolution of the gravity darkening exponent β_1 of a $2 M_\odot$ star. The inset figure shows the position of the star on the HR diagram. Some points in the evolution of the star are labelled on both figures. Taken from Claret (1998).

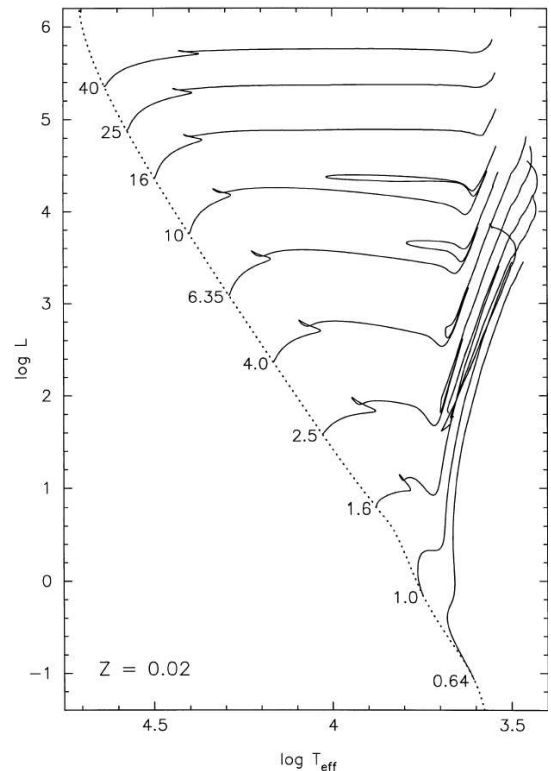


Figure 19. HR diagram showing the theoretical evolutionary tracks of stars for an approximately solar chemical composition. The numbers give the initial stellar mass for each evolutionary track, in $M_\odot$. Taken from Pols et al. (1998).

Hayashi line. This may even extend beyond the ZAMS for O-type stars as their evolution is so quick (Maeder 1998).

The protostars continue to contract and lose energy by radiating light. This evolution occurs along the Hayashi track and continues until the core of the protostar attains a sufficient temperature and density for large-scale thermonuclear reactions to occur. The star has reached the ZAMS, and is in equilibrium between the generation of energy by thermonuclear reactions (the ‘burning’ of hydrogen) and the emission of the energy in the form of radiation from its surface. The pre-MS (PMS) stage is shown in the HR diagram in Fig. 21.

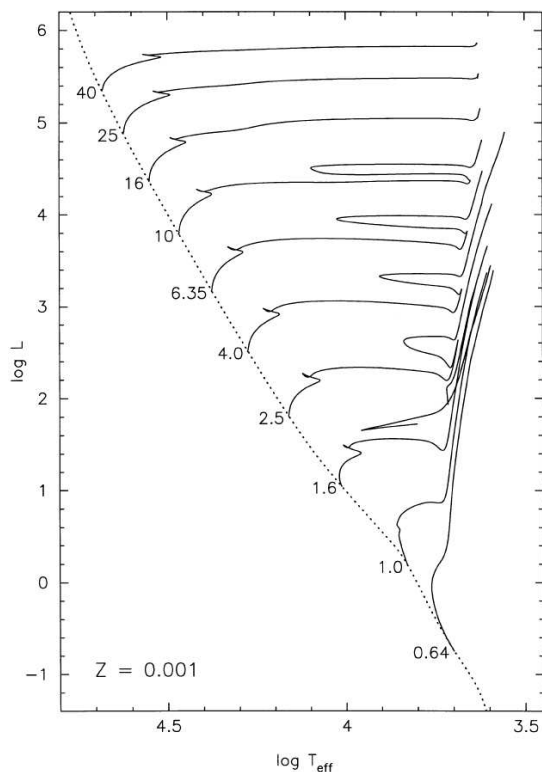


Figure 20. HR diagram showing the theoretical evolutionary tracks of stars for a low metal abundance of $Z = 0.001$. The numbers give the initial stellar mass for each evolutionary track, in $M_{\odot}$. Taken from Pols et al. (1998).

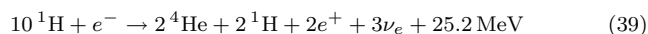
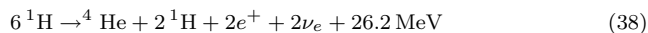
Below a mass of approximately $0.08 M_{\odot}$, an object is not capable of reaching a sufficient temperature and pressure in its core for major thermonuclear reactions to occur, so it does not become a star. The object becomes a brown dwarf if its mass is between about 0.08 and $0.01 M_{\odot}$; its core is electron-degenerate and can only burn deuterium. Young brown dwarfs radiate significant energy as they gravitationally contract, but become very faint as they age and the gravitational contraction ends.

2.1.2 Main sequence evolution

The ZAMS is the point at which a protostar becomes a star, but is not precisely defined (e.g., Torres & Ribas 2002). Alternative definitions include the point at which the radius of a stellar object reaches a minimum after PMS contraction (Lastennet & Valls-Gabaud 2002) and the point at which 99% of the energy emitted by the stellar object is generated from thermonuclear reactions in the core (e.g., Marques, Fernandes & Monteiro 2004).

Whilst on the MS, thermonuclear fusion in the cores of stars converts hydrogen into heavier elements. The energy produced in this way is transported through the envelope of the star by radiative and convective processes. Once it reaches the surface it is emitted, causing the star to be bright.

Several types of nuclear reactions convert hydrogen to helium. The proton-proton chain has four branches, the first of which produces 85% of the Sun's power and the second of which produces almost all the remainder (Phillips 1999, p. 118). These two branches can be summarised as



The reaction rates of the proton-proton chain are proportional to approximately the fourth power of temperature for solar-type stars (and greater/lesser than this for hotter/cooler stars).

The CN and CNO nuclear reaction chains (Phillips 1999, p. 121) become important in stars more massive than about

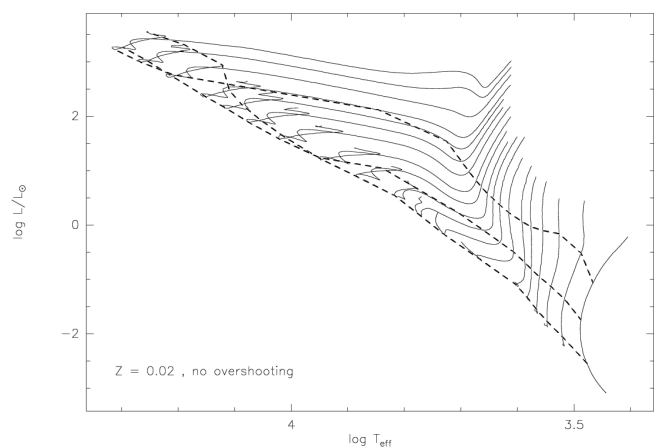
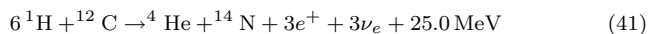
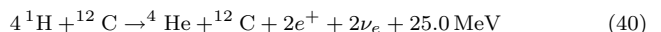


Figure 21. Theoretical predictions of PMS evolution for stars of 0.1 to $7 M_{\odot}$ (larger masses are higher in the figure) and an approximately solar metal abundance. Evolution begins at the top right and proceeds to the MS. Dashed lines connect the evolutionary tracks of different stellar masses for ages of 10^6 , 10^7 and 10^8 years. Taken from Siess, Dufour & Forestini (2000).

$1.2 M_{\odot}$ as they require a higher temperature and pressure to occur in large quantities, but have reaction rates which are proportional to temperature to the power of between 13 and 18. These chains require ${}^{12}\text{C}$ for catalysis and can be summarised as



Stars with masses lower than about $0.4 M_{\odot}$ are completely convective throughout their PMS and MS evolution. Stars with masses below about $1.1 M_{\odot}$ have radiative cores and convective envelopes (Hurley, Pols & Tout 2000). Stars with masses above about $1.3 M_{\odot}$ develop radiative envelopes (Hurley, Tout & Pols 2002) and the convective zone moves towards the centre of the star. More massive stars have convective cores and radiative envelopes. These mass limits are valid for a solar chemical composition; different chemical abundances will change the limits.

As the conversion of hydrogen into helium increases the mean molecular mass of the core of an MS star, the density increases. This causes the amount of thermonuclear fusion to increase, so the core temperature and energy production rise. The increased energy production causes both the luminosity and the radius of the star to increase. The T_{eff} s of low-mass stars rise as a result of this; high-mass stars get cooler (Hurley, Pols & Tout 2000).

2.1.3 Evolution of low-mass stars

At the end of their MS lifetimes, low-mass stars (those with radiative cores) run out of hydrogen in their core. As the core is mainly helium, it is denser and so becomes hotter. The region of hydrogen burning moves outwards to a shell, and the radius of the star increases. It now spends significant time as a red giant.

The shell hydrogen burning produces helium, which causes the core to increase in density and temperature. The core becomes degenerate and, once a sufficient temperature has been reached, helium burning abruptly starts in the core in an episode termed the 'helium flash' (Kaufmann 1994, p. 385). The star is now a horizontal-branch giant powered by the thermonuclear fusion of helium in its core. Once helium has been exhausted, it goes through the AGB and planetary nebula evolutionary phases before ending its life cooling slowly as a white dwarf.

2.1.4 Evolution of intermediate-mass stars

For stars which have convective cores on the MS ($M \gtrsim 1.2 M_{\odot}$), the end of their MS evolution is more extreme than for low-mass stars. The exhaustion of hydrogen occurs almost simultaneously

over the well-mixed core, leading to a rapid contraction of the core and large increase in radius. As the star climbs the giant branch in the HR diagram, the envelope of the star becomes convective and hydrogen burning moves outwards in a shell, depositing more helium on the core.

Once the conditions in the core have reached a threshold, helium burning commences. For stars of masses above about $2M_{\odot}$, whose helium cores have not become degenerate, this occurs gently. The star returns along the giant branch to the ‘blue loop’ in the HR diagram and consumes helium in its core and hydrogen in a shell around the core. Once core helium is exhausted, it goes through the AGB phase and either the planetary nebula or supernova phases, ending its life as a white dwarf.

2.1.5 Evolution of massive stars

The evolution of massive stars is strongly dependent on the initial chemical composition of the star, mass loss, rotation, magnetic effects and the different mixing processes which occur inside a star. Some of these physical phenomena will be discussed later.

Massive stars ($\gtrsim 12M_{\odot}$) undergo helium burning before reaching the giant branch stage of evolution. The progressively more extreme conditions in the core allow the burning of carbon, oxygen and other elements up to and including iron. Further thermonuclear fusion reactions are endothermic, causing loss of the pressure which was supporting the stellar envelope. The envelope collapses, rebounds, and is ejected in a supernova explosion. The core finishes up as a neutron star or a black hole.

3 MODELLING OF STARS

Much of the progress in our understanding of stars has required the construction of theoretical models of their structure and evolution. The intention of a theoretical model is that, for an input mass and chemical composition, it should be able to predict the radius, T_{eff} and internal structure of a star for an arbitrary age. It has recently become clear that the initial rotational velocity is also important (see below) and there remain some physical phenomena which are not incorporated into the current generation of available theoretical models.

The predictive power of the current generation of stellar models is very good for MS and giant stars of spectral types between approximately B and K. The predicted properties of more massive or evolved stars are strongly dependent on several physical phenomena which are simplistically treated, for example convective efficiency and mass loss. Models of less massive stars continue to require work to correct the apparent disagreement between the observed and predicted properties of M dwarfs (Ribas 2003; Maceroni & Montalbán 2004).

Theoretical stellar models generally begin from a reasonable approximation of a ZAMS or slightly pre-ZAMS stellar structure. The initial chemical composition is decided by assuming a fractional metal abundance, Z , using a chemical enrichment law to find the corresponding helium abundance, Y , and making up the rest with hydrogen, X (see Sec. 1.3.4). The metal abundance is normally distributed between the different elements according to the relative elemental abundances of the Sun (‘scaled solar’) although some models have enhanced α -elements.

One-dimensional models are generally used, in which the properties of matter are followed on a radial line from the core of the star to its surface, with the use of roughly 500 discrete ‘mesh points’ (e.g., Bressan et al. 1993) for which the instantaneous temperature, pressure and chemical abundances are calculated. Numerical integration is then used to follow the conditions at these mesh points when physical processes occur. The subsequent evolution of the star is followed until a certain point in its later evolution where it is known that the model has insufficient physics implemented to be able to follow the evolution further. Typically several thousand timesteps are required to follow the evolution of a star (e.g., Bressan et al. 1993).

Theoretical model sets contain several parameterisations of

physical effects. The choice of parameter values for these is generally made by forcing the models to match the radius and T_{eff} of the Sun for its mass, chemical composition, and an age of 4.6 Gyr. Helioseismological constraints can also be applied, mainly in specifying the solar helium abundance (Schröder & Eggleton 1996).

The parameterisations incorporated into theoretical models compromise the predictive ability of such models. This predictive power is important to almost all areas of astrophysics (Young & Arnett 2004).

3.1 Details and shortcomings of some of the physical phenomena included in theoretical stellar evolutionary models

3.1.1 Equation of state

A central part of a theoretical stellar model is the equation of state, which relates the electron and gas pressure to the temperature and density. Once the pressures have been calculated from the temperature and density, the excitation and ionisation state of each element can be calculated. As the pressures themselves depend on the elemental states, the equation of state must be dealt with using iterative calculation.

3.1.2 Opacity

The main effect of most of the species in a stellar interior is to retard the progress of radiative energy from the core of the star to the surface. Photons can be scattered or absorbed and re-emitted by ions and electrons, retarding the photons and causing radiation pressure. The size of this opacity depends on the cross-section of interaction of each chemical species and is an important ingredient in theoretical models. This has a large influence on the predicted stellar radius and core conditions, for stars which have large zones where energy transport is radiative.

Several different investigations have provided opacities for use in theoretical models. Earlier models used the opacities of Cox & Stewart (1962, 1965, 1970a, 1970b), which were the first to include bound-bound as well as bound-free transitions. The Los Alamos group has continued to update their opacity calculations (Cox & Tabor 1976; Huebner 1977) and the most recent results are available from their homepage¹. Two separate opacity investigations were started in the late 1980s and their results are commonly used in the current theoretical stellar models. The Opacity Project (OP) at University College London is led by M. Seaton; further details can be found in Seaton et al. (1994) and Seaton (1997). The Lawrence Livermore National Laboratory has an opacity project called OPAL; further details can be found in Rogers & Iglesias (1992), Rogers, Swenson & Iglesias (1996), Rogers & Nayfonov (2002) and at the OPAL homepage².

Determinations of the strength of stellar opacities have generally increased over time. In the 1980s the properties of massive stars (predominantly in dEBs) often required models with $Z \approx 0.04$ to match their properties despite having approximately solar chemical compositions found from spectroscopic measurements (Stothers 1991; Andersen et al. 1981). An increase of opacity causes the effect of metals to be increased, so fewer metals are needed to give the same effect. The effect of opacity and metal abundance are difficult to separate when comparing model predictions to observations (Cassisi et al. 1994).

3.1.3 Energy transport

Stars are made up of plasma which is at high temperatures and generally at high pressures. The transport of energy through this medium, from its generation in thermonuclear reactions to its escape from the stellar surface, is of fundamental importance to the characteristics of stars. Energy transport in stars occurs in

¹ <http://www.t4.lanl.gov/>

² <http://www-phys.llnl.gov/Research/OPAL/opal.html>

two ways: by radiative diffusion and by convective motion. The latter is a particularly complex process to model.

The diffusion of energy can occur by random motion of electrons and of photons. In the typical conditions of a stellar envelope, the energy diffusion by electrons is several orders of magnitudes smaller than the radiative diffusion due to the movement of photons (Phillips 1999, p. 91).

Radiative diffusion is the dominant source of energy transport below a certain critical temperature gradient. When the temperature gradient rises above this value, radiative diffusion is unable to cope with the amount of energy which is being transported, and convective motions occur. The critical temperature gradient is given by (Phillips 1999, p. 91)

$$\frac{dT}{dx} = \frac{\gamma - 1}{\gamma} \frac{T}{P} \frac{dP}{dx} \quad (42)$$

where T and P are the temperature and pressure, x is a spatial scale and γ is the adiabatic index of the gas.

Once the critical temperature gradient has been reached (from below), large-scale motions occur. These convective currents are very efficient at transporting energy but their nature makes them very difficult to model.

3.1.4 Convective core overshooting

Massive stars tend to have convective cores and radiative envelopes, but there is evidence that the transition between these two modes of energy transport occurs somewhat further out from the core than the point at which the critical temperature gradient is reached. This phenomenon is called convective core overshooting, and may have an important effect on the properties and lifetimes of massive stars. The physical explanation for the effect concerns a pile of material which is undergoing convective motion outwards from the core of the star. Once it reaches the point at which the temperature gradient drops below the critical value, it enters a volume which is formally expected to be free of convective motions. However, the kinetic energy of this pile of material causes it to rise further before it cools sufficiently to sink back towards the core.

The effect of overshooting is to make a larger proportion of the matter in a star available for thermonuclear fusion in the core. This increases the MS lifetime of the star as it has more hydrogen to burn. The luminosity of the star also increases, its T_{eff} changes more during its MS lifetime (e.g., Alongi et al. 1993; Schröder & Eggleton 1996), and it becomes more centrally condensed (Claret & Giménez 1991). Overshooting has a large effect on the evolution of stars beyond the terminal-age MS (TAMS; e.g., Pols et al. 1997). This means that the amount of convective core overshooting can be deduced by comparing observations of stars with the predictions of theoretical stellar evolutionary models (Sec. 3.2). These models generally incorporate overshooting by parameterisation, where the overshooting parameter, α_{OV} , is equal to the length of penetration of convective motions into radiative layers in units of the pressure scale height:

$$\alpha_{\text{OV}} = \frac{l_{\text{overshoot}}}{H_p} \quad (43)$$

Another effect of overshooting is to modify the surface chemical abundances of evolved stars (Maeder & Meynet 1989).

Maeder & Meynet (1989) summarised some evidence for the existence of overshooting to investigate the amount to incorporate into their theoretical stellar models (Maeder & Meynet 1988) and concluded that a moderate amount of overshooting ($\alpha_{\text{OV}} \approx 0.2$) was required in their models to match observations of intermediate-age open clusters, including the MS width in T_{eff} , the ‘blue loop’ positions of stars undergoing core helium burning, the number ratio of red to blue giants and the luminosity difference between yellow giants and the MS turnoff. Andersen, Clausen & Nordström (1990b) also found strong evidence for the presence of overshooting from consideration of the properties of dEBs. Component stars in dEBs with masses of about $1.2 M_{\odot}$, which have small convective cores, are well matched by the predictions of theoretical models but those with masses not much

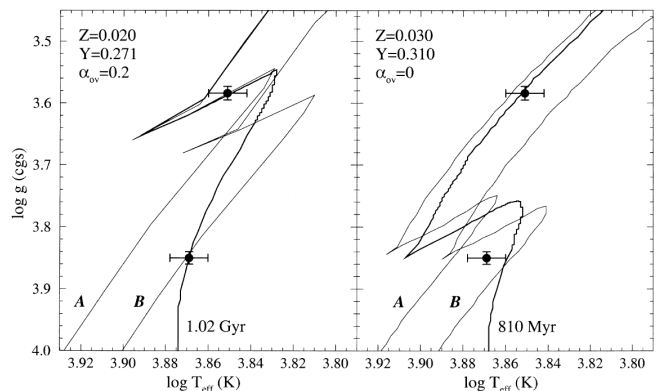


Figure 22. T_{eff} - $\log g$ plot showing the observed properties of the dEB AI Hya (Andersen 1991). The panel on the left shows evolutionary tracks and isochrones from the Granada theoretical models (Claret 1995 and subsequent works) for $\alpha_{\text{OV}} = 0.20$. The panel on the right shows the predictions for standard models ($\alpha_{\text{OV}} = 0$). Taken from Ribas et al. (2000).

greater than this (here between 1.5 and $2.5 M_{\odot}$) clearly require models with overshooting to match their properties.

Stothers & Chin (1991) found that the adoption of newer opacity data in their stellar evolutionary code eliminated the need for convective core overshooting when attempting to match predictions to observations. They quoted the maximum amount of overshooting to be $\alpha_{\text{OV}} = 0.20$. Stothers (1991) detailed the results of fourteen tests for the presence of overshooting in medium- and high-mass stars. The results of every test were consistent with $\alpha_{\text{OV}} = 0$, four tests produced the constraint of $\alpha_{\text{OV}} < 0.4$ and one test allowed this constraint to be strengthened to $\alpha_{\text{OV}} < 0.2$. However, Stothers states that matching the amount of apsidal motion exhibited by some well-studied dEBs may continue to require a small amount of overshooting in the evolutionary models.

Castellani, Chieffi & Straniero (1992) claim that the improved physics in their theoretical models means that overshooting is not required. Daniel et al. (1994) studied the open cluster NGC 752 and found that this was not the case.

Woo et al. (2003) found that overshooting was necessary for models to match the morphology of the CMDs of intermediate-age open clusters in the LMC. Nordström, Andersen & Andersen (1997) made an extensive study of the open cluster NGC 3680 and found that a small amount of overshooting was needed to match its properties with the predictions of stellar evolutionary models. Lebreton (2000) states that overshooting is suggested to become important at masses of about $1.6 M_{\odot}$ from counts of stars observed by Hipparcos.

In their study of the F-type dEB EI Cephei, Torres et al. (2000a) required overshooting to match the properties of the dEB with models. The evolved components of several dEBs can be matched by theoretical models without overshooting, but only in a short-lived state beyond the TAMS (Fig. 22). If the models include overshooting, these stars can be matched by MS models in an evolutionary phase which lasts much longer (Andersen 1991; Ribas, Jordi & Giménez 2000). Evolved dEBs therefore provide strong evidence that overshooting is significant. Fig. 22 also shows that the value of α_{OV} derived from consideration of the properties of a dEB is correlated with metal abundance.

Ribas et al. (2000) have found evidence that α_{OV} has a dependence on stellar mass (Fig. 23). This claim is based on the existence of several dEBs with component masses around $2 M_{\odot}$ for which the best match is for theoretical models with $\alpha_{\text{OV}} \approx 0.2$, and two dEBs with larger component masses and a good match for $\alpha_{\text{OV}} \approx 0.6$. It is also thought that overshooting is unimportant for lower-mass stars. On closer examination, though, this work presents very little new significant evidence of such a mass dependence for α_{OV} . Young et al. (2001) found that overshooting is needed to explain the apsidal motion of massive dEBs and that the best match to the observations may require an α_{OV} dependent on mass.

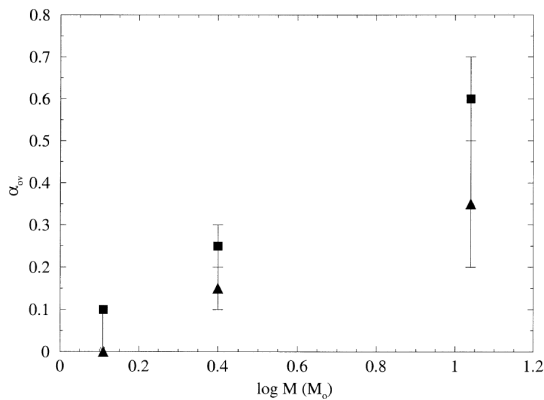


Figure 23. Plot of the best-fitting values of α_{OV} for dEBs against stellar mass. Taken from Ribas et al. (2000).

Cordier et al. (2002) have presented evidence that α_{OV} depends on chemical composition, with larger metal abundances being accompanied by a smaller amount of overshooting (Fig. 24). This result is not very robust and could be modified by the inclusion of other effects, such as rotation, in theoretical stellar models (Cordier et al. 2002).

The existence of convective core overshooting seems to be accepted by most of the astronomical community, and it has been included as a free parameter (i.e., fixed at several values) in all major theoretical stellar evolutionary models since the late 1980s. Further work is required to increase our understanding of this effect; for example the ages of globular clusters have an uncertainty of 10% simply due to uncertainty in the treatment of convection in theoretical stellar models (Chaboyer 1995).

3.1.5 Convective efficiency

As convection in stars is very difficult to model successfully, the efficiency of convective energy transport in stellar envelopes is normally parameterised using the mixing length theory (MLT) of Böhm-Vitense (1958). The parameter α_{MLT} is defined to be

$$\alpha_{MLT} = \frac{l_{\text{mixing}}}{H_p} \quad (44)$$

where l_{mixing} is the mixing length. Convective efficiency is proportional to α_{MLT}^2 (Lastennet et al. 2003).

MLT affects stars whose external layers are convective, which is between $B-V \approx 0.4$ (the boundary with a radiative envelope) and $B-V \approx 1.2$ (where adiabatic convection becomes dominant) (Castellani et al. 2002). In theoretical evolutionary models, α_{MLT} is generally calibrated using the Sun, the only star for which we have an accurate age. However, there is dispute over whether the solar value of α_{MLT} is directly applicable to other stars. Fernandes et al. (1998) state that α_{MLT} is independent of mass, age and chemical composition, so that $\alpha_{MLT\odot}$ is valid for all low-mass Population I stars, whereas D’Antona & Mazzitelli (1994) note that $\alpha_{MLT\odot}$ is not directly relevant to other stars.

Ludwig & Salaris (1999) modelled the dEB AI Phoenicis and found α_{MLT} values which were larger than the solar value, but consistent within the uncertainties. Lastennet et al. (2003) found mixing length values for the component stars of the dEB UV Piscium of $\alpha_{MLT}(A) = 0.95 \pm 0.12 \pm 0.30$ and $\alpha_{MLT}(B) = 0.65 \pm 0.07 \pm 0.10$ (where the uncertainties are random and systematic, respectively), which are significantly smaller than the solar value of approximately 1.6. These authors note that α_{MLT} may decrease with mass, and that it may even not be constant throughout the structure of one star. Palmieri et al. (2002) have investigated whether α_{MLT} is dependent on metallicity, but found no evidence for this. However, Chieffi, Straniero & Salaris (1995) have found evidence that α_{MLT} may depend on metallicity.

An alternative parametric theory for convective efficiency has been proposed by Canuto & Mazzitelli (1991, 1992) and is called the Full Spectrum of Turbulence (FST). Whilst MLT as-

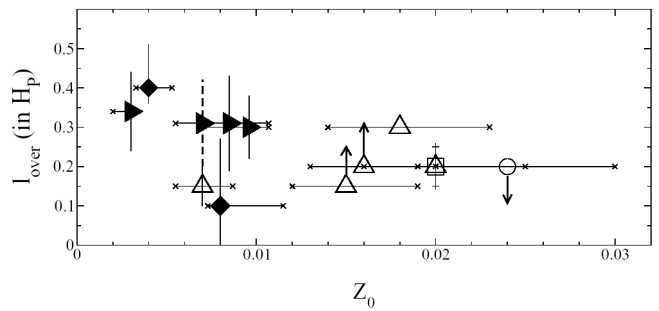


Figure 24. Variation of convective core overshooting parameter, α_{OV} , with fractional metal abundance, Z . Cordier et al. (2002).

sumes that there is one large eddy in stellar convection, FST considers the full spectrum of eddies, using convective theory, and provides an alternative equation for the mixing length. Although the replacement of MLT by FST causes one parametric theory to be replaced by a more modern parametric theory (B. Smalley, 2004, private communication), D’Antona & Mazzitelli (1994) state that the new adjustable parameter, a , can only be varied between 0.5 and 2.0 for physical reasons, and that this makes very little difference to the predictions of theoretical models.

3.1.6 The effect of rotation of stellar evolution

It is now known that the properties of a star depend not only on mass, initial chemical composition and age but also on its initial rotational velocity. Chiosi & Maeder (1986) stated that the next major piece of physics which needed incorporation into theoretical stellar evolutionary models is rotation. It has been included in most recent sets of theoretical models. The usual way to include rotation in theoretical models is to modify the coordinate system from spherical to equipotential (Maeder & Meynet 2000).

Rotation affects stars because (Claret & Giménez 1993):–

- It lowers the effective surface gravity.
- It produces aspherical equipotential surfaces.
- It affects the flux emitted by the star as the equipotential surfaces are not spherical, causing a scatter in the mass-luminosity relation (Maeder & Meynet 2000).
- It stops some modes of convection occurring.

The rotation of stars causes their brightness to increase (Gray, Napier & Winkler 2001) and their T_{eff} to fall (Lastennet, Fernandes & Lejeune 2002). The nearby A-type star Vega is about 0.7 mag brighter than expected because it has a high rotational velocity and is seen pole-on from the Earth (Gray, Napier & Winkler 2001). Rotation in stars causes increased mass loss. It also increases chemical mixing, mimicking the effect of a small amount of overshooting. These two effects have a large influence on the later evolution of massive stars (Maeder & Meynet 2000). The effects of rotation are stronger for stars with lower metallicities (Meynet & Maeder 2002); partly because the ZAMS radii of metal-deficient stars are smaller so they rotate more quickly (Meynet, Maeder & Ekström 2004).

Stellar rotation also influences the MS lifetimes of stars for several reasons (Maeder & Meynet 2000):–

- It increases the amount of hydrogen available to the core (MS lifetime is increased).
- It increases the helium abundance in the outer envelope (luminosity increases so MS lifetime decreases).
- It causes the star to behave as if its mass were lower (MS lifetime is increased).

3.1.7 The effect of mass loss on stellar evolution

Mass loss occurs because radiation pressure, magnetism, convective effect and temperature gradients at the surface of a star cause some particles to be pushed out into interstellar space. For most

stars the effect of this is quite small; for example, the Sun's mass loss rate is about $10^{-14} M_{\odot} \text{ yr}^{-1}$ (Kaufmann 1994, p. 392). Massive stars have a much greater radiation pressure so have much larger mass loss rates; the most massive stars lose most of their mass during their MS lifetime (Meynet et al. 1994) and may end up as Wolf-Rayet stars (Chiosi & Maeder 1986).

Mass loss is thought – and often assumed – to be proportional to the square root of metal abundance: $\dot{M} \propto \sqrt{Z}$ (e.g., Bressan et al. 1993; Maeder 1997) but many other functional forms have been proposed (Chiosi & Maeder 1986). One goal of stellar astrophysics is the determination of the coefficients α , β , γ , δ and ϵ in the equation

$$\dot{M} = \alpha M^{\beta} R^{\gamma} L^{\delta} Z^{\epsilon} \quad (45)$$

The structure of stars is sensitive to the present and past mass loss experienced by them. Mass loss causes MS lifetime to increase (as the star becomes less massive), the luminosity to decrease and the T_{eff} to fall (Underhill 1980; Chiosi & Maeder 1986). The lifetime as a giant is also significantly increased. Mass loss can be dramatic and episodic in evolved massive stars, for example luminous blue variables, so a smooth parameterisation of mass loss in theoretical models is only an approximation for these stars (Massey et al. 1995).

3.1.8 The effect of diffusion on stellar evolution

Diffusion occurs in radiative zones inside stars and is a result of different chemical species having different opacities and masses. Radiation pressure exerts a smaller force on species with lower opacity, and the gravitational force depends on the mass of the species. Because of this, some species are pushed outwards and other species preferentially settle inwards, causing the chemical composition to vary throughout the radiative zone.

Diffusion causes surface chemical composition anomalies in A-type stars, which have radiative envelopes but less mass loss than more massive stars (lower-mass stars have convective envelopes), creating chemically peculiar objects such as Am, Ap and λ Boötis stars. Thus diffusion causes the spectroscopic chemical composition of stars to differ from the actual envelope chemical composition (e.g., Vauclair 2004).

Diffusion is an essential physical ingredient in theoretical models of the Sun. Whilst the solar envelope is convective towards the surface, the radiative lower layer undergoes diffusion processes. This affects the convective layer by changing the chemical abundances at the boundary between the two layers. The depth of a convective envelope depends on its chemical composition (R. D. Jeffries, 2004, private communication), so the radius of the Sun depends on diffusion processes in the solar interior. Diffusion of hydrogen and helium must be included in solar models, and metal diffusion is also desirable (Weiss & Schlattl 1998).

3.1.9 The effect of magnetic fields on stellar evolution

The next major piece of physics to be included in stellar evolutionary models may be magnetism. Maeder & Meynet (2003, 2004) have begun investigating this effect and implementing it in the Geneva stellar evolutionary code. Magnetic fields can be generated by turbulent convection and by differential rotation in radiative layers of a star. Initial results suggest that magnetic fields can significantly enhance chemical mixing in stars.

3.2 Available theoretical evolutionary models

Some of the most commonly used current theoretical models are detailed below, along with some from the recent past. Some characteristics of the current models are given in Table 3.

3.2.1 Hejlesen (1980)

The theoretical models of Hejlesen (1980a, 1980b) were the most popular for the comparison with properties of dEBs through the

1980s. The opacities of Cox & Stewart (1969) and Cox & Tabor (1976) were used and ten different chemical compositions were adopted. The mixing length was $\alpha_{\text{MLT}} = 2.0$ and no overshooting was considered. Internal structure constants, which may be compared to the properties of dEBs which exhibit apsidal motion (see Sec. 7.2) were given by Hejlesen (1987).

3.2.2 Granada theoretical models

Claret & Giménez (1989) published a set of evolutionary calculations using a code based on that of Kippenhahn (1967). The opacities were taken from the Los Alamos group and the mixing length was $\alpha_{\text{MLT}} = 2.0$. Five chemical compositions were considered and the internal structure constants were given.

Claret & Giménez (1992) updated their previous study by adopting the opacities of OPAL (Iglesias & Rogers 1991). The mixing length was $\alpha_{\text{MLT}} = 1.5$, overshooting was included with $\alpha_{\text{OV}} = 0.2$, and four chemical compositions were given. The internal structure constants were also included and mass loss was incorporated.

The current set of theoretical models was published by Claret (1995, 1997) and Claret & Giménez (1995, 1998) and their characteristics are given in Table 3. One major advantage of these calculations is that three helium abundances are available for each of the four metal abundances.

Updated theoretical models have been given by Claret (2004a) for an approximately solar chemical composition only. They are optimised for comparison with the properties of dEBs. The effects of stellar rotation have been included.

3.2.3 Geneva theoretical models

The Geneva models were developed by Maeder (1976, 1981; Maeder & Meynet 1989). The current generation of theoretical models was introduced by Schaller et al. (1992) and are currently by far the most popular with astrophysicists, with over 1400 citations for the Schaller et al. work alone. They use the opacities of Rogers & Iglesias (1992); characteristics and successive references are given in Table 3. Additional consideration has been given to massive star evolution with high mass loss rates (Meynet et al. 1994), evolved intermediate-mass stars (Charbonnel et al. 1996) and an alternative magnetohydrodynamical equation of state for low-mass stars (Charbonnel et al. 1999).

3.2.4 Padova theoretical models

The main rivals to the Geneva models have been developed by the Padova group, culminating in Alongi et al. (1993). The next generation, which remains the current generation for the massive stars, was initiated in Bressan et al. (1993) and uses the OPAL opacities. Further works are given in Table 3. The overshooting formalism is different to that in other models in that it is calculated *across* rather than *above* the convective boundary (Girardi et al. 2000). More recent model predictions have been given by Girardi et al. (2000) for masses between 0.15 and $7 M_{\odot}$.

3.2.5 Cambridge theoretical models

The original models were produced by Eggleton (1971, 1972; Eggleton, Faulkner & Flannery 1973) and incorporate a simple equation of state which allows evolutionary calculations to be relatively inexpensive in terms of computing time (Pols et al. 1995). The models have been extensively tested using the astrophysical properties of dEBs, and moderate convective core overshooting has been found to best fit the observations (Pols et al. 1997).

The current generation of theoretical models (Pols et al. 1998) uses OPAL opacities. Convective core overshooting is formulated differently to other evolutionary codes; the adoption of δ_{OV} is equivalent to $\alpha_{\text{OV}} = 0.22$ and 0.4 for 1.5 and $7.0 M_{\odot}$ stars, respectively. This implicitly includes a mass dependence in α_{OV} . Commendably, the Cambridge models are available both

Table 3. Some characteristics of the current generation of theoretical stellar evolutionary models.

Reference	Mass ($M_{\odot}$)	Y	Z	α_{MLT}	α_{OV}
Claret (1995)	1.0 to 40	0.380 0.280 0.180	0.020	1.52	0.20
Claret & Giménez (1995)	1.0 to 40	0.360 0.260 0.190	0.010	1.52	0.20
Claret (1997)	1.0 to 40	0.420 0.320 0.220	0.030	1.52	0.20
Claret & Giménez (1998)	1.0 to 40	0.346 0.252 0.196	0.004	1.52	0.20
Claret (2004a)	0.8 to 125	0.280	0.020	1.68	0.20
Schaller et al. (1992)	0.8 to 120	0.300 0.243	0.020 0.001	1.60	0.20
Schaerer et al. (1993a)	0.8 to 120	0.264	0.008	1.60	0.20
Charbonnel et al. (1993a)	0.8 to 120	0.252	0.004	1.60	0.20
Schaerer et al. (1993b)	0.8 to 120	0.264	0.040	1.60	0.20
Mowlavi et al. (1998)	0.8 to 60	0.480	0.100	1.60	0.20
Bressan et al. (1993)	0.6 to 120	0.280	0.020	1.63	0.50*
Fagotto et al. (1994a)	0.6 to 120	0.240 0.250	0.004 0.008	1.63	0.50*
Fagotto et al. (1994b)	0.6 to 120	0.230 0.352	0.0004 0.050	1.63	0.50*
Girardi et al. (1996)	0.6 to 120	0.230	0.0001	1.63	0.50*
Fagotto et al. (1994c)	0.6 to 9	0.475	0.100	1.63	0.50*
Girardi et al. (2000)	0.15 to 7	0.23 0.23 0.24 0.25 0.273 0.30	0.0004 0.001 0.004 0.008 0.019 0.030	1.68	0.50*
Pols et al. (1998)	0.5 to 50	0.240 0.240 0.242 0.248 0.260 0.280 0.300	0.0001 0.0003 0.001 0.004 0.01 0.02 0.03	2.00	0 and 0.12 [†]

* The overshooting formalism differs in the Padova theoretical models. Their overshooting of $\Lambda_{\text{OV}} = 0.50$ is equivalent to $\alpha_{\text{OV}} = 0.25$

[†] The overshooting formalism in the Cambridge theoretical models is different to normal. Their overshooting of $\delta_{\text{OV}} = 0.12$ is equivalent to $\alpha_{\text{OV}} = 0.22$ and 0.40 for 1.5 and 7 $M_{\odot}$ stars.

with and without convective core overshooting over their entire mass range. Details of the models are given in Table 3.

Approximate analytical formulae which reproduce the results of the models are given in Hurley, Pols & Tout (2000).

3.2.6 Other theoretical models

Many other theoretical stellar evolutionary models exist:-

- Y^2 models (Yi, Kim & Demarque 2003; Demarque et al 2004)
- CESAM models (Morel 1997)
- Grenoble models (Siess, Dufour & Forestini 2000), which include a PMS phase
- FRANEC (Chieffi & Straniero 1989; Castellani et al. 2003)
- the models of Vandenberg (1985) and Vandenberg et al. (2003), intended mainly for metal-poor stars
- TYCHO models (Young & Arnett 2004)
- ATON models (Mazzitelli 1989; D'Antona & Mazzitelli 1994)

Models for low-mass stars are more challenging, due to the low temperatures and high pressures encountered compared to more massive stars, and will not be detailed here.

3.3 Comments on the currently available theoretical models

Several approximations and parameterisations of complicated physical phenomena allow the construction of theoretical models which are very successful at reproducing the bulk physical properties of many types of stars. However, these approximations and parameterisations are masking a lack of knowledge of the underlying physical processes, and can introduce ‘theoretical uncertainties’ into the results of research which uses theoretical models. As this is often not appreciated, and because observers would then be able to investigate the effects, it should be a priority of theoretical researchers to publish the predictions of models with several different values of α_{MLT} and α_{OV} . Currently, only the Cambridge models are available both with and without convective core overshooting. Also, the assumption of one helium abundance for each metal abundance is even more difficult to support, particularly as

different researchers assume different relations between the abundances of helium and metals. As with the 1995 Granada models, several helium abundances should be considered for each metal abundance. Also, an increased sampling in metal abundance and mass would be useful for most model sets, to limit the need and the difficulty in interpolating between predictions for different chemical abundances and masses. Finally, models should be published using several different competing radiative opacity sets.

4 SPECTRAL CHARACTERISTICS OF STARS

4.1 Spectral lines

When the light from an object is dispersed by a prism or grating, the variation of the brightness of the light with wavelength can be seen. The form of this variation is generally a continuous change in brightness, which depends on the temperature of the object, with the superimposition of sharp peaks, which may rise from the continuum (spectral emission lines) or drop below the continuum (spectral absorption lines), at places in the spectrum dependent on the chemical composition of the object.

Empirical rules of the appearance of spectra of objects were formulated by G. Kirchoff, from terrestrial experiments undertaken with R. Bunsen in the middle of the 19th century:

First law A hot, opaque solid, liquid or compressed gas produces a continuous spectrum.

Second law A hot, transparent gas produces a spectrum containing emission lines whose strength and wavelength depend on which elements are present in the gas.

Third law A cool, transparent gas in front of a source of a continuous spectrum produces an absorption line spectrum with lines whose strength and wavelength depends on the chemical composition of the gas.

The spectra of stars obey these rules, which allows us to derive a lot of information about a star simply by studying its spectrum. The spectral characteristics of a star depend on the conditions in its photosphere. Stellar photospheres are generally composed of plasma which produces a continuous spectrum with absorption lines superimposed. At the centre of an absorption

line, the photosphere has a greater opacity than in the continuum. This means that the light at this wavelength which escapes the star comes from outer, cooler, parts of the photosphere. As cooler objects produce less light than hotter objects, less light is produced in the centre of the absorption line. The central wavelength of an absorption line is the wavelength at which there is an increase in opacity due to the presence of many atoms or ions of one type in the stellar photosphere.

As the velocity of light is not infinite, a difference in velocity between the light emitter and the observer causes the wavelength of the detected light to be different to its wavelength on leaving the emitter. C. Doppler showed that the shift in wavelength, $\Delta\lambda$, depends on the rest wavelength, λ_0 , the relative velocity between the emitter and observer, v , and the speed of light, c :

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c} \quad (46)$$

if $v \ll c$. Light from an emitter which is travelling away from the observer is thus found to increase in wavelength, an effect called 'redshift'. The opposite effect is called blueshift. This is the same effect as that which causes the pitch of a siren on a police car to change as the car drives past you.

4.1.1 Spectral line broadening

Spectral line broadening occurs for many reasons and the amount of broadening is a useful indicator of stellar properties.

Natural broadening There is a minimum width to a spectral line which is set by the Heisenberg Uncertainty Principle (Gray 1992, p. 207). This width is of the order of $5 \times 10^{-4} \text{ \AA}$ at optical wavelengths (Zeilik & Gregory 1998, p. 169).

Pressure broadening Collisional interaction between the light-absorbing species and other particles causes a change in the energy of an energy level, ΔE , which depends on the separation, d , between the interacting particles of the form $\Delta E \propto d^{-n}$ where n depends on the type of interaction. Hydrogen is affected by Linear Stark pressure broadening, for which $n = 2$, which is caused by collisions with protons and electrons. Quadratic Stark broadening is caused by collisions with electrons, has $n = 4$, and affects most spectral lines, particularly in hot stars. Van der Waals broadening has $n = 6$ and affects most spectral lines, particularly in cool stars (Gray 1992, p. 209).

Thermal broadening Atomic species have a range of velocities due to the thermal motion in gases and plasmas. The Doppler effect causes this to result in a broadening of spectral lines.

Rotational broadening As stars rotate, part of the surface of a star approaches the Earth and part recedes from the Earth during an observation. The Doppler effect means that this broadens spectral lines. It is the dominant effect for metal lines under most conditions. The broadening is roughly Gaussian in shape except for large rotational velocities, when spectral lines become more parabolic with less pronounced wings (Collins & Truax 1995).

Turbulence broadening Convection causes an additional variation in the velocities of different species in a stellar photosphere. The effects have been arbitrarily separated into microturbulence, due to small-scale convective motions, and macroturbulence, due to large-scale convective motions (Gray 1992, p. 401). Microturbulence is discussed in Sec. 4.4.1. Macroturbulence is important in early-type giants (Trundle et al. 2004).

Magnetic broadening The Zeeman effect causes spectral lines under the influence of a magnetic field to split into several components. Except for very large magnetic fields, the separation of the components is much smaller than rotational and instrumental broadening so appears as a minor broadening mechanism (Pace & Pasquini 2004).

Instrumental broadening The spectra of stars suffer from a smoothing effect due to the way in which they are observed. Instrumental broadening is discussed in Sec. 10.3.

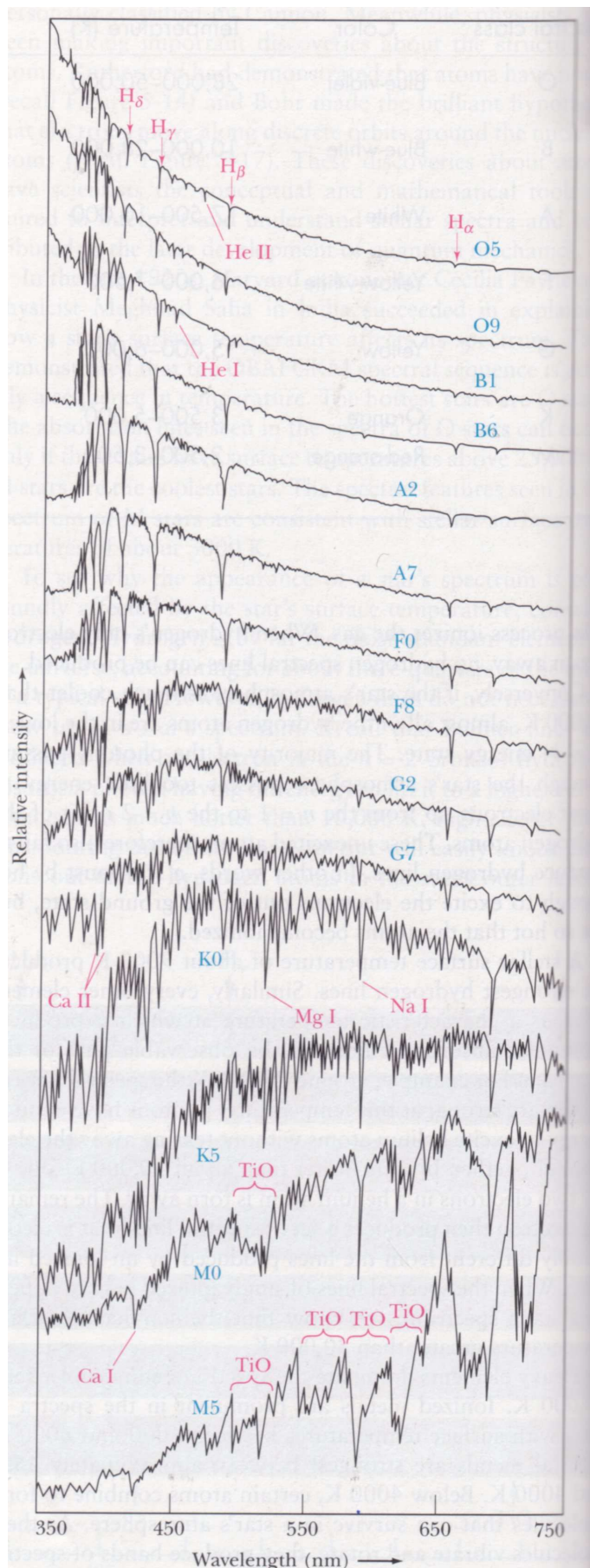


Figure 25. Intensity plot of the optical spectra of a wide range of MS stars (spectral types are given on the diagram) with important spectral lines labelled. Taken from Kaufmann (1994, p. 350).

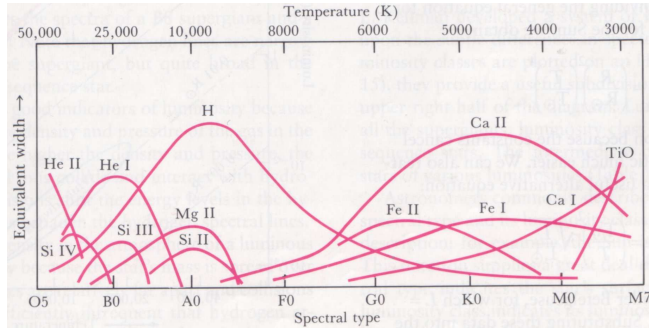


Figure 26. The variation of equivalent widths of some important spectral lines with T_{eff} . Taken from Kaufmann (1994, p. 351).

4.2 Spectral features in stars

Early-type stars have relatively few spectral lines in the optical whereas late-type stars have many lines. A representative set of spectra is shown in Fig. 25. The blue part of the spectrum is the optical region with the most spectral lines. The phenomenon of ‘line blanketing’ arises when this region contains a sufficient number of lines to significantly affect the amount of flux emitted by the star from over these wavelengths. The flux is redistributed to longer wavelengths and is emitted in the red part of the spectrum, affecting the spectral energy distribution of the star (e.g., Kubát & Korčáková 2004). This effect can cause the T_{eff} s of O stars derived from spectral energy distributions to change by up to 3000 K (Mokiem et al. 2004). A similar blanketing effect due to stellar winds is significant in very hot stars (Kudritzki & Hummer 1990). It can also have an effect on the temperature structure of a star due to the ‘backwarming’ effect (Smalley 1993).

The spectral classification of stars depends on the relative strengths of different lines in their spectra. A representation of how the strengths of some lines vary over T_{eff} is given in Fig. 26 and the important lines for each spectral type are given in Table 4. Some important line pairs for classification are in Table 5.

Spectral atlases to aid the classification of stars have been given by Walborn (1980; optical spectral atlas of early-type stars), Walborn, Nichols-Bohlin & Panek (1984; UV atlas for hot stars), Walborn & Fitzpatrick (1990; OB stars), Kilian, Montenbruck & Nissen (1991; early-B stars), Carquillat et al. (1997; IR atlas for late-type stars), Walborn & Fitzpatrick (2000; peculiar early-type stars) and on the internet by R. O. Gray³.

Late-type stars display wide spectral absorption features due to the presence of molecules in their photospheres. The presence of more than one nucleus in a molecule causes electronic energy levels to split into a large number of closely-spaced rotational and vibrational energy levels (Eisberg & Resnick 1985, p. 422). This results in a large number of very close spectral lines which blend together and cause absorption features which extend over several tens of Ångströms in the spectra of cool stars (for example, the features labelled TiO in Fig. 25).

4.3 Stellar model atmospheres

Atmospheric models of stars simulate the conditions in a stellar photosphere and predict the variation of the physical conditions throughout the photosphere as a function of optical depth (Gray 1992, p. 146). Important physical conditions include temperature, pressure, density, geometrical depth and various plasma velocity characteristics. These results can then be used to interpret the characteristics of observed stellar spectra in terms of the physical conditions in the outer layers of the star.

Most model atmospheres are calculated with the assumption of local thermodynamic equilibrium (LTE), where the electronic energy level populations of atomic species are dependent entirely

Table 4. Indication of characteristics of strong lines in spectra of stars of different spectral types (Zeilik & Gregory 1998, p. 258).

Type	Optical spectral characteristics
O	Very few lines. Hydrogen Balmer lines and He II lines are prominent; He I lines increase in strength to lower T_{eff} s. Other lines include Si IV, O III, N III and C III.
B	Hydrogen Balmer lines and He I lines dominate but helium lines become very weak towards B9. Other lines include Si II and Mg II.
A	Hydrogen Balmer line strength peaks at A0 and helium lines disappear entirely. Metallic lines strengthen, particularly Ca II. Many classes of spectral peculiarity occur in A-type stars.
F	Hydrogen lines are much weaker than for A stars but Ca II H and K are strong. Neutral metal lines become stronger than ionised metal lines by late-F.
G	Hydrogen lines are very weak but Ca II H and K reach their maximum strength at G2. Neutral metal lines are strong, ionised metal lines are weak, and the CH molecular G band is quite strong.
K	Hydrogen lines are almost gone, neutral metal lines are strong, TiO molecular bands become visible by late-K.
M	Neutral metal lines and molecular bands are very strong; TiO dominates by M5 and VO bands appear.

on collisional excitation. The Saha equation (which expresses ionisation equilibrium; see Zeilik & Gregory p. 167) and the Boltzmann equation (which expresses excitation equilibrium; see Zeilik & Gregory p. 166) can then be used to determine the excitation and ionization characteristics of the species present.

If radiation, excitation and ionisation pressure becomes significant compared to collisional pressure then the assumption of LTE breaks down. The excitation and ionization of atomic species depends on both the radiation and the collisional pressure. Unfortunately, the amount of radiation pressure depends on the excitation and ionization characteristics of the plasma. Model atmospheres which do not assume LTE are complex, so a large number of iterative calculations are required in order to construct them. The assumption of LTE breaks down between 10 000 and 20 000 K, depending on surface gravity and the accuracy required.

4.3.1 The current status of stellar model atmospheres

The first of the modern generation of theoretical model atmospheres are the ATLAS models which were produced by Kurucz (1979). These are plane-parallel LTE models; they do not contain any contribution to opacity from molecules so significant systematic errors appear at T_{eff} s below about 6000 K (Smalley & Kupka 1997). The currently most popular version of the Kurucz atmospheres is ATLAS9 (Kurucz 1993b); more details can be found on R. L. Kurucz’s homepage⁴. The main competition to the Kurucz model atmospheres is MARCS, developed by the Uppsala (Sweden) group (Gustafsson et al. 1975; Asplund et al. 1997).

The first non-LTE model atmospheres were produced by Auer & Mihalas (1972) and Kudritzki (1975, 1976) but these were relatively unrealistic as they did not contain metals (Massey et al. 2004). Several more recent non-LTE model atmospheres have been successfully used to interpret the spectra of hot stars. These model atmospheres employ spherical geometry and include the effects of line blanketing and stellar winds, so are far more advanced than the atmospheres of Kurucz. They include FASTWIND

³ <http://nedwww.ipac.caltech.edu/level5/Gray/frames.html>

⁴ <http://kurucz.harvard.edu/>

Table 5. Spectral line pairs which give MK spectral classes for stars (Cox 2000, p.383).

Spectral class	Line pairs	
O5 to O9	He I λ 4471	/ He II λ 4541
B0 to B1	Si III λ 4552	/ Si IV λ 4089
B2 to B8	Si II λ 4128-30	/ He I λ 4121
B8 to A2	He I λ 4471	/ Mg II λ 4481
	He I λ 4026	/ Ca II λ 3934
A2 to F5	Mn I λ 4030-34	/ Mn I λ 4128-32
	CH λ 4300	/ CH λ 4385
F5 to G5	Fe I λ 4045	/ H δ λ 4101
	Ca I λ 4226	/ H γ λ 4340
G5 to K0	Fe I λ 4144	/ H δ λ 4101
K0 to K5	Ca I λ 4226	/ Ca I λ 4325
	Ca I λ 4290	/ Ca I λ 4300

(Santolaya-Rey, Puls & Herrero 1997), CMFGEN (Hillier & Miller 1998) and WM-BASIC (Pauldrach, Hoffmann & Lennon 2001).

4.3.2 Convection in model atmospheres

Models of stellar atmospheres are similar to evolutionary models of stars (Sec. 3) in that convection must be accounted for to provide a more realistic description of the stellar properties. The overshooting of convection zones in the envelope and the efficiency of convective energy transport are both important for stars with $T_{\text{eff}} \lesssim 8500$ K (Smalley 2004). The treatment of convection affects the photometric colours of stars calculated using theoretical model atmospheres (Smalley 1998).

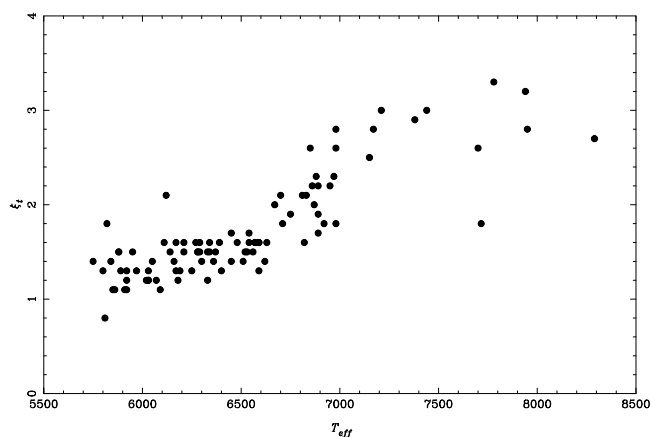
Mixing length theory (MLT, Sec. 3.1.5) is commonly used to model convective effects but MLT model atmospheres are generally unable to match the observed helioseismological oscillation frequencies (Kupka 1996). The Kurucz (1993b) ATLAS9 model atmospheres optionally employ ‘approximate overshooting’, which is more successful in matching some observations (Castelli, Gratton & Kurucz 1997) but not others (Smalley & Kupka 2003). The Canuto & Mazitelli (1991, 1992) turbulent convection theory has been implemented into ATLAS9 by F. Kupka and is generally an improvement on MLT and approximate overshooting (Montalbán et al. 2001; Smalley & Kupka 2003; Smalley 2004).

4.3.3 The future of stellar model atmospheres

Model atmospheres are currently in need of a much better treatment of convection (Kurucz 1998). One-dimensional model atmospheres cannot reproduce convective stellar atmospheres (Kurucz 1998). There is also a need for greater knowledge of the energy levels of atoms and ions so more complete spectral line lists can be constructed (Kurucz 2002a). Molecular opacity is also an area where a large amount of work is required – for example, R. L. Kurucz uses line lists for the H₂O and TiO molecules with 38 million and 66 million lines respectively (Kurucz 2002a). CH₄ is yet more complex but is very important in the study of the spectral characteristics of brown dwarfs and planets. Kurucz (2003) states that “We can produce more science by investing in laboratory spectroscopy rather than by building giant telescopes that collect masses of data that cannot be correctly interpreted.” Kurucz (2002b) states that microturbulence velocity is not constant even in one star, and that half the lines in the spectrum of the Sun remain unidentified.

The effects of magnetic fields have been included in model atmospheres for A and B stars by Kochukhov, Khan & Shulyak (2005), who find that energy transport, diffusion and spectral line formation are significantly modified. They note that the effect of a magnetic field on metal lines can be roughly approximated by a ‘pseudo-microturbulence’.

Three-dimensional hydrodynamical model atmospheres are being developed by several research groups (see Ludwig &

**Figure 27.** The variation of microturbulence velocity with T_{eff} . Taken from Smalley (2004).

Kučinskas 2004). The advantage of these models is that convective energy transport can be modelled directly, so microturbulence and macroturbulence are no longer required (Asplund, Grevesse & Sauval 2004). Mixing length theory is also bypassed, so the parameter α_{MLT} (Sec. 3.1.5) is no longer relevant and the predictive capability of the atmospheres is enhanced. Synthetic spectra calculated using current hydrodynamical model atmospheres provide an ‘almost perfect’ match to the solar spectrum (Ludwig & Kučinskas 2004). The drawback is that a typical three-dimensional hydrodynamical model atmosphere requires about 100 grid points per dimension and some resolution in wavelength, so a lot of computer processor time is required to perform the calculations (of the order of one month for one atmosphere using a desktop PC; Ludwig & Kučinskas 2004).

4.4 Calculation of theoretical stellar spectra

Once a theoretical model atmosphere has been constructed for a star, the formation of spectral lines can be modelled using the atmospheric conditions derived using the model. Apart from a theoretical model atmosphere, the calculation of synthetic spectra requires detailed lists of spectral lines and their characteristics.

Synthetic spectra can be compared to observed spectra in order to derive the atmospheric parameters of stars. The main problem with this approach is that synthetic spectra are calculated using model atmospheres, so the resulting T_{eff} s, surface gravities and chemical abundances are dependent on theoretical calculations. This problem should usually be minor because model atmospheres are generally successful, and many T_{eff} s in the literature are actually on the T_{eff} scale of the ATLAS9 model atmospheres.

For B and early-A stars, Balmer lines are sensitive both to T_{eff} and to surface gravity, and a $T_{\text{eff}} - \log g$ diagram will have an almost straight line of best fit (Kilian et al. 1991) pointed towards increasing T_{eff} and increasing $\log g$. This degeneracy can be broken by including silicon lines or helium lines in the analysis to provide a measure of T_{eff} through the ionisation balances. The degeneracy can also be avoided if the analysed star is in an EB because its surface gravity may then be accurately known.

For stars with $T_{\text{eff}} \lesssim 8000$ K the Balmer lines have very little dependence on $\log g$ so can provide accurate values of T_{eff} (Smalley 1996) if there are sufficiently few metal lines to allow the Balmer line shapes to be well defined. This is because the Balmer lines are formed at a wide range of depths in the atmospheres of stars (Smalley & Kupka 2003).

4.4.1 Microturbulence velocity

Microturbulence is an effect which is generally required to improve the match between synthetic spectra and observed stellar spectra. It is a line-broadening mechanism caused by small-scale turbulent motions in the photospheres of stars, and in the Sun

may result from granulation (Smalley 2004). Microturbulence was originally introduced to make elemental abundances derived from weak and strong spectral lines of the same species agree (Smalley 1993). It can be determined by forcing the abundances from strong and weak lines to agree. Microturbulence increases spectral line widths so affects the opacity in stars (Kurucz 2002b).

A microturbulent velocity of about 2 km s^{-1} is generally found for B and A dwarfs (Smalley 1993; Fig. 27), but more evolved stars have larger microturbulent velocities (Lennon, Brown & Dufton 1988) which can be up to 12 km s^{-1} for B giants (Rolleston et al. 2000). Magain (1984) noted that observational errors generally increase a derived value of microturbulence.

Non-LTE model atmospheres have been claimed not to need microturbulence (Becker & Butler 1988), but Gies & Lambert (1992) found that microturbulence is important in non-LTE analyses (Smartt & Rolleston 1997). Trundle et al. (2004) also find that microturbulence is required when using non-LTE codes.

Hydrodynamical model atmospheres directly simulate convective effects so render the concepts of microturbulence and macroturbulence obsolete (Asplund, Greves & Sauval 2004), and are very successful in matching the observed line profiles of stars (Ludwig & Kučinskas 2004).

4.4.2 The UCLSYN spectral synthesis code

The UCLSYN (University College London SYNthesis) code uses theoretical model atmospheres and atomic data to calculate synthetic spectra. It also has a binary-star mode (BINSYN) for composite spectra and can calculate telluric-line spectra (TELSYN). UCLSYN was produced from a code at UCLA by Smith (1992) and is maintained by B. Smalley (Smalley, Smith & Dworzesky 2001). It uses the LTE ATLAS9 model atmospheres of Kurucz (1993b) and the atomic line information lists of Kurucz & Bell (1995). The profiles of some of the helium lines are calculated using the work of Barnard, Cooper & Shamey (1969) and Shamey (1969), with $\log gf$ values from Wiese, Smith & Glennon (1966).

4.4.3 Abundance analysis of stellar spectra

Once the T_{eff} , surface gravity and microturbulence velocity have been found for a star, the abundances of individual chemical elements and ions may be derived from high-resolution and high signal-to-noise spectroscopic observations of it. This can only be done for those elements which exhibit easily identifiable spectral lines, so it is not possible to directly observe the helium abundance of low-mass stars, including the Sun (Fernandes et al. 1998). The equivalent widths of spectral lines can be calculated given atomic data and the atmospheric parameters of a star. Comparison between the observed and calculated equivalent widths gives the chemical abundances of the star relative to the abundances used to find the calculated equivalent widths.

There is a significant correlation between microturbulence velocity and observed stellar abundances (Chaffee 1970). The two effects cannot be separated without the use of high-resolution spectroscopy (Kurucz 1975). An increased microturbulence velocity of 0.5 km s^{-1} can cause a decrease in derived abundances of 0.1 dex (Smalley 1993). Chemical abundances are also correlated with T_{eff} for F, G and K stars in the sense that an increase in adopted T_{eff} causes the derived abundances to increase (Ribas et al. 2003). It must also be remembered that spectroscopically-derived chemical abundances are strictly only valid for stellar photospheres and may not reflect the internal chemical composition of a star (e.g., Vauclair 2004).

4.5 Spectral peculiarity

The atmospheres of A stars are relatively quiet because they do not have significant winds, like O and B stars, or convection, which occurs in stars later than F0 (Kubát & Korčáková 2004). There is also a large range of formation depths for spectral lines in A stars (Kubát & Korčáková 2004). Most A stars which do not rotate quickly develop peculiar spectra due to elemental diffusion,

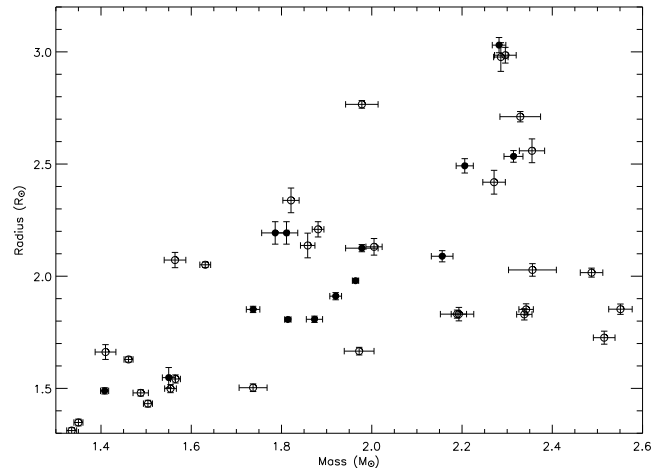


Figure 28. Mass-radius plot of the components of well-studied dEBs with normal spectra (open circles) and with metallic-lined spectra (filled circles). Data taken from Andersen (1991) with updates from more recent works.

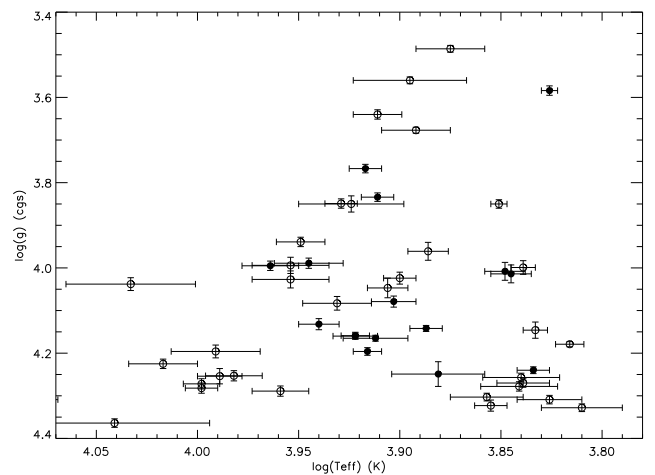


Figure 29. Same as Fig. 28 for surface gravity and T_{eff} .

gravitational settling or the presence of magnetic fields, but are believed to have essentially the same atmospheric structure as normal stars (Bikmaev et al. 2002). Element settling is now included in many theoretical stellar evolutionary codes in order to explain spectrally peculiar stars (Sec. 3.1.8; Vauclair 2004).

4.5.1 Metallic-lined stars

Metallic-lined stars (often referred to as Am stars) are dwarfs of spectral types between A4 and F0 (Popper 1971) which show weak calcium and scandium spectral lines but enhanced lines of other metals. The F0 cut-off is linked to the onset of surface convection (Smalley & Dworzesky 1993). The first Am stars were discovered in the Pleiades by Titus & Morgan (1940) as a group of A stars for which spectral types found from the calcium lines and from the metallic lines were earlier and later, respectively, than those found from the Balmer lines. ρ Puppis stars are subgiant and giant Am stars (Frémat, Lampens & Hensberge 2005).

Am stars have rotational velocities below about 100 km s^{-1} (Budaj 1996); they are often members of short-period binary systems because these stars have the rotation slowed by tidal interactions (Smalley 1993; Abt & Morrell 1995; Budaj 1996, 1997). Am stars appear slightly redder than expected for their Balmer-line spectral types because their enhanced metal lines cause increased line blanketing.

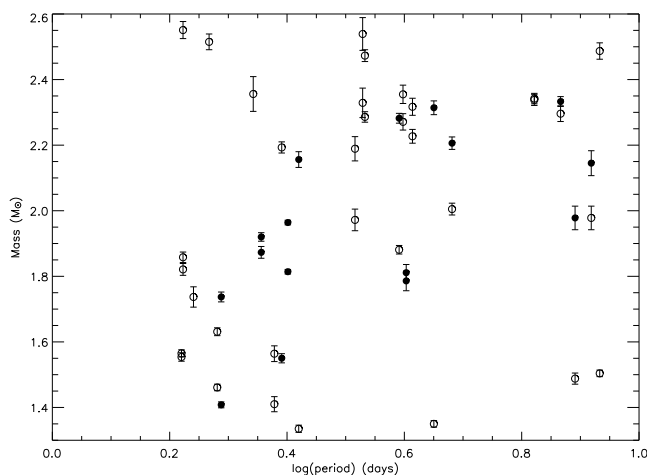


Figure 30. Same as Fig. 28 for orbital period and mass.

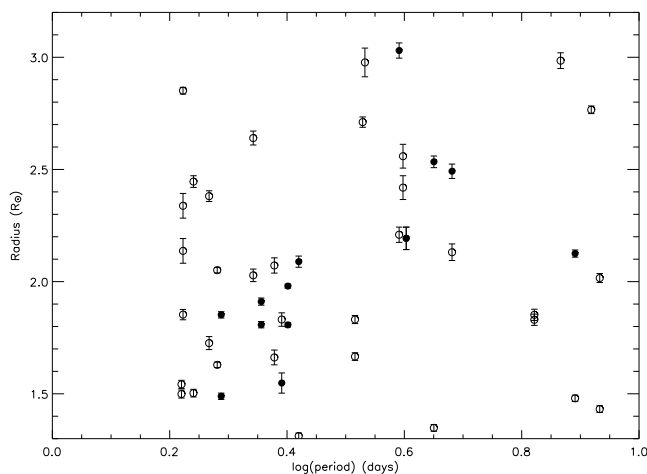


Figure 31. Same as Fig. 28 for orbital period and radius.

Am stars have often been found to be slightly evolved (e.g., Kitamura & Kondo 1978) but Dworetzky & Moon (1986) found that Am stars in clusters had surface gravities which were negligibly different to those of normal A stars. The Am phenomenon ends at $\log g \approx 3.05$ due to the onset of convection (Richer, Michaud & Turcotte 2000).

The metallic-lined phenomenon in stars arises because diffusion and gravitational settling cause metallic ions and atoms to migrate towards the stellar surface. The phenomenon can therefore be likened to a ‘skin disease’ (J. Andersen, private communication) in which the surface chemical composition does not reflect the interior chemical composition of the star. Several well-studied dEBs show metallic-lined spectral characteristics, so their properties can be used to shed light on the Am phenomenon. Figs. 28, 29, 30 and 31 compare the characteristics of metallic-lined dEB components to those which exhibit normal spectra. It can be seen that there is no obvious region in parameter space where all stars are Am, which is consistent with the phenomenon being a surface effect which depends partially on physical properties which have not been considered here.

4.5.2 Other chemically peculiar stars

Three categories of chemically peculiar (CP) A stars have been introduced. CP1 stars are metallic lined stars (see above). CP2 stars are Ap (A-type peculiar) stars. CP3 stars are also termed HgMn stars (North, Studer & Kunzli 1997).

Ap stars have variable spectral line profiles and also ex-

hibit photometric variability. They are slow rotators so are able to exhibit strong magnetic fields, creating starspots which cause the spectral and photometric variability. The magnetic fields also cause extreme abundance anomalies in which the surface abundances of some elements are a thousand times different from normal stars (Abt & Morrell 1995), affecting their spectral energy distributions (Napiwotzki, Schönberger & Wenske 1993).

HgMn stars have T_{eff} s between 11 000 and 15 000 K (spectral types B6 to B9) and display abnormally strong mercury and manganese lines (Wolff 1983, p.144). These enhancements can be up to 6 dex for mercury and 3 dex for manganese (Castelli & Hubrig 2004). They are not magnetic stars and their abundance anomalies are thought to be due to elemental diffusion.

λ Boötis stars are A-type dwarfs which show normal abundances of C, N, O and S, but abnormally weak spectral lines of all other metallic elements (Turcotte 2002). They are spectroscopically similar to Population II stars but have the kinematics of Population I stars, including no dependence of the phenomenon on rotational velocity. They may occur due to the accretion of gas which has been depleted by the formation of dust grains (Abt & Morrell 1995). Most λ Boo stars are photometrically variable due to pulsations of the δ Scuti type (Turcotte 2002). Diffusion models can simulate λ Boo stars, but with difficulty (Turcotte 2002).

5 MULTIPLE STARS

The processes by which stars form naturally also create systems which contain two or more stars. Data on stellar multiplicity can be used to constrain the theories of the formation of star clusters, single stars, and of other celestial objects such as planets. The evolution of stars in multiple systems can be very different to the evolution of single stars. Close binary systems are the sole progenitors of many exotic objects, so their study and characterisation can be very rewarding. The study of binary systems can be regarded as a window through which we can study single stars.

Reasons for studying the multiplicity of stars and the characteristics of multiple stars include:–

- constraining star formation theory by statistical study of the distribution of orbital elements (e.g., Mazeh et al. 1992)
- characterisation of large stellar populations and the light they produce (which contains a significant contribution from objects which are only formed by interaction between stars in a binary system)
- finding the age of large stellar systems from comparison of the eccentricities of binary systems with tidal evolution theories (section 7.1.5)
- investigating the physics of the evolution of close binaries
- calibrating the $M - L$ relation (Duquennoy & Mayor 1991)
- finding high-mass stellar remnants (Duquennoy & Mayor 1991)
- studying how our Galaxy formed (Duquennoy & Mayor 1991)
- multiple stars play an important role in the evolution of gravitationally bound stellar systems (Mermilliod et al. 1992)

5.1 Dynamical characteristics of multiple stars

From the radial velocity (RV) study of Duquennoy & Mayor (Duquennoy & Mayor 1991; Duquennoy, Mayor & Halbwachs 1991), the fraction of nearby F, G and K-type dwarfs which have one or more companion star is 0.43 and each star has on average 0.50 companions. The completeness of this study will drop towards low mass ratios as lower-mass secondary components will have a progressively smaller effect on the radial velocities of the primary stars. The orbital elements were derived for 37 binary systems and studied statistically. The orbital period distribution was found to approximate a Gaussian with a peak at about 180 years, and the median orbital eccentricity was found to be 0.31. This last finding is relevant only for systems with periods greater than about eleven days, which have not been significantly affected by tidal effects. The binary fraction of evolved stars is much lower

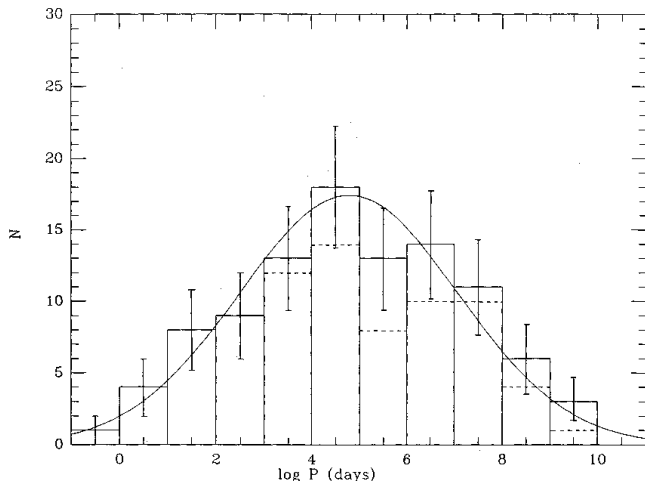


Figure 32. The period distribution of nearby G-dwarf spectroscopic binary systems. The solid histogram represents the observations and the dashed histogram includes a correction for detection biases. The solid line is the normalised distribution $f(e) = 2e$. Taken from Duquennoy & Mayor (1991).

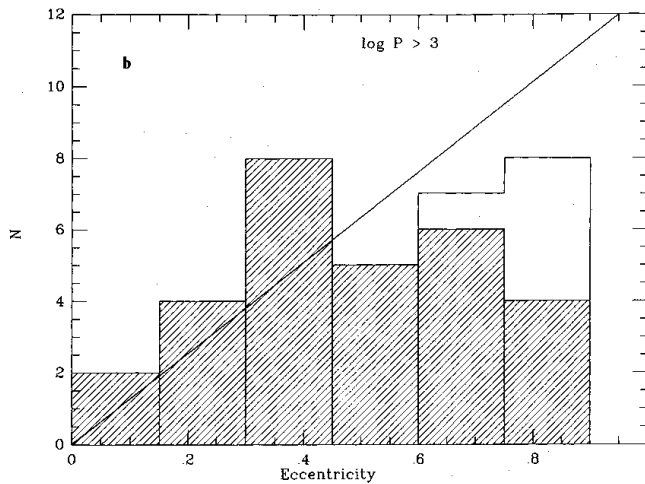


Figure 33. The eccentricity distribution of nearby G-dwarf spectroscopic binary systems with periods greater than 1000 days. The hatched histogram represents the observations and the clear histogram includes a correction for detection biases. Taken from Duquennoy & Mayor (1991).

as the close binaries will have evolved through phases of mass transfer to make exotic objects (Abt & Levy 1973; Nordström, Andersen & Andersen 1997). The distribution of orbital periods and eccentricities is shown in Figs. 32 and 33.

The mass ratio distribution of binary systems has been found by many researchers to peak at mass ratios near to unity. There are two selection effects which encourage this finding. Firstly, binaries with mass ratios close to unity are brighter than similar binaries with lower mass ratios due to the light contribution from the secondary star. Secondly, radial velocities are difficult to derive when the secondary star is much fainter than the primary star, so for these systems the mass ratio is more difficult to obtain (Prato et al. 2002). The study of Duquennoy & Mayor found no peak, as the completeness was much improved over previous works, but instead an increasing number of systems as the mass ratio became smaller. This has recently been confirmed by Mazeh et al. (2003), who found that the number of systems is approximately constant between mass ratios of 0.1 and 1.0 (Fig. 34). Mazeh et al. (1992) suggested that the mass ratio distribution is quite different for close binaries than for wide binaries, probably as a consequence of the formation process of the systems.

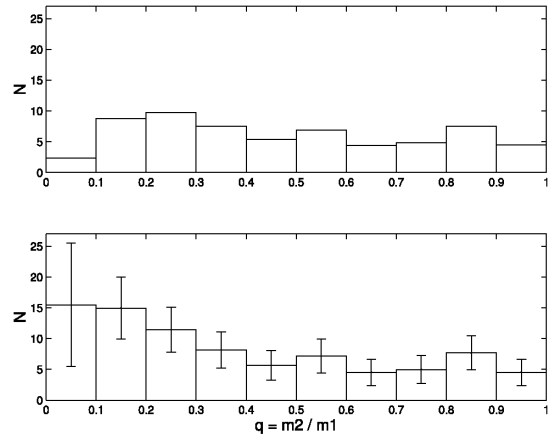


Figure 34. The mass ratio distribution found for G and K-type double-lined spectroscopic binaries in the solar neighbourhood. The upper panel shows the results whilst the lower panel shows the results after correction for incompleteness, which is important for mass ratios below 0.3. Taken from Mazeh et al. (2003).

The evolution of the components of binary systems is different to the evolution of single stars which are otherwise similar. This phenomenon seems to arise during formation, where a binary system may have quite different energy characteristics to a single star (Tohline 2002). This is manifested in the fact that even stars in young binary systems rotate more slowly than single stars of the same type (e.g., Levato & Morrell 1983). During evolution as a detached binary, the presence of a companion star affects evolution through tidal effects (which modify the rotational characteristics of the star), irradiation (the reflection effect) and mass transfer in close binary systems. The conditions under which this becomes significant are not accurately known and will not be the same for different research projects.

5.2 Binary star systems

Binary star systems present many possibilities for discovering the physical laws which govern the existence of stars. Direct measurements of the characteristics of stars can be made by studying several different types of binary system.

Visual binaries are long-period systems which are situated sufficiently close to the Earth that the individual component stars can be observed separately. With the current generation of stellar interferometers, many more binary systems fall into this category, although some researchers call these “interferometric binaries”. Knowledge of the positions of the stars on the sky, as a function of orbital phase, coupled with RV observations, allow the masses of the stars to be measured directly, along with their luminosity ratio. These stars are therefore good for determining the mass–luminosity relation of stars, but, more importantly, they provide an essentially geometric determination of the distance to the system which is very reliable (Paczynski 2003). Perhaps the best-known studies of such stars allowed Torres, Stefanik & Latham (1997a, 1997b, 1997c) to determine the distance of the Hyades open cluster to be 47.6 ± 1.1 pc from analysis of the visual binaries 51 Tauri, 70 Tauri, θ^1 Tauri and θ^2 Tauri. The data for these visual binaries were also compared to stellar evolutionary models to derive an age and metal abundance from their absolute masses and luminosities.

Spectroscopic binary systems are those for which their binarity is apparent from variation of their RV. The secondary component may also produce spectral lines strong enough to be visible in the spectrum of the system, in which case the spectroscopic binary is “double-lined”. Spectroscopic observations of these systems allows calculation of the orbital period and eccentricity, the mass ratio, and the minimum masses of the components, $M \sin^3 i$, where i is the inclination of the orbit relative to the line of sight of the observer (see Sec. 11). These can be studied statistically to constrain tidal evolution theories (Sec. 7) but the other uses are

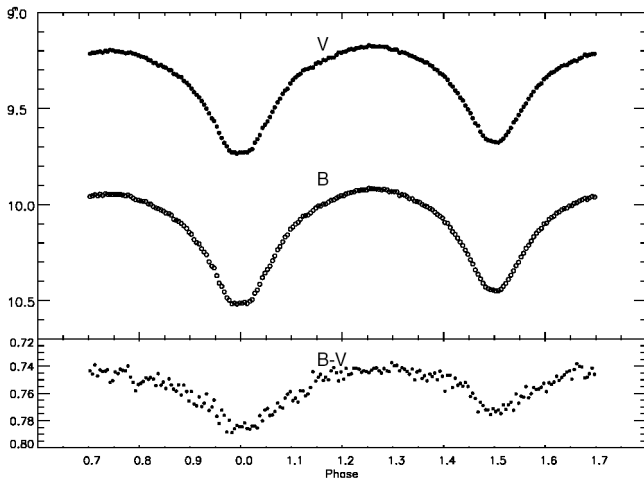


Figure 35. Example light curve of a W UMa system. This shows the light variation of V1128 Tauri, which has a period of 0.3 days and a total primary eclipse. Taken from Taçs et al. (2003).

minor. It is useful to know which stars are binary when studying the photometric properties of open clusters (see Sec. 8.1).

5.3 Eclipsing binary systems

EBs consist of two stars whose orbit periodically causes one star to eclipse the other star, as seen from Earth. As the other star also eclipses the first star once per orbit (except for a few EBs which have very eccentric orbits and orbital inclinations significantly below 90° , for example NY Cephei, Holmgren et al. 1990), there are two different eclipses for every orbital period. EBs are classified into a wide variety of types, depending on the shape of their light curve and their evolutionary status.

As approximately 0.2% of stars are EBs, it is expected that about 5×10^6 exist in the Milky Way Galaxy, of which about four thousand have been discovered (Guinan 2004). The Hipparcos space satellite found 917 nearby EBs, of which 347 were previously undiscovered (Turon 1997).

W Ursae Majoris systems are very close binaries composed of two stars which are in contact with each other at the inner Lagrangian point. The W UMa systems divide into two distinct groups (Van't Veer 1975) with an approximately equal frequency. It has been suggested that the group composed of systems with spectral types between F8 and G2 are formed by fission in the PMS stage. These have light curves disturbed by starspots. The second group contains systems with spectral types from early-A to F8 and are created from detached binaries which lose angular momentum through evolution and magnetic braking (Mazur, Krzemiński & Kałużny 1995). Rucinski (1994) has provided a calibration for the absolute visual magnitude, M_V , of W UMa systems versus their orbital period, P in days, and their $B-V$ photometric colours:

$$M_V = (-2.38 \pm 1.20) \log P + (4.26 \pm 0.87)(B-V) + (0.28 \pm 1.01) \quad (47)$$

where the scatter around the best fit is about 0.5 mag. The absolute masses and radii of W UMa systems can be derived from light curves and RV curves, but the photometric mass ratio and orbital inclination are strongly correlated unless the eclipses are total (Kałużny & Thompson 2003). They are quite common EBs. An example light curve is shown in Fig. 35. Starspots can cause the light curve to be asymmetric around the phases of maximum light, a phenomenon called the O'Connell effect (O'Connell 1951; Linnell 1986; Milone, Wilson & Hrivnak 1987).

Algol systems are created from a close binary consisting of two MS stars. The more massive component evolves past the TAMS, increases in radius and overflows its Roche Lobe. The secondary star accretes much of the mass lost by the primary star, and becomes more massive. Algol systems therefore consist of an evolved low-mass star (usually a subgiant), which fills its

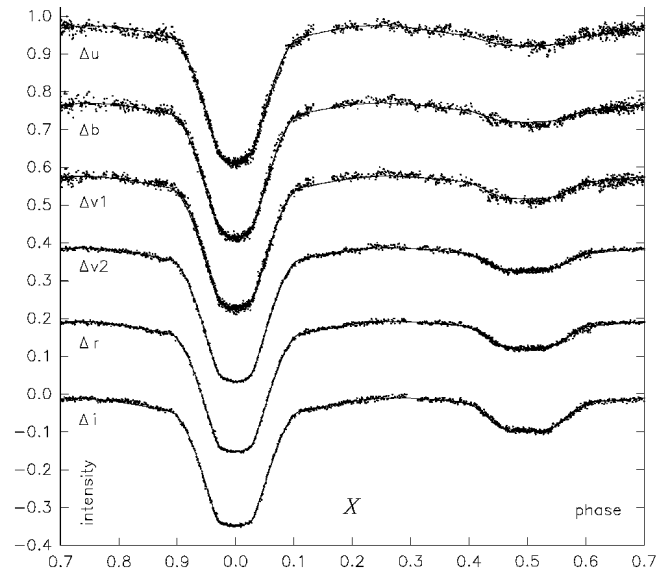


Figure 36. Example of the light curve of an Algol EB. This shows the light variation of AG Phoenicis, which has a period of 1.5 days and undergoes total eclipses. Taken from Cerruti (1996).

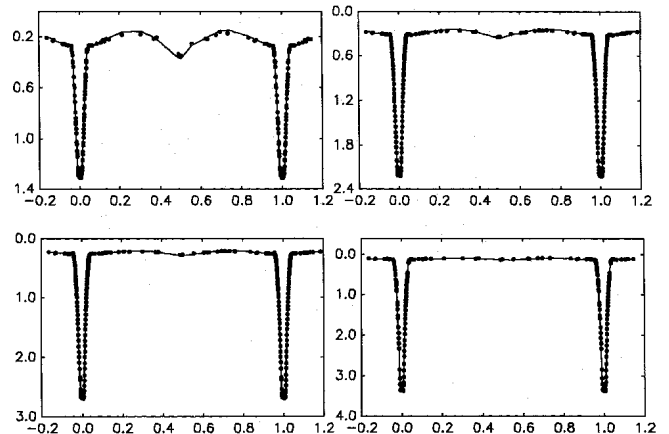


Figure 37. Example of the light curve of an Algol EB. This shows the light variation of S Cancri, which has a period of 9.5 days, in the I (top left), y (top right), b (bottom left) and v (bottom right) passbands. Taken from Olson & Etzel (1993).

Roche Lobe, orbiting an early-type MS star. They are relatively common systems and have mass ratios of the order of 0.3 (Hilditch 2001, p. 288). Example light curves are shown in Figs. 36 and 37.

dEBs are composed of two stars which have not interacted by mass transfer and are effectively gravitationally bound single stars. They differ from single stars in their formation (Tohline 2002), and due to tidal interactions, mutual irradiation and interception of each other's stellar winds. dEBs for which these effects are negligible are very important because they allow the direct measurement of absolute masses, radii, T_{eff} s and luminosities of stars which have evolved as single stars. A full characterisation of a dEB requires a significant number of radial velocities to determine a spectroscopic orbit (Fig. 38) and a large number of photometric observations to derive the radii of the stars (Fig. 39). These systems will be discussed further in the next section.

RS Canum Venaticorum and BY Draconis systems are chromospherically active detached binaries (Hilditch 2001, p. 286). One or both components has a deep convective envelope and displays starspots, spectral emission lines and X-ray emission from magnetic activity in the stellar corona. The light curves of such objects show significant distortion, compared to dEBs containing stars with radiative envelopes, which change over time (Hilditch 2001, p. 221). The absolute dimensions of RS CVn and BY Dra ob-

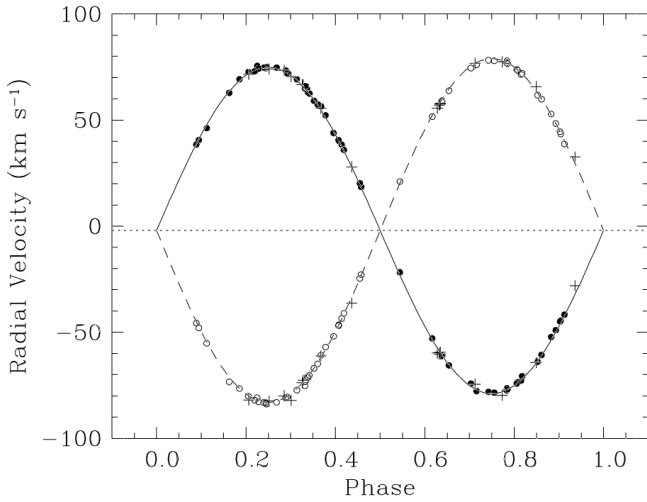


Figure 38. Example RV curve of the dEB V364 Lacertae. RVs for the primary and secondary stars are given by filled and open symbols respectively. Taken from Torres et al. (1999).

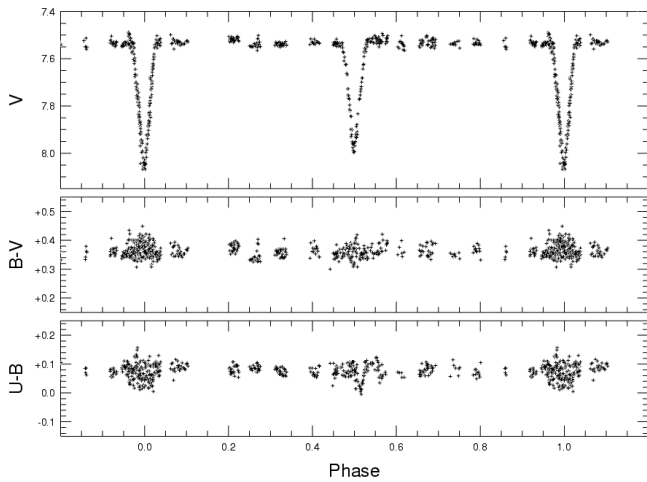


Figure 39. Example light curve of the dEB V364 Lacertae by Padalia & Srivastava (1975). Taken from Torres et al. (1999).

jects are difficult to determine accurately due to the distortions present in the light curves.

Many exotic objects are exclusively binary systems. PG 1336-018 (Fig. 40) is an EB with a period of 0.10 days, containing a pulsating sdB star (Kilkenny et al. 1998). Subdwarf B stars are composed of $0.5 M_{\odot}$ helium cores covered by a thin envelope of hydrogen, and are thought to be created from red giants which lose their envelope due to binary interactions or winds on the red giant branch (Maxted et al. 2000). EC 13471-1258 is a white dwarf-M dwarf binary which displays total eclipses (O'Donoghue et al. 2003) (Fig. 41). It is thought to have been created by the ejection of a common envelope during binary evolution on the asymptotic giant branch.

6 DETACHED ECLIPSING BINARY STARS

Double-lined dEBs are of fundamental importance to astronomy and astrophysics as they represent one of the main links between theoretical stellar astrophysics and what happens in the real world (Andersen 1991). Excluding the Sun and a few nearby visual binaries, dEBs are the only systems from which accurate and absolute stellar masses can be found. Accurate absolute stellar radii can also be determined using entirely empirical methods, and they are also amenable to determination of photospheric metal abundance using the same techniques as for single stars (Sec. 4.4.3).

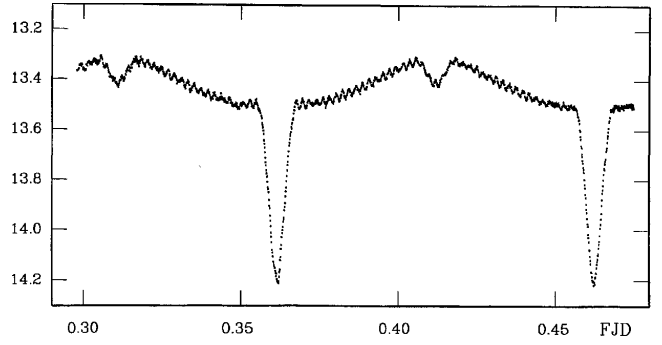


Figure 40. Light curve of PG 1336-018, an EB containing a pulsating sdB star. Taken from Kilkenny et al. (1998).

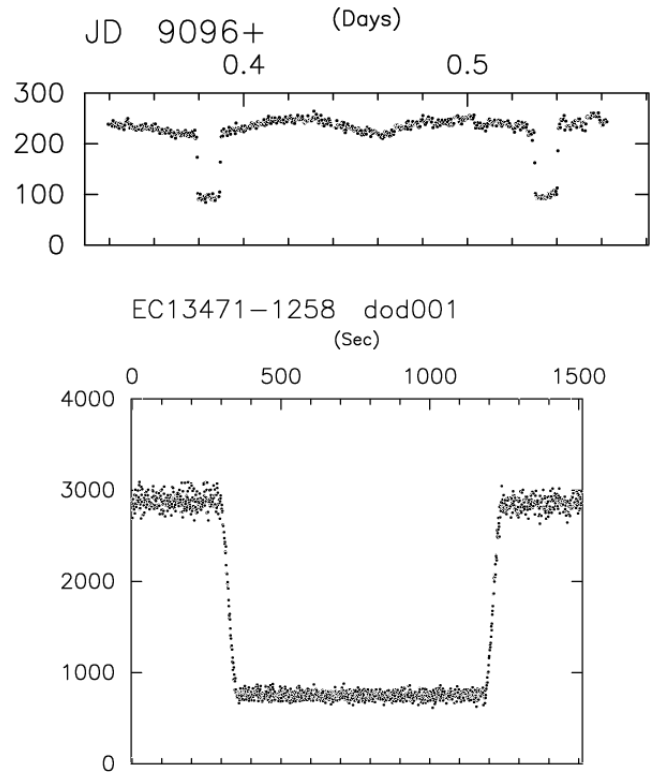


Figure 41. Light curve of EC 13471-1258, an EB containing a white dwarf and an M dwarf star. The upper panel contains observations acquired with exposure times of 30 s and the lower panel shows the primary eclipse observed with exposure times of 1 s. Taken from O'Donoghue et al. (2003).

The derivation of accurate T_{eff} s can be more tricky (Sec. 4.4), but this knowledge allows the calculation of the luminosities of the two stars and, ultimately, the distance (Sec. 6.3).

Excellent reviews of the then-available data on dEBs, techniques for their observation and analysis and general results obtained from their study, have been published by Popper (1967, 1980) and by Andersen (1991). Harmanec (1988) has collected an exhaustive database of the absolute dimensions of dEBs. Whilst the reviews of Popper (1967, 1980) concentrated on the determination of stellar masses and radii, the celebrated work of Andersen (1991) considered only those dEBs for which the masses and radii were known with uncertainties below 2% and T_{eff} s to within 5%, the final total being 45 dEBs (containing 90 stars). The reason for the outright rejection of data on dEBs with more uncertain parameters is that such systems generally have only a limited use compared to the most well-studied dEBs (Andersen, Clausen & Nordström 1980, 1984; Andersen 1993, 1998). In fact, knowledge

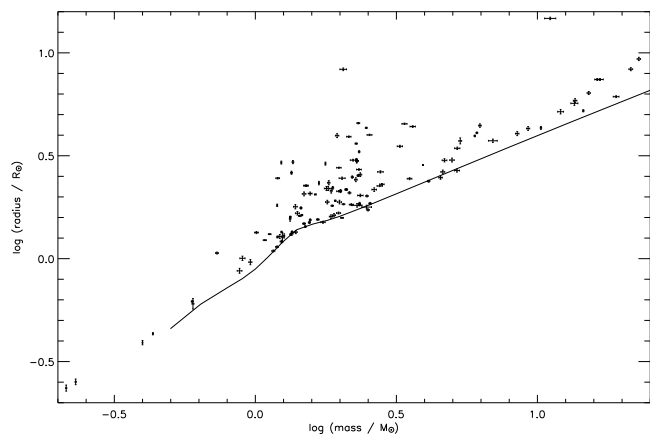


Figure 42. Logarithmic mass-radius diagram containing the components of well-studied dEBs. Uncertainties are shown as errorbars and the theoretical ZAMS for a solar composition, taken from the Cambridge stellar evolutionary models (Pols et al. 1998), is given by a solid line.

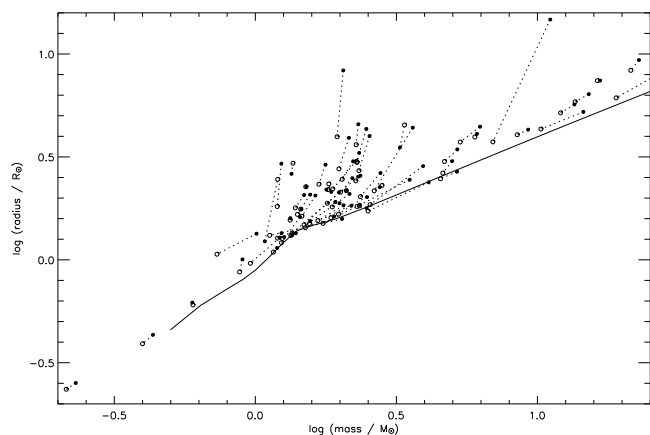


Figure 43. Logarithmic mass-radius diagram containing the components of well-studied dEBs. The primary components of dEBs are shown using filled circles and the secondary components by open circles. Dotted lines connect the components of individual dEBs. The theoretical ZAMS for a solar composition, taken from the Cambridge stellar evolutionary models (Pols et al. 1998), is given by a solid line.

of the dimensions of a dEB to within 5% is no longer in general useful (e.g., Andersen 1991, Giménez 1992).

Whilst there are a good number of well-studied MS dEBs of spectral types between B and G, very few exist outside these boundaries. Whilst several O star dEBs have been studied (e.g., V1007 Scorpii, Sana, Rauw & Gosset 2001), these systems exhibit complications which makes determination of accurate parameters very difficult. dEBs known to contain K or M dwarfs are very rare as the small sizes of these stars means that few exhibit deep eclipses (Popper 1993). Starspots can also be problematic when analysing the light curves of such systems (see e.g., Torres & Ribas 2002; Ribas 2003). There is also a shortage of dEB components which are close to the ZAMS (Andersen 1991, Giménez 1992), particularly for high-mass stars, and beyond the TAMS (with the important exceptions of the giant system TZ Fornacis, Andersen et al. 1991, and SZ Centauri, Andersen 1975c). This is because dEBs which contain an evolved star tend to exhibit single-lined spectra as the unevolved star is a lot dimmer than the evolved star. Therefore dEBs which contain an evolved star but are double-lined must have a mass ratio close to unity (Andersen 1975c), so are very rare. Recent work has begun to focus on dEBs containing substellar objects (e.g., 2MASS

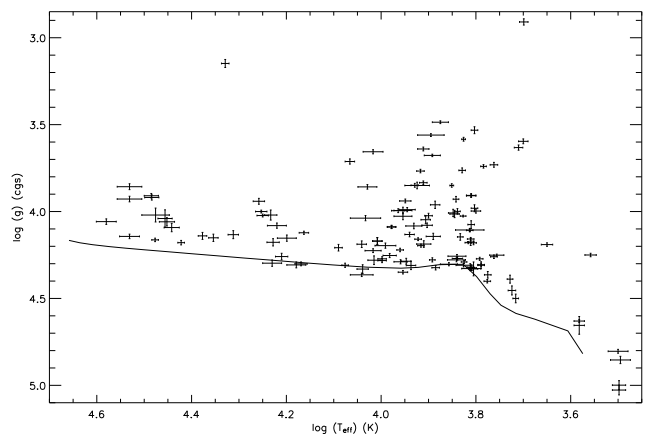


Figure 44. Logarithmic T_{eff} - $\log g$ diagram containing the components of well-studied dEBs. Symbols are as in Fig. 42.

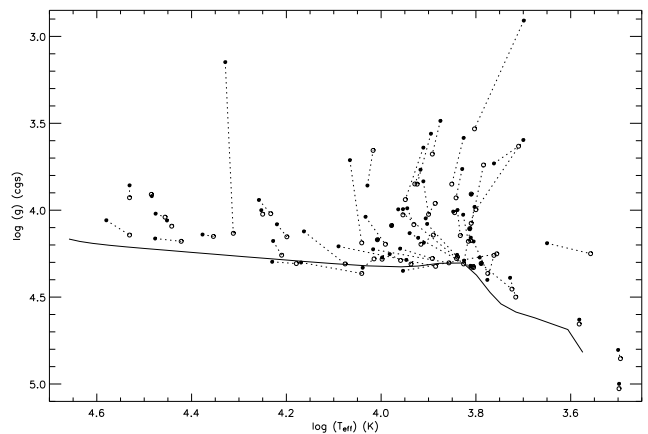


Figure 45. Logarithmic T_{eff} - $\log g$ diagram containing the components of well-studied dEBs. Symbols are as in Fig. 43.

J0516288+260738, which appears to be an eclipsing M dwarf – brown dwarf system, Schuh et al. 2003).

The available data on dEBs with masses and radii accurate to 2%, and T_{eff} s to within 5%, has been collected from Andersen (1991). Results from more recent publications have been added, with an emphasis on the inclusion of interesting dEBs rather than those which conform precisely to the above limits on accuracy. These have been plotted in mass-radius diagrams (Figs. 42 and 43) with errors (Fig. 42) or with lines connecting different stars in one dEB (Fig. 43). The corresponding T_{eff} - $\log g$ diagrams and HR diagrams have also been plotted in Figs. 44, 45, 46 and 47.

There are two main uses of the fundamental astrophysical parameters of dEBs: as calibrators and checks of theoretical models, and as standard candles. The methods of determining the distance to different types of dEB will be covered in the next section; other uses of dEBs will be discussed here.

Knowledge of the masses and radii of the components of dEBs has allowed Ribas et al. (1997) to construct photometric calibrations which predict the masses and radii of single stars using Strömgren photometric indices (Sec. 12.1.3). This study updates the calibration contained in the UVBYBETA code of Moon & Dworetzky (Moon 1985a), which was based on finding the absolute magnitude and surface brightness of a star in order to predict its radius. Calibrations of surface brightness could be aided by study of the dEBs suggested by Kruszewski & Semeniuk (1999).

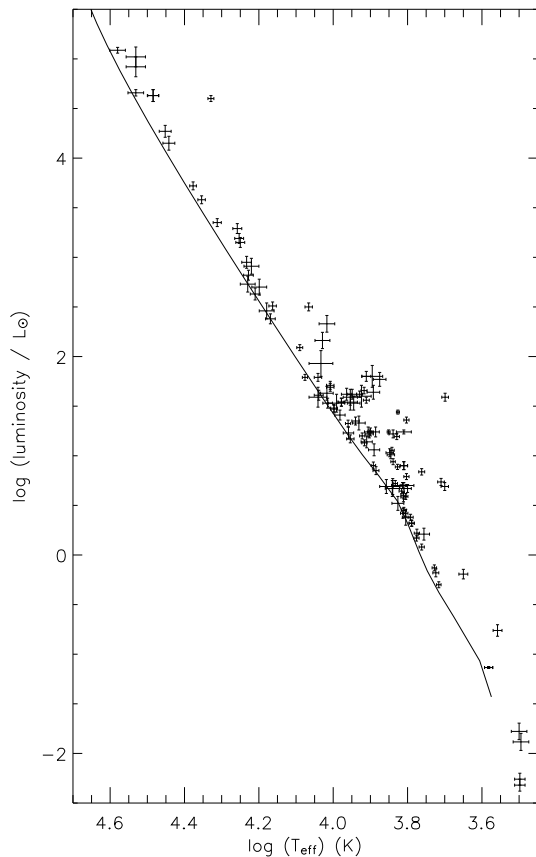


Figure 46. HR diagram containing the components of well-studied dEBs. Symbols are as in Fig. 42.

6.1 Comparison with theoretical stellar evolutionary models and model atmospheres

The basic stellar properties are mass, radius, luminosity and chemical composition (Andersen 1991). In principle, the mass and chemical composition determine all other stellar properties throughout the lifetime of a star, but the predictions of stellar evolutionary theory are not completely reliable and so must be tested by comparison with observed properties of stars (Andersen 1991). This is because many physical processes are simplistically treated (e.g., convection, mass loss, magnetic fields) and some atomic data (e.g., reaction rates, opacities) are poorly known.

Theoretical stellar evolutionary models are usually calibrated to predict the radius and T_{eff} of the Sun given its known mass, age and approximately known chemical composition. They are therefore very successful at predicting the properties of solar-type stars. Extension to higher masses, however, depends a lot on the observed properties of well-studied dEBs. Theoretical models for stars much less massive than the Sun can be extremely complex, and the current generation of models do not show a good agreement with each other and with the few well-studied dEBs in this mass range (e.g., Maceroni & Montalbán 2004). A particular advantage of dEBs is that accurate masses, radii and T_{eff} s can be found for two stars which have a common age, initial chemical composition and distance (according to most star formation theories). This provides a more detailed test of theoretical predictions, as models must match the astrophysical properties of both stars for the same age and chemical composition.

The predictions of stellar evolutionary models are often calibrated, or checked, with the use of accurate astrophysical parameters of dEBs (e.g., Claret 1995; Pols et al. 1995). In particular, the amount of convective core overshooting to use has sometimes been decided using studies of dEBs (e.g., Pols et al. 1997; Hurley, Pols & Tout 2000; Ribas, Jordi & Giménez 2000). Other physical effects incorporated into theoretical models for which the study

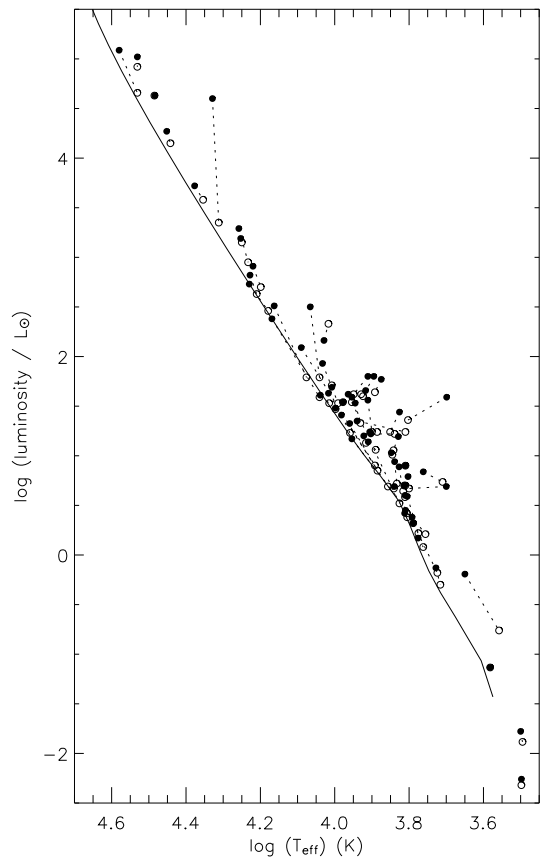


Figure 47. HR diagram containing the components of well-studied dEBs. Symbols are as in Fig. 43.

of dEBs may provide constraints include opacity, mass loss, and characteristic mixing lengths (Shallis & Blackwell 1980).

The use of spectral disentangling (Sec. 11.3.5) has made it more straightforward to critically test the success of model atmospheres in predicting stellar spectra. The study of dEBs provides a fundamental and accurate determination of the surface gravity of both stars. The other main atmospheric parameter, T_{eff} , can be inferred in several ways. Given these properties, model atmospheres should enable the calculation of theoretical spectra which are in good agreement with the individual spectra of the two stars, found by disentangling the observed composite spectra (Smalley, private communication; Ribas 2004).

6.1.1 The methods of comparison

Once the properties of a dEB have been accurately calculated, they can be compared to the predictions of stellar models. As the two stars are expected to have the same age and chemical composition, stellar models should be able to simultaneously fit their properties for one age and composition. Further constraints can be provided by knowledge of the central concentrations of the stars (from apsidal motion studies) and by the derivation of the chemical compositions of the stars from high-resolution spectroscopic observations (Andersen 1993, 1998; see for example Ribas, Jordi & Torra 1999). The determination of the chemical compositions of well-studied dEBs is suggested to be important in the near future to aid the careful study of the success of different sets of stellar model predictions (Andersen 1993, 1998).

An example of the graphical representation of the properties of a dEB compared to theoretical stellar models is shown in Fig. 48. This variant on the HR diagram shows the components of the dEB AI Phoenicis (Andersen et al. 1988) compared to the predictions of the stellar evolutionary models of VandenBerg (1983). Figs. 49 and 50 show comparisons between the properties of the low-mass dEBs YY Geminorum (Leung & Schneider 1978) and CM Draconis (Lacy 1977b) and the low-mass stellar models of

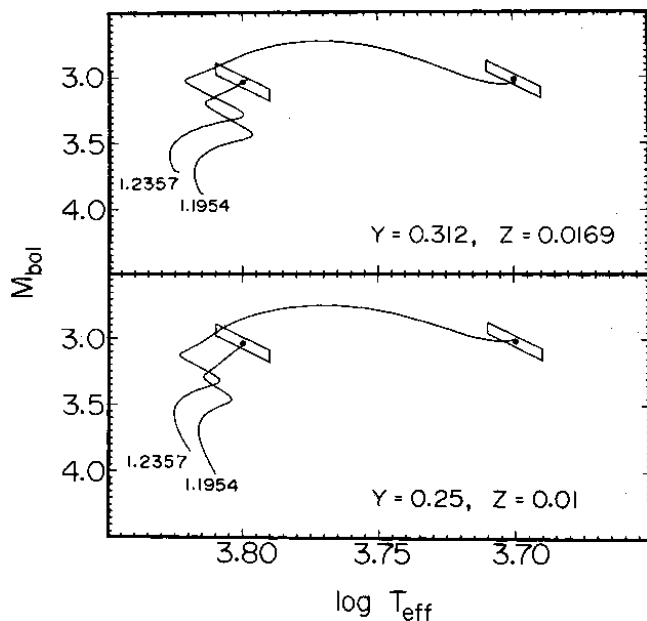


Figure 48. HR diagram showing the components of AI Phoenicis (Andersen et al. 1988) compared to predictions of Vandenberg (1983) theoretical models computed for the masses of AI Phe (indicated on the diagram) and two chemical compositions (Y and Z as shown). Taken from Andersen et al. (1988).

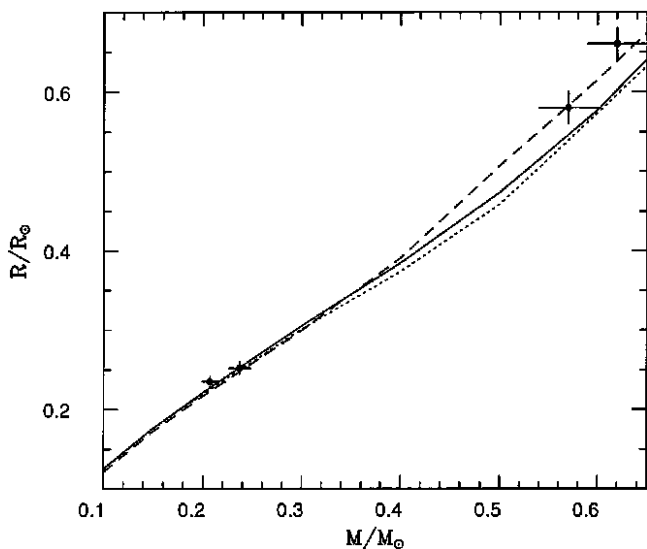


Figure 49. An example of a mass-radius plot for the comparison of the properties of the low-mass dEBs YY Geminorum and CM Draconis versus the stellar models of Baraffe et al. (1995). The solid and dashed lines show predictions for metal abundances of $[M/H] = 0.0$ and -0.5 , respectively. The dotted line is for $[M/H] = 0.0$ under the Eddington approximation. Taken from Chabrier & Baraffe (1995).

Baraffe et al. (1995). Figs. 51 and 52 show comparisons between the properties of the high-mass dEB V3903 Sagittarii (Vaz et al. 1997) and the Claret & Giménez (1992) theoretical models.

The comparison between models and stellar properties is commonly undertaken using HR diagrams, as this method resembles that often used in the photometric study of stellar open clusters (see Sec. 8). However, the most directly known fundamental parameters of a dEB are the masses and radii, and the surface gravities which are calculated from them. T_{eff} s must be found using less straightforward methods such as spectral analysis or application of photometric calibrations. Logically, there-

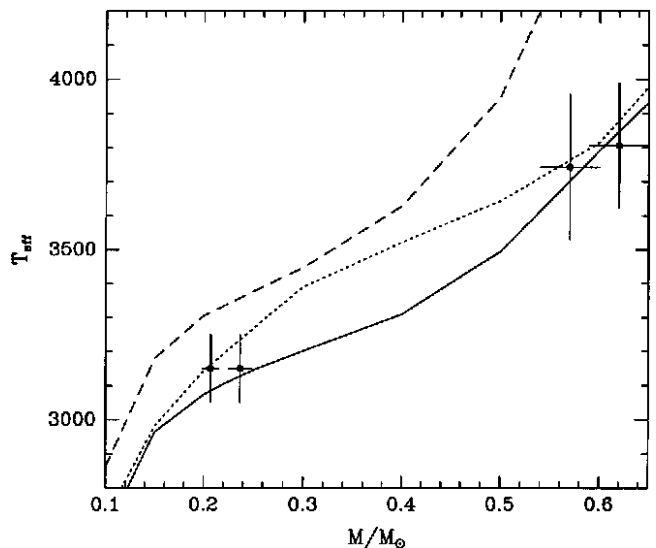


Figure 50. An example of a mass- T_{eff} plot for the same data and models as Fig. 49. Taken from Chabrier & Baraffe (1995).

fore, the best comparisons are between mass, radius and surface gravity, with comparisons using T_{eff} , luminosity or M_{bol} being of secondary importance. As stellar radii and surface gravities are quite sensitive to evolution and convection, they are particularly useful properties against which to compare theoretical predictions (Lacy et al. 2003). More detailed comparisons are, however, possible using T_{eff} s or luminosities.

6.1.2 Further work

Further work should be concentrated on low-mass stars (Shallis & Blackwell 1980; Clausen, Helt & Olsen 2001), high-mass stars (Herrero, Puls & Najarro 2002), metal-poor stars (such as those found in the LMC and SMC; see Sec. 6.3.4) and other types of stars which are poorly represented in the compilation of Andersen (1991). In particular, there exists a discrepancy between the masses of high-mass single stars found from spectroscopic and photometric observations, and the masses inferred from comparison with theoretical evolutionary models (Herrero et al. 1992; Herrero, Puls & Villamariz 2002). Burkholder, Massey & Morrell (1997) studied seven high-mass spectroscopic binaries, of which five are EBs, and found that careful analysis did not support this mass discrepancy. However, their study extended only to masses of about $15 M_{\odot}$, because EBs more massive than this usually exhibit major observational complications. Hilditch (2004) has found that the mass discrepancy disappears when several effects, including difficulties related to spectroscopic analysis and RV and T_{eff} determination, are allowed for. Major improvements in theoretical model atmospheres of high-mass stars has also helped the situation, but Herrero, Puls & Najarro (2002) find that there are still extremely large random differences between masses found using the two methods. This does suggest that the previous systematic effect has been explained and removed.

There are some dEBs known to contain stars of extremely high mass, for example WR 20a. This was discovered to be a WR star by Shara et al. (1991) and classified as WN7:h + WC by Shara et al. (1999). A spectroscopic orbit was provided by Rauw et al. (2004), who suggested that it should be monitored for eclipses as its minimum masses ($M \sin^3 i$; see Sec. 11.4) are very large. Bonanos et al. (2004) obtained a light curve and fitted it using the Wilson-Devinney code (Sec. 13.1.4), finding absolute masses of 83 and $82 M_{\odot}$ and radii of $21 R_{\odot}$, to accuracies of about 5%. Further observations and analyses of WR 20a are expected to be published very soon, but it must be remembered that the stars in this system are probably not detached and therefore not representative of single stars. Three more very high mass EBs have been found in the R 136 cluster in the LMC by Massey, Penny &

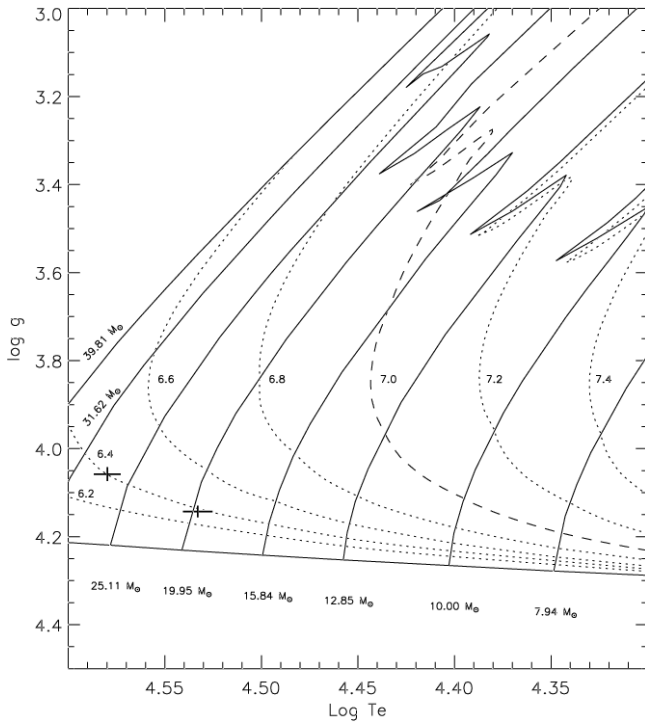


Figure 51. An example of a $T_{\text{eff}}\text{-log } g$ plot for the comparison between the properties of the high-mass dEB V3903 Sagittarii and the stellar models of Claret & Giménez (1992). Solid lines show the predictions of evolutionary models (initial masses are labelled) and dashed lines are isochrones (the ages are given as $\log_{10}(\text{years})$). Taken from Vaz et al. (1997).

Vukovitch (2002), but these may be very difficult to observe further due to the extreme stellar crowding in the cluster as viewed from ground-based telescopes.

Very few late-type dEBs have been found because such stars are small, so are less likely to eclipse, and dim, so it is less likely that their eclipsing nature is discovered (Clausen et al. 1998). An additional problem is that the light curves of late-type dEBs exhibit complexities due to the presence of large starspots, making accurate photometric parameters more difficult to obtain (Fig. 53). The Copenhagen Group has a research project to discover and analyse late-type dEBs (Clausen et al. 1998; Clausen 1998; Clausen, Helt & Olsen 2001). Initial results suggest that the mass-radius relation given by low-mass dEBs is shallower than that predicted by theoretical models (Clausen et al. 1999). This result is confirmed by Lastennet & Valls-Gabaud (2002), who found that this problem exists for well-studied low-mass dEBs. In many cases the masses and radii of the two components can be fitted by adopting a large metal abundance, suggesting that the observation of atmospheric metal abundances for these EBs may allow further conclusions to be drawn.

6.1.3 The difference between stars in binary systems and single stars

The properties of close binary stars and single stars cannot be assumed to be identical. Due to the effects of mutual irradiation, gravitationally generated tides and mass transfer, single stars are quite simply different to the stars in multiple systems. Therefore the comparison between the properties of dEBs and theoretical models of single stars must be restricted to the cases where it is reasonable to assume that the difference between the components of the dEB and single stars of the same mass, age and chemical composition are negligible. This should be the case for well-separated systems, but even for close binaries the modification of the properties of the stars can be minor.

Malkov (2003) found that the single-star mass-luminosity relation cannot be determined from dEBs. This analysis is open

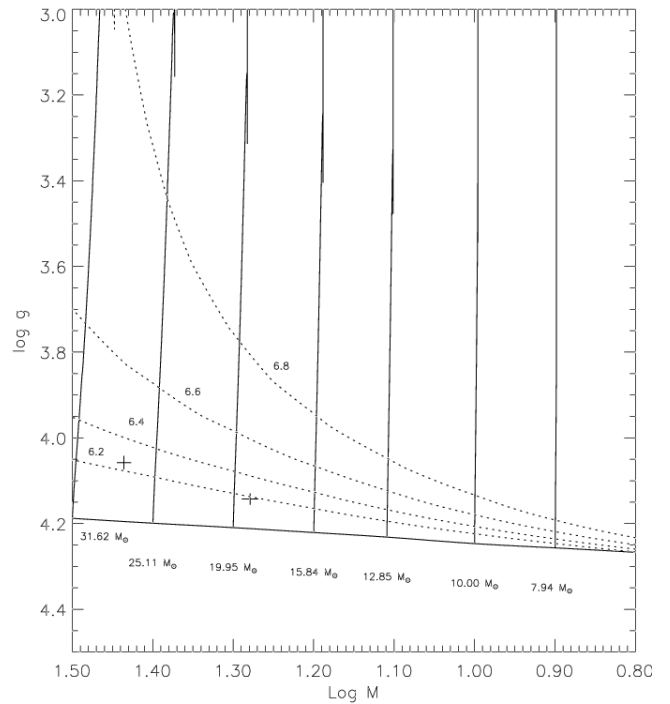


Figure 52. An example of a mass- $\log g$ plot for the same data and models as Fig. 51. Taken from Vaz et al. (1997).

to criticism for three reasons. Firstly, it was assumed that the components of wide binaries are representative of single stars, although the formation scenarios must have been a little different (Tohline 2002). Secondly, single and binary stars of similar spectral types were directly compared despite spectral type classifications being overly coarse for such a comparison. Thirdly, the components of well-studied dEBs were assumed to be representative of all dEBs, so no corrections for biases were made.

In a study of the discrepancies between theoretically predicted and observed apsidal motion rates, Claret & Giménez (1993) noted that this discrepancy was significant only for a small subset of stars, for which the components occupy more than about 60% of the volume of their Roche lobes at periastron (when the Roche lobes are at their minimum volume) (Fig. 54).

Lacy, Frueh & Turner (1987) have discovered that the secondary components of some dEBs (with late-A spectral types) have an anomalously low surface brightness compared to the primary components. This suggests that a systematic effect may exist which could be caused by binarity, but the study was based on only six dEBs, none of which had definitive light curves. Further investigation is required to confirm or disprove this anomaly.

6.2 Metal and helium abundances of nearby stars

The astrophysical parameters of dEBs allow the age and chemical composition to be derived from comparison with theoretical evolutionary models. This means that the chemical evolution of the Galaxy can be mapped by studying dEBs of different ages.

The low-mass dEB CM Draconis is important to the study of galactic chemical evolution because of its age. It has a motion through space characteristic of a Population II system, so is expected to be very old. As both components have very low masses (both about $0.2 M_{\odot}$; Lacy 1977b) they are completely convective, allowing an accurate determination of the helium abundance from the absolute dimensions of the stars (Paczynski & Sienkiewicz 1984). This has allowed the primordial helium abundance of the Galaxy to be found. An updated study, including YY Geminorum (in which the component masses are close to $0.6 M_{\odot}$) was given by Chabrier & Baraffe (1995).

Popper et al. (1970) used a similar method to find the helium

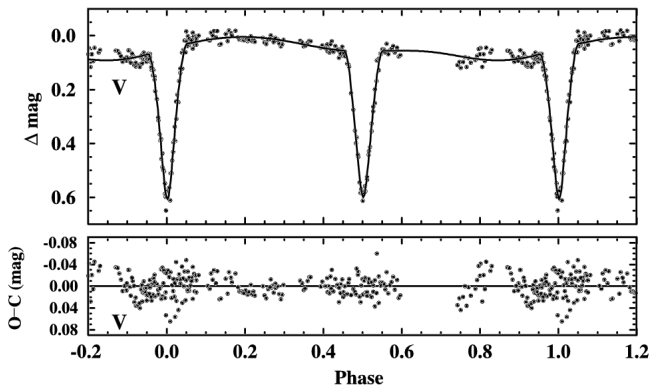


Figure 53. Wilson-Devinney model light curve fit of YY Gem to the data of Kron (1952). The out-of-eclipse variations are caused by large starspots. Taken from Torres & Ribas (2002).

abundance of seven more massive, nearby, young dEBs, finding a ratio of 0.12 between the number of helium and hydrogen atoms.

Ribas et al. (2000) used astrophysical properties of the Andersen (1991) list of well-studied dEBs to determine the chemical enrichment law – the relation between the metal abundance and helium abundance of the interstellar medium – and primordial helium abundance in the solar neighbourhood. They found that the chemical enrichment law, $Y(Z)$ where Y and Z are the abundances of helium and metals respectively, is $\Delta Y/\Delta Z = 2.2 \pm 0.8$. The corresponding primordial helium abundance is $Y_p = 0.225 \pm 0.013$. The advantages of their approach over the more usual method of determining both Y and Z from high-resolution spectroscopy is that it reflects the overall chemical composition of the stars, rather than the atmospheric composition, and that Y is difficult or impossible to determine for many stars (including the Sun) from spectroscopic observations alone.

6.3 dEBs as standard candles

An important astrophysical function of dEBs is that accurate distances can be calculated for them from their astrophysical properties. There are several ways to determine distances to dEBs, and the most reliable of these are calibrated directly from trigonometrical parallax measurements and/or interferometric observations of nearby stars. Using current telescopes, dEBs can give reliable and empirical distances for stellar systems from nearby star clusters to Local Group galaxies. This distance limit is currently being pushed out to more remote galaxies, for example M33 and M31 (see below). The methods of determining the distance to dEBs are discussed below.

All methods of distance determination require the measurement of reliable reddening-free apparent magnitudes. The effect of interstellar reddening on the final distance can be large, but can be minimised by using IR photometry (see Sec. 12.1). The apparent magnitudes used must be both precise and accurate. The best sources for these data are well-calibrated large-area surveys, for which the data is both precise and very homogeneous. One good source is the Tycho experiment on board the Hipparcos space satellite (Perryman et al. 1997), which observed the entire sky in the broad-band B_T and V_T passbands down to a limiting magnitude of $V \approx 11.5$. B_T and V_T data can be transformed to the standard Johnson system using the calibration of Bessell (2000). An excellent source of near-IR JHK photometry is the Two Micron All Sky Survey (2MASS; Kleinmann et al. 1994) which has web-based database access⁵. Another source of near-IR data is the DEep Near Infrared Survey⁶ (DENIS), which has surveyed the entire Southern Hemisphere sky in the IJK pass-

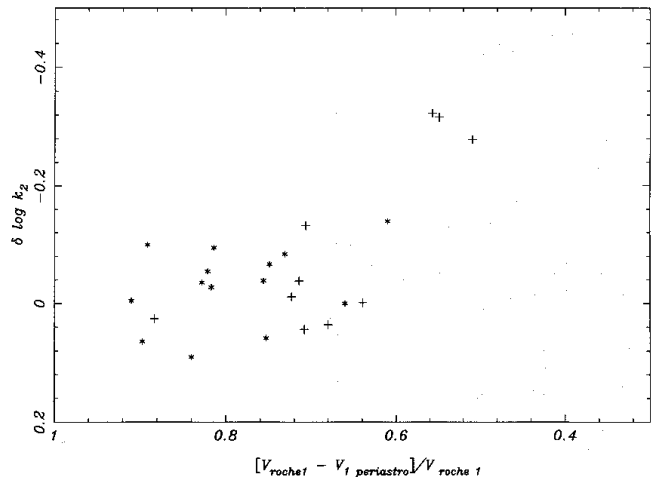


Figure 54. Plot of the difference between the theoretically predicted and observed central condensations of the components of dEBs against the fraction of the volume of the Roche lobe not filled by the primary star at periastron (Claret & Giménez 1993).

bands to limiting magnitudes of $J = 16.5$, $K = 14.0$ and $I = 18.5$, and also has web-based database access⁷.

6.3.1 Distance determination using bolometric corrections

The most common way of finding the distance to a dEB involves the use of BCs (Sec. 1.3.5), e.g., Munari et al. (2004) and Hensberge, Pavlovski & Verschueren (2000). Knowledge of the stellar T_{eff} s and the radii, R , of the stars means that the luminosities, L , can be calculated using the formula which defines T_{eff} :

$$L = 4\pi\sigma_{\text{SB}}R^2T_{\text{eff}}^4 \quad (48)$$

where $\sigma_{\text{SB}} = 5.67040(4) \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan-Boltzmann constant. The M_{bol} s of the two stars can then be calculated using the formula

$$M_{\text{bol}} = M_{\text{bol},\odot} - 2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right) \quad (49)$$

where $M_{\text{bol},\odot}$ and $L_{\odot}$ are the M_{bol} and luminosity of the Sun. Whilst there are no defined values for $M_{\text{bol},\odot}$ and $L_{\odot}$, they are usually taken to be $M_{\text{bol},\odot} = 4.75$ and $L_{\odot} = 3.826 \times 10^{26} \text{ W}$ (Zombeck 1990). M_{bol} s only have significance if they are accompanied by the values of $M_{\text{bol},\odot}$ and $L_{\odot}$ used to calculate them.

The M_{bol} s of the two stars then must be transformed to the absolute magnitudes of the stars in a passband for which the apparent magnitude of the system is available. This is done using

$$M_{\lambda} = M_{\text{bol}} - BC_{\lambda} \quad (50)$$

where BC_{λ} represents the bolometric correction and M_{λ} is the absolute magnitude in passband λ . The absolute magnitudes of the two stars, M_{λ}^A and M_{λ}^B , are then combined to determine the passband-specific absolute magnitude of the overall dEB using

$$M_{\lambda}^{\text{TOT}} = -2.5 \log_{10}(10^{-0.4M_{\lambda}^A} + 10^{-0.4M_{\lambda}^B}) \quad (51)$$

The distance, d (pc), is then calculated from the absolute magnitude of the dEB and the apparent magnitude, m_{λ} , using

$$\log_{10} d = \frac{m_{\lambda} - A_{\lambda} - M_{\lambda}^{\text{TOT}} + 5}{5} \quad (52)$$

where A_{λ} is the total interstellar extinction in passband λ .

The difficulty with this method lies mainly in complications in obtaining BCs, which depend on T_{eff} and surface gravity, but also on the photospheric metal abundance of a star. BCs can be found empirically by means of interferometric observations of

⁵ <http://www.ipac.caltech.edu/2mass/>

⁶ <http://www-denis.iap.fr/denis.html>

⁷ <http://cdsweb.u-strasbg.fr/DENIS/catDENISP.html>

nearby stars with accurate trigonometrical parallaxes, or by UV, optical and IR spectrophotometry (e.g., Code et al. 1976; Habets & Heintze 1981; Malagnini et al. 1986; Flower 1996), but the resulting BCs are subject to observational uncertainties and are often only relevant to stars with an approximately solar metal abundance. Alternatively, theoretical BCs can be calculated using model atmospheres (e.g., Bessell, Castelli & Plez 1998; Girardi et al. 2002). The advantage of this approach is that observational uncertainties are entirely avoided and BCs can be calculated for a wide range of chemical compositions. However, using theoretical BCs does introduce a dependence on stellar models into the resulting distance, so theoretical BCs should be avoided, if possible, or carefully compared to empirical BCs. Further discussion on BCs can be found in Sec. 1.3.5.

BCs are quite uncertain for hot stars because these stars emit a large fraction of their radiation in the UV, where it is strongly absorbed by the interstellar medium. Therefore this distance determination method is not a good one for OB stars (Harrises, Hilditch & Howarth 2003).

The zeropoint of the BC scale is set by the assumption of certain values for $M_{\text{bol},\odot}$, $L_{\odot}$ and $\text{BC}_{\odot}$. This means that BCs are only relevant if they are applied to M_{bol} s which have been calculated using the same values for $M_{\text{bol},\odot}$ and $L_{\odot}$. In this case, the zeropoints coincide and a meaningful answer is found (Bessell, Castelli & Plez 1998). The use of different zeropoints means that the final answer is meaningless (e.g., Munari et al. 2004).

The measurement of distances using BCs requires that the T_{eff} s of the stars must be derived consistently with the fundamental definition of T_{eff} ; so the T_{eff} scale has known small or negligible systematic errors. Determination of T_{eff} is discussed in Sec. 4.4. This constraint is important as it can be very difficult to quantify systematic errors in T_{eff} scales.

A good example of this distance determination technique, using empirical BCs, is for V578 Monocerotis (Hensberge, Pavlovski & Verschueren 2000). This method has also been discussed by Clausen (2004); he finds that the main uncertainty comes from the calibrations for empirical BCs.

6.3.2 Distances using surface brightness calibrations

In this method, the angular diameter of each component of an EB is estimated using the apparent magnitude of the star and calibrations between surface brightness and one or several photometric properties. The linear radius of each star is known from a light and RV curve analysis, and comparison between this and the angular diameter of each star gives its distance (Lacy 1977a). One distance estimate is obtained for each star – the two estimates should be in agreement – and these can be combined using weighted means, although careful consideration of the uncertainties and the correlations in the results is necessary.

Surface brightness relations have been discussed in Sec. 1.3.6 and generally comprise a calibration between some measure of the visual surface brightness of a star (in magnitudes) and an observed photometric index. Lacy (1977a) adopted the Barnes-Evans relation (Barnes & Evans 1976) between F_V and the $V-R$ index. F_V is defined to be

$$F_V = \log T_{\text{eff}} + 0.1\text{BC} = 4.2207 - 0.1V_0 - 0.5 \log \phi \quad (53)$$

where BC is the bolometric correction, V_0 is the dereddened apparent visual magnitude and ϕ is the angular diameter in milliarcseconds. F_V is given as linear functions of $(V-R)_0$ for different types of star. The distance (in parsecs) is then found from

$$d = 9.3048 \frac{R}{\phi} \quad (54)$$

where R is the stellar radius (in solar units). Individual V_0 magnitudes and $(V-R)_0$ indices must be determined for the two stars in the dEB from the apparent V magnitude, light curves in the V and R passbands, and a known visual absorption, A_V .

Lacy (1977a) applied the above method to nine dEBs for which accurate parallaxes were available, finding that distance moduli derived using the Barnes-Evans relation had accuracies of

about 0.2 mag and were in agreement with distances found using the parallax measurements. Lacy (1978) applied the method to CW Cephei, V453 Cygni and AG Persei, all members of nearby open clusters or associations, and found that the distances derived were in agreement with, although slightly larger than, distances found from main-sequence fitting analyses of the stellar associations of which the dEBs were members. Lacy (1979) then applied the tested method to 48 dEBs, finding that their absolute magnitudes were in good agreement with theoretical predictions.

Semeniuk (2000) has compared the method of Lacy (1977a) to other distance determination methods and found that it is robust as long as it is used on well-behaved dEBs; single-star surface brightness relations are not applicable to interacting binaries.

6.3.3 Distance determination by modelling stellar spectral energy distributions

This distance determination method was introduced by Fitzpatrick & Massa (1999) and has been used to find the distance to four EBs in the LMC: HV 2274 (Guinan et al. 1998; Ribas et al. 2000), HV 982 (Fitzpatrick et al. 2002), EROS 1044 (Ribas et al. 2002) and HV 5936 (Fitzpatrick et al. 2003).

The principle of this method is to determine the physical parameters of an early-type EB by fitting Kurucz ATLAS9 theoretical model atmospheres to UV and optical spectrophotometry. The observed spectral energy distribution of an EB at the Earth is a function of wavelength, λ ;

$$f_{\lambda,\oplus} = \frac{R_A^2 F_{A,\lambda} + R_B^2 F_{B,\lambda}}{d^2} \times 10^{-0.4A_\lambda} \quad (55)$$

where $F_{i,\lambda}$ ($i = A, B$) are the emergent fluxes at the surfaces of the two stars, R_i are the radii of the stars and A_λ is the total extinction along the line of sight of the EB. Therefore

$$f_{\lambda,\oplus} = \left(\frac{R_A}{d} \right)^2 \left[F_{A,\lambda} + \left(\frac{R_B}{R_A} \right)^2 F_{B,\lambda} \right] 10^{-0.4E_{B-V} - k(\lambda-V) + R_V} \quad (56)$$

where E_{B-V} is the reddening, $k(\lambda - V) \equiv \frac{E(\lambda-V)}{E_{B-V}}$ is the extinction curve and $R_V = \frac{A_V}{E_{B-V}}$ is the ratio of selective to total absorption in the V passband.

Synthetic spectra from the model atmospheres are fitted to the observed $f_{\lambda,\oplus}$, using nonlinear least squares algorithms, to derive values for $\left(\frac{R_A}{d} \right)^2$, $F_{i,\lambda}$, E_{B-V} and $k(\lambda - V)$. The distance estimate is found directly from $\left(\frac{R_A}{d} \right)^2$ and the radius of the primary star. The ATLAS9 model atmospheres, which represent the surface fluxes, $F_{i,\lambda}$, depend on T_{eff} , surface gravity, metallicity and microturbulence velocity. Fitzpatrick & Massa (1999) found that the ATLAS9 predictions provide a match to observations at a level consistent with current uncertainties in spectrophotometric observations. In addition, it can be assumed that the metallicity and microturbulence velocity of both components is the same. The ratio of the T_{eff} s of the stars is also known from light curve analysis. Thus there are only five parameters needed to specify ATLAS9 model spectral energy distributions of the two stars.

6.3.4 Recent results for the distance to EBs

The main research area currently involving the observation and analysis of EBs is to use their properties as standard candles to determine the distances to Local Group galaxies. The first detailed photometric study of a dEB outside the Milky Way Galaxy was that of Jensen, Clausen & Giménez (1988), who provided the first CCD light curves of dEBs in the Magellanic Clouds.

The Copenhagen (Denmark) group has continued to study dEBs in the Magellanic Clouds (see Clausen 2000 and Clausen et al. 2003) in order to test the predictions of theoretical stellar evolutionary models in the low-metallicity environment of the Magellanic Clouds. The Villanova (USA) group (Guinan et al. 1998; Ribas et al. 2000, 2002; Fitzpatrick et al. 2002, 2003) are continuing their efforts (detailed above). The Mount John (New Zealand) group are also running an observing program to obtain

good CCD light curves of Magellanic Cloud EBs (e.g., Bayne et al. 2004). An impressive observing program has been undertaken by Harries, Hilditch & Howarth (2003; Hilditch, Harries & Howarth 2004, 2005), who used the 2dF multi-object spectrograph at the Anglo-Australian Telescope to obtain RV curves of about 100 high-mass short-period EBs in the SMC.

Recent large-scale photometric surveys have targeted the Magellanic Clouds, obtaining a large number of light curves of distant stars in order to detect and analyse the brightening effects caused by gravitational microlensing phenomena. The Optical Gravitational Microlensing Experiment (OGLE⁸) group have obtained a huge amount of data, through three phases of increasingly sophisticated instrumentation, which is of sufficient quality to derive preliminary results for several thousand EBs. Additional data have also been obtained by the Microlensing Observations in Astrophysics (MOA⁹), Expérience pour la Recherche d'Objets Sombres (EROS¹⁰) and MAssive Compact Halo Objects (MACHO¹¹) groups. As a byproduct of these searches, over five thousand EBs have been detected in the Magellanic Clouds.

Wyithe & Wilson (2001, 2002) have investigated EBs found in the SMC and suggested that close binaries, including semidetached systems, are very good distance indicators. They are better than dEBs because, given the same quality and quantity of photometric observations, the properties of the system tend to be more accurately determined (Wilson 2004). Graczyk (2003) agrees that close EBs are more useful as the proximity effects in their light curves give useful constraints on the properties of the systems, in particular third light and mass ratio. It is also clear that close binaries spend a greater fraction of their time in eclipse, so a given set of photometric observations will contain more datapoints inside eclipses, and that RV curves are more easy to obtain as the velocity semiamplitudes are greater.

The determination of distance from the study of EBs is being applied to more distant galaxies as observing time on large telescopes becomes more easily available. The large Local Group galaxies M31 and M33 (which are gravitationally bound; Guinan 2004) have been targeted by the DIRECT project¹² (Kalužny et al. 1998, 1999; Stanek et al. 1998, 1999; Mochejska et al. 1999; Macri et al. 2001) and about 130 EBs have been detected, along with about 600 Cepheids (Macri 2004a). The DIRECT group have begun RV observations of four dEBs in M31 and M33, using the 10 m Keck telescopes (Macri 2004b). I. Ribas is also independently leading a research program to further study some EBs discovered by DIRECT, using the 2.5 m Isaac Newton Telescope to obtain light curves and the 8 m Gemini telescopes for spectroscopic observations (Ribas et al. 2004).

6.4 dEBs in stellar systems

The metal abundance, helium abundance, age or distance are often known for nearby stellar open clusters and associations (see Sec. 8). If a dEB is a member of the cluster, then it is possible to derive accurate masses, radii and T_{eff} s for two stars of known age, distance or chemical composition. These data can then be used to provide a detailed and discriminating test of theoretical stellar evolutionary models. Alternatively, the properties of the dEB can be used to find the age, chemical abundance or distance of the cluster of stars as a whole (e.g., Clausen & Giménez 1991).

The properties of stellar open clusters are generally derived by comparison with the predictions of stellar evolutionary models. The same set of models should be adopted for comparison with the properties of dEBs as are used for the derivation of the properties of their parent cluster. Ideally, models of the same age and chemical composition should be able to simultaneously accurately predict the photometric properties of the cluster and the physical properties of the dEB.

The study of EBs has long been known to be facilitated by their membership of a stellar cluster. Lists of EBs in open clusters have been presented by Kraft & Landolt (1959), Sahade & Dávila (1963) and Clausen & Giménez (1987; Clausen 1996b; Giménez & Clausen 1996).

6.4.1 Results from the literature on dEBs in open clusters

A research project on EBs in open clusters has been undertaken by Milone & Schiller (1991) and collaborators at the Rothney Astrophysical Observatory (Canada). The status of the project has been discussed by Milone & Schiller (1984, 1988). They have studied the dEBs V818 Tauri (HD 27130) in the Hyades (Schiller & Milone 1987) and DS Andromedae in NGC 752 (Schiller & Milone 1988); OX Cassiopeiae was discovered to be a nonmember of NGC 381 by Crinklaw & Etzel (1989). The contact binary Heinemann 235, in NGC 752, was also studied (Milone et al. 1995) as was the curious case of SS Lacertae (Milone et al. 2000), a dEB member of NGC 7209 which no longer shows eclipses due to the perturbations of a third body in the system (Torres 2001). It was stated by Milone & Schiller (1991) that analyses of QX Cassiopeiae (NGC 7790) and CN Lacertae (NGC 7209) were close to completion, but these are yet to be published.

The study of well-detached binaries in open clusters was stated to be able to provide strong constraints on stellar evolutionary theory by Lastennet, Valls-Gabaud & Oblak (2000). These authors considered the Hyades visual binaries 51 Tauri and θ^2 Tauri (Sec. 5.2), and the dEBs V818 Tauri (a Hyades member) and CW Cephei (a member of the Cepheus OB3 association). They found that predictions of the Padova stellar evolutionary models (Sec. 3.2.4) were unable to fit the components of V818 Tau in the mass-radius diagram, a conclusion also reached by Pinsonneault et al. (2003). From consideration of the photometric study of this dEB (Schiller & Milone 1987) I would suggest that the problem is probably caused by the analysis of low-quality observations with inadequate consideration of the uncertainties of the resulting photometric parameters.

Lebreton, Fernandes & Lejeune (2001) derived the helium content and the age of the Hyades open cluster from a comparison between the predictions of the CESAM stellar evolutionary models (see Sec. 3.2.6) and a mass-luminosity relation derived from three double-lined spectroscopic visual binaries (51 Tauri, Finsen 342 and θ^2 Tauri; Torres, Stefanik & Latham 1997a, 1997b, 1997c), a single-lined spectroscopic visual binary (θ^1 Tauri; Torres, Stefanik & Latham 1997c) and the dEB V818 Tauri (referred to as vB 22 by the authors). They were hampered by a correlation between the helium and metal abundances and by the influence of the mixing length parameter, α_{MLT} , but were able to conclude that the helium abundance was somewhat lower than expected for a given metal abundance, suggesting that the chemical enrichment law in the Hyades is slightly anomalous.

Hurley, Pols & Tout (2000) have found that an overshooting parameter value of $\alpha_{\text{OV}} \approx 0.12$ is supported by the consideration of dEBs in open clusters.

Probably the best-known analysis of a dEB in a stellar cluster is that of OGLE GC 17 in the globular cluster ω Centauri (Thompson et al. 2001). From a relatively limited amount of observational data – due to the dEB being dimmer than 17th magnitude in the *I* passband – these authors were able to derive masses accurate to 7% and radii accurate to 3%, partially because the dEB exhibits total eclipses. Thompson et al. calibrated several IR surface brightness relations and used these to find a distance to OGLE GC 17 of 5360 ± 300 pc. Comparison with theoretical stellar evolutionary models gave the age of the dEB to be between about 13 and 17 Gyr. Note that very accurate masses are not vital for the determination of distance because the masses of the stars are not needed for distance calculation. The need for spectroscopy is to find the separation of the two stars, which is better determined than the masses for the same observational data. Accurate masses are needed for a comparison between the properties of the dEB and the predictions of theoretical stellar evolutionary models. Thompson et al. state that improved observations will be able to give a significantly more accurate distance to ω Cen from

⁸ <http://bulge.princeton.edu/~ogle/>

⁹ <http://www.physics.auckland.ac.nz/moa/>

¹⁰ <http://eros.in2p3.fr/>

¹¹ <http://www.macho.mcmaster.ca/>

¹² <http://cfa-www.harvard.edu/~kstanek/DIRECT/>

study of the dEB OGLE GC 17, and these authors have obtained further observations (Kaluźny et al. 2002).

7 TIDAL EFFECTS

The mutual gravitational attraction between binary stars causes several dynamical phenomena to occur:–

- The orbits of binary stars continuously decrease in eccentricity, so close binary orbits can become circular.
- The angular rotational velocities of the component stars move towards that of the orbit. As stars are always born with rotational velocities greater than this value (due to the conservation of angular momentum as the stellar radii decrease during evolution towards the ZAMS) their rotational velocities decrease towards synchronization.
- Eccentric binary orbits change orientation continuously (the longitude of periastron increases). This effect is called apsidal motion and can be very useful as it depends on the internal structure of the stars, so the degree of central condensation of stars can be determined observationally.
- The axes of rotation and orbital motion tend to align perpendicular to the plane of the orbit.

7.1 Orbital circularization and rotational synchronization

Several theories exist of the magnitude, and indeed reality, of the dynamical effects which cause orbital circularization and rotational synchronization. These theories, however, do not in general agree with each other or with all observations, and additional effects exist which have not yet been quantitatively investigated.

The equilibrium shapes of the surfaces of single stars are accurately described by equipotential surfaces, where the potential due to gravitational attraction is modified by the effects of rotation. Binary stars have an additional potential due to the gravitational attraction of the other component, causing the stellar surfaces to bulge outwards in two places: towards and away from the other star. If the orbit is circular and the star's rotation is synchronous with the orbit, this bulge is static and has no effect on the dynamics of the stars. If the orbit is eccentric and/or the star has an asynchronous rotational velocity, this bulge does not point straight to the companion star. As stars consist of viscous material, the bulge is pushed by rotation away from the other star and so exerts a force on its own star, due to the gravitational attraction between the bulge and the companion star. This force acts to bring the rotation of the stars towards the synchronous velocity, and to decrease the eccentricity of elliptical orbits.

It has been known for many years that binaries with short periods tend to have circular orbits (e.g., Campbell 1910) and several theories have been developed to explain and quantify this phenomenon.

7.1.1 The theory of Zahn

Zahn (1970, 1975, 1977, 1978) considered several physical mechanisms which produce tidal friction in close binary stars. The equilibrium tide is the hydrostatic adjustment of the structure of the star to the perturbing force from the companion. The dynamical tide is the response to the equilibrium tidal force; it depends on the properties of the star and may be resonant over the volume of the star. The most important tidal evolution mechanism in stars with a convective envelope is turbulent viscosity retarding the equilibrium tide. The most important mechanism in stars with a radiative envelope is radiative damping on the dynamical tide (Zahn 1984).

The timescales of orbital circularization and rotational synchronization for stars with convective envelopes are derived, for a single star (in years) to be

$$\tau_{\text{circ}}^{\text{conv}} = \frac{1}{84q(1+q)k_2} \left(\frac{MR^2}{L} \right)^{\frac{1}{3}} \left(\frac{a}{R} \right)^8 \quad (57)$$

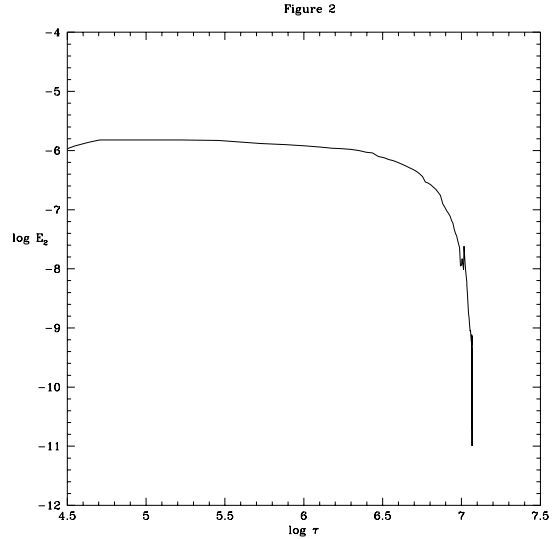


Figure 55. Evolution of the logarithm of the tidal constant E_2 , for a $15.8 M_{\odot}$ model star with core overshooting. Taken from Claret & Cunha (1997).

$$\tau_{\text{synch}}^{\text{conv}} = \frac{1}{6q^2k_2} \left(\frac{MR^2}{L} \right)^{\frac{1}{3}} \frac{I}{MR^2} \left(\frac{a}{R} \right)^6 \quad (58)$$

where q is the mass ratio, M is the mass, R is the radius, L is the luminosity, I is the moment of inertia, a is the semimajor axis, M , R , L and I are in solar units, and k_2 is the apsidal motion constant of the star (Zahn 1977, 1978). Note the very strong dependence on the fractional stellar radius, $\frac{a}{R}$.

Due to uncertainties in the treatment of several physical effects, the formulae are inexact. These approximations are “probably well within the error margin” (Zahn 1977, 1978):

$$\tau_{\text{circ}}^{\text{conv}} \approx 10^6 \frac{1}{q} \left(\frac{1+q}{2} \right)^{\frac{5}{3}} P^{\frac{16}{3}} \quad (59)$$

$$\tau_{\text{synch}}^{\text{conv}} \approx 10^4 \left(\frac{1+q}{2q} \right)^2 P^4 \quad (60)$$

where the orbital period, P , is in days.

For stars containing a convective core and a radiative envelope, the theory is more complex and gives the equations

$$\tau_{\text{circ}}^{\text{rad}} = \frac{1}{5} \frac{1}{2^{5/3}} \left(\frac{R^3}{GM} \right)^{\frac{1}{2}} \frac{I}{MR^2} \frac{1}{q^2(1+q)^{5/6}} \frac{1}{E_2} \left(\frac{a}{R} \right)^{\frac{17}{2}} \quad (61)$$

$$\tau_{\text{synch}}^{\text{rad}} = \frac{2}{21} \left(\frac{R^3}{GM} \right)^{\frac{1}{2}} \frac{1}{q(1+q)^{11/6}} \frac{1}{E_2} \left(\frac{a}{R} \right)^{\frac{21}{2}} \quad (62)$$

where G is the gravitational constant and the constant E_2 depends on the tidal torque and must be determined from stellar structure theory. No suitable approximations for E_2 exist, mainly because it is very sensitive to the mass and evolutionary state of the star (see Fig. 55). In fact E_2 is proportional to the seventh power of the ratio of the radii of the convective core and the whole star. Tabulations of E_2 are provided by Zahn (1975) and more extensively and accurately by Claret & Cunha (1997).

Zahn (1989) revisited the theory of the equilibrium tide and updated the resulting timescale equations. He suggested that convective effects could cause the orbital circularization timescale to depend on the orbital period according to $\tau_{\text{circ}} \propto P^{\frac{10}{3}}$ for stars with convective envelopes. Goldman & Mazeh (1991) have developed this further and found that it may be a better match to observations.

Zahn & Bouchet (1989) investigated the problem of dynamical evolution of binary stars during the PMS evolutionary phase. This is an important effect because of the strong dependence of the magnitude of tidal forces on the separation of the component stars. During PMS evolution the stars have much greater radii,

and it appears that the majority of the dynamical evolution of close binary stars occurs during the PMS phase rather than the MS phase. Fig. 56 shows the evolution in time of the orbital period, P , eccentricity, e and the ratio of the orbital and rotational velocities, $\frac{\Omega}{\omega}$, for a close binary composed of two $1 M_{\odot}$ stars. The initial parameters were arbitrarily selected and correspond to a well separated system. It is notable that e decreases from an initial value of 0.3 to 0.005 by the time the stars have evolved to the ZAMS. The local maximum of $\frac{\Omega}{\omega}$ at that point is due to the ZAMS being (by definition) the point at which stellar radii attain their minimum value.

7.1.2 The theory of Tassoul & Tassoul

Tassoul (1987) developed a theory based on a purely hydrodynamical mechanism which causes orbital circularization and rotational synchronization. The derived spin-down timescale can be expressed in two equivalent ways:

$$\tau_{\text{sd}} = \frac{1.44 \times 10^{-N/4}}{q(1+q)^{3/8}} \left(\frac{L_{\odot}}{L}\right)^{\frac{1}{4}} \left(\frac{M_{\odot}}{M}\right)^{\frac{1}{8}} \left(\frac{R}{R_{\odot}}\right)^{\frac{9}{8}} \left(\frac{a}{R}\right)^{\frac{33}{8}} \quad (63)$$

$$\tau_{\text{sd}} = 535 \times 10^{-\frac{N}{4}} \frac{1+q}{q} \left(\frac{L_{\odot}}{L}\right)^{\frac{1}{4}} \left(\frac{M}{M_{\odot}}\right)^{\frac{5}{4}} \left(\frac{R_{\odot}}{R}\right)^3 \left(\frac{P}{\text{d}}\right)^{\frac{11}{4}} \quad (64)$$

where N depends on the turbulent viscosity. If eddy viscosity in radiative envelopes is ignored then $N = 0$. For turbulent convective envelopes, N is probably between 8 and 12 (Tassoul 1988). Tassoul states that τ_{sync} can be conservatively assumed to be about one order of magnitude larger than $\tau_{\text{spin down}}$. This mechanism is a relatively long-range force [proportional to $(\frac{a}{R})^{33/8}$] compared to the theory of Zahn.

Tassoul (1988) considered the timescale for orbital circularization. This can be obtained by multiplying τ_{sync} by the ratio of the orbital and rotational angular momenta of the stars, to give

$$\tau_{\text{circ}} = \frac{14.4 \times 10^{-N/4}}{(1+q)^{11/8} r_g^2} \left(\frac{L_{\odot}}{L}\right)^{\frac{1}{4}} \left(\frac{M_{\odot}}{M}\right)^{\frac{1}{8}} \left(\frac{R}{R_{\odot}}\right)^{\frac{7}{8}} \left(\frac{a}{R}\right)^{\frac{49}{8}} \quad (65)$$

$$\tau_{\text{circ}} = 9.4 \times 10^{4-\frac{N}{4}} \frac{(1+q)^{\frac{2}{3}}}{r_g^2} \left(\frac{L_{\odot}}{L}\right)^{\frac{1}{4}} \left(\frac{M}{M_{\odot}}\right)^{\frac{23}{12}} \left(\frac{R_{\odot}}{R}\right)^5 \left(\frac{P}{\text{d}}\right)^{\frac{49}{12}} \quad (66)$$

where r_g is the radius of gyration of the star (for a homogeneous sphere $r_g^2 = \frac{2}{5}$; for centrally condensed stars $r_g^2 \approx 0.01$ to 0.1).

Tassoul (1990, 1995, 1997) and Tassoul & Tassoul (1990) consider the tidal evolution theory of Tassoul and conclude that its main features are generally confirmed by observations, particularly of high-mass circular-orbit binary stars, with orbital periods of tens of days, which disagree with the theory of Zahn.

7.1.3 The theory of Press, Wiita & Smarr

Press, Wiita & Smarr (1975) considered the turbulence induced in the radiative envelopes of binary stars to derive:

$$\tau_{\text{circ}}^{\text{rad}} \approx \frac{125}{242} \frac{R_T}{K_{\mu} \delta \omega_1} (1-e^2)^5 \left(\frac{a}{R}\right)^{11} \left[\frac{M_1^3}{M_2^2(M_1+M_2)} \right] \quad (67)$$

$$\tau_{\text{sync}}^{\text{rad}} \approx \frac{75}{224} \frac{R_T \alpha}{K_{\mu} \delta n} (1-e^2)^{\frac{9}{2}} \left(\frac{a}{R}\right)^9 \left(\frac{M_1}{M_2}\right)^3 \quad (68)$$

where R_T is a dimensionless constant approximately equal to unity, ω_1 is the rotational frequency of the star, K_{μ} is a function of the mean turbulent viscosity and is roughly equal to 0.025, $n = \frac{2\pi}{P}$ where $\delta \approx \max[(\frac{\omega_1}{n} - 1), e]$, M_1 and M_2 are the masses of the component stars, and α is the internal structure constant which defines the star's moment of inertia through the equation $I = \alpha M_1 r_1^2$. It is notable that all quantities, except R_T (which is of order unity), are directly observable in eclipsing systems.

7.1.4 The theory of Hut

Hut (1981) studied the tidal evolution of close binary stars for a 'weak friction' model where the stars' shapes are static but

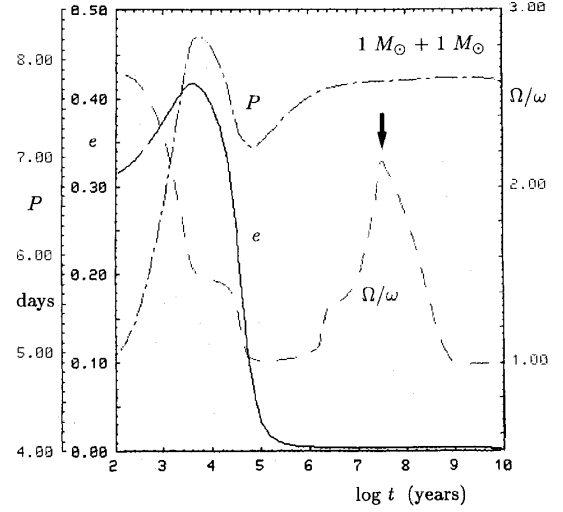


Figure 56. The evolution of the orbital period, P , eccentricity, e and the ratio of the orbital and rotational velocities, $\frac{\Omega}{\omega}$, for a close binary composed of two $1 M_{\odot}$ stars. The downward arrow indicates the time at which the ZAMS is reached. Taken from Zahn & Bouchet (1989).

the bulge is displaced from its position in static equilibrium. This model is simple but allows the derivation of straightforward equations for the timescales of orbital circularization, rotational synchronization and alignment of the axes of rotation and orbital motion. The derived timescales are dependent on the ratio of orbital and rotational angular momenta at the stable equilibrium configuration, γ_0 , where

$$\gamma_0 = \frac{q}{1+q} \frac{1}{r_g^2} \left(\frac{a_0}{R}\right)^2 \quad (69)$$

where a_0 is the orbital semimajor axis at the equilibrium state.

The characteristic timescale for the change of dynamical parameters is

$$T_* = \frac{1}{k_2 q (1+q)} \left(\frac{a}{R}\right)^8 \frac{P_0}{\tau} P_0 \quad (70)$$

where P_0 is the orbital period of the equilibrium state and τ is the (constant) time lag of the tides. Using the quantities γ_0 and T_* , the timescales of rotational synchronization, orbital circularization, and axial alignment are

$$\tau_{\text{sync}} = \frac{1}{3(\gamma_0 - 3)} T_* \quad (71)$$

$$\tau_{\text{circ}} = \frac{2}{21} T_* \quad (72)$$

$$\tau_{\text{inc}} = \frac{2}{3(\gamma_0 - 1)} T_* \quad (73)$$

7.1.5 Comparison with observations

Firstly, the above timescales are applicable to individual stars. The overall timescale for a binary star must be calculated using

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{prim}}} + \frac{1}{\tau_{\text{sec}}} \quad (74)$$

(Claret, Giménez & Cunha 1995) where τ is the characteristic timescale and τ_{prim} and τ_{sec} are the timescales for the stars.

Several attempts have been made to compare tidal theories with observations, concentrating mainly on the age-dependent cutoff period, P_{cut} , below which all binary stars in a co-evolutionary sample exhibit circular orbits. This cutoff period has been determined for populations of binaries in the nearby intermediate-age open clusters Hyades and Praesepe (Mayor & Mermilliod 1984; Burki & Mayor 1986) and M67 (Mathieu, Latham & Griffin 1990), the old open cluster NGC 188 (Mathieu, Meibom & Dolan 2004) and for Galactic Population I stars

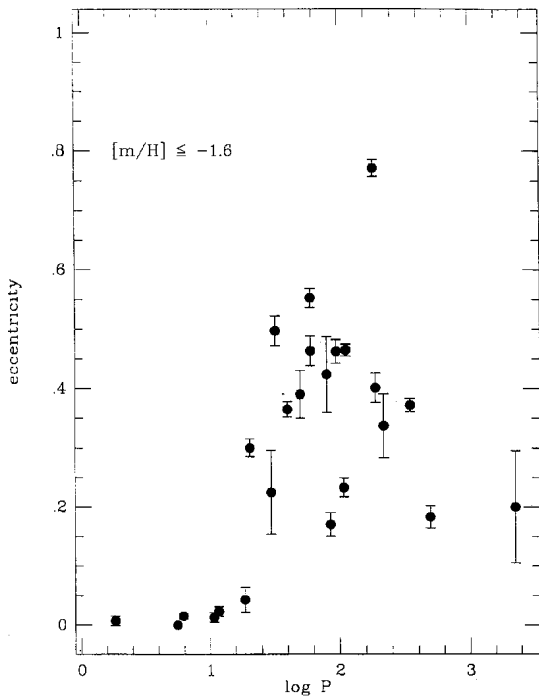


Figure 57. Eccentricity versus logarithmic period distribution of 22 metal-poor halo binary stars from Latham et al. (1992), who concluded that P_{cut} is around 19 days for Population I stars.

(Latham et al. 1992; see Fig. 57). The PMS tidal evolution described by Zahn & Bouchet (1989), twinned with the MS evolution theorised by Zahn (1977), would cause all these groups of binaries to display very similar values of P_{cut} , between around seven and nine days, as almost all tidal changes occur before the ZAMS. The observations display a greater range of values of P_{cut} , particularly for NGC 188 and the Pop I stars, for which the cutoff periods are 15 and 19 days respectively. It is therefore clear that tidal effects are important on the MS as well as before the ZAMS.

Giuricin, Mardirossian & Mezzetti (1984a, 1984c, 1984d, 1985) compiled lists of eclipsing and non-eclipsing binary stars from the literature and compared their rotational properties to predictions from the theory of Zahn. They found good agreement for late-type stars (with convective envelopes). They also found that there existed early-type binaries in a state of rotational synchronization with periods greater than that allowed by the theory of Zahn. Giuricin, Mardirossian & Mezzetti (1984b) investigated the orbital circularization characteristics of the same binaries and concluded that the observations were compatible with the theory of Zahn. Koch & Hrivnak (1981) found that Zahn's theory could explain the dynamics of radiative-envelope binaries with small eccentricities and orbital periods below about 20 days.

Claret, Giménez & Cunha (1995) investigated the theory of Tassoul by integration of the relevant differential equations, and concluded that it was in satisfactory agreement with the observations of rotational synchronization and orbital circularization. However, they indicate that the validity of the Tassoul theory has not yet been fully confirmed. Claret & Cunha (1997) treated the Zahn theory in the same way and found that it predicted the majority of the observational results, but was unable to explain some early-type systems which have circular orbits despite τ_{circ} being greater than the MS lifetime of the primary components.

Pan (1997) studied rotational synchronization timescales for 48 early-type detached binaries. Pan, Tan & Shan (1998) studied orbital circularization timescales for a similar list of 37 systems. These authors found that the theory of Zahn was in agreement with the observations of most of the sample binaries, but were unable to explain the characteristics of three binaries in each sample. Whilst stellar evolution may be a solution to this disagreement for rotational synchronization and PMS dynamical evolution may

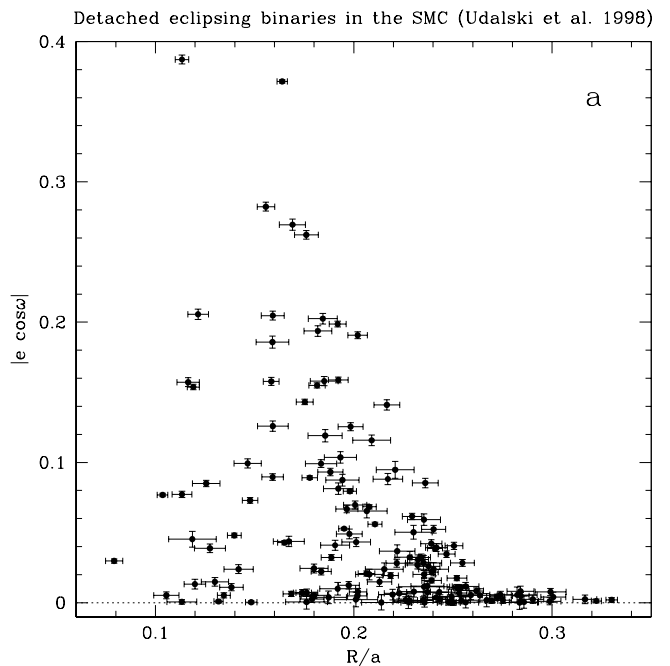


Figure 58. Fractional stellar radius versus orbital eccentricity for a selection of high-mass EBs in the SMC (taken from Udalski et al. 1998). Each fractional radius is an average over the two stars, and the quantity $e \cos \omega$ is plotted as it is better determined than e due to its strong dependence on the time interval between successive primary and secondary eclipses (North & Zahn 2003).

be important, other theories of tidal evolution were also unable to explain all the observations.

North & Zahn (2003) investigated the critical fractional stellar radii, $\left(\frac{R}{a}\right)$, for which binary stars in the LMC and SMC had circular orbits (see Fig. 58). Their results are consistent with the tidal theory by Zahn and confirm that there is negligible dependence on metallicity, as the binary samples in the LMC and SMC are both consistent with the findings by Giuricin et al. (1984b).

Mathieu & Mazeh (1988) proposed that observations of P_{cut} could be used to determine the age of stellar groups. However, tidal theory uncertainties and the difficulty of determining an accurate value of P_{cut} do not allow ages to be derived to good accuracy. Zahn & Bouchet (1989) suggested that PMS tidal interaction makes such a method impossible, but that rotational synchronization could be used instead. However, as stated above, the results of Zahn & Bouchet (1989) are not fully supported by observations.

7.1.6 Summary

There exist further problems which are not in general incorporated into the various tidal evolution theories:–

- Stellar magnetic fields may be important contributors to the overall tidal torque on a star.
- Orbital evolution at the PMS stage appears to be more important than evolution after the ZAMS.
- The axes of revolution of the stars may not be parallel to the orbital axis, an effect which some theories neglect.
- Differential rotation in stars may cause them to appear rotationally synchronized when their interior is not. Synchronization has been suggested to proceed from the surface of a star towards the core (Goldreich & Nicholson 1989).
- Tidal frequencies which are resonant in the stars, and pulsations, have not been included in the above theories, but see Papaloizou & Savonije (1997).
- Binary stars are created with a range of orbital characteristics but current tidal evolution theories do not fully account for this, although PMS dynamical evolution will reduce the effect.

- The timescales above are valid for unchanging stars (no stellar evolution) which are, except for the theory of Hut (1980), in almost circular orbits and rotating close to synchronously.

- As the conditions of circular orbit and synchronous rotation are approached asymptotically, the tidal timescales are estimates of the amount of time taken for stars to become much closer to these conditions. They are not the time taken for the orbit to become perfectly circular and the rotation to become perfectly synchronous for any initial conditions.

- The circularization timescale depends on rotation in a way not explicitly incorporated into models (Claret & Cunha 1997).

- The time taken to reach circular orbits and synchronous rotation for a particular system must be calculated by integrating its orbital characteristics from their initial values to the present age of the system (Claret & Cunha 1997). As we do not know the initial conditions, this can only be approached in a statistical manner (Tassoul & Tassoul 1992).

- Tidal timescales generally have an abrupt discontinuity at the boundary between radiative and convective envelopes, so at this point the timescales are very uncertain (Claret et al. 1995).

- The binary components of hierarchical triple systems can have their orbital characteristics significantly modified by the third star. This can cause small eccentricities to exist when tidal theories predict that the orbit should be circular (Mazeh 1990).

In conclusion, several sophisticated tidal theories exist which predict degrees of orbital circularization and rotational synchronization which are in acceptable agreement with the majority of observed binary systems. Of the two commonly investigated – and somewhat controversial – theories, the basic premise of the theory of Tassoul is not yet fully accepted despite this theory being probably the most successful overall, and the theory of Zahn considers forces which are too weak to explain some observations. Until researchers are able to solve several of the problems listed above, tidal theories are unlikely to become significantly more successful. A vital part of any implementation of the theory is the time integration of specific systems rather than dependence on one equation intended for all systems (Claret & Cunha 1997).

7.2 Apical motion

The tidal forces which cause orbital circularization and rotational synchronization also affect the orientation of binary orbits, resulting in a constant increase in the value of the longitude of periastron, ω , over time. The apical motion period, U , is the time taken for one complete revolution of the line of apsides, and in observed systems varies from a few years, for the very close binaries, to many centuries for well-separated systems. Beyond apical periods of about one thousand years the effect becomes too small to be noticed in the comparatively short time interval in which humans have had access to good observing equipment.

Apical motion is caused by the fact that stars are not point masses and its strength depends sensitively on how centrally condensed the stars are. Knowledge of the apical motion period and the absolute dimensions of an EB allows us to calculate the internal structure constant $\log k_2$, which can then be compared with theoretical models to see if their internal structure predictions match observations (Hilditch 1973).

The apical motion period can be derived spectroscopically by analysing the increase in the values of ω derived from spectroscopic orbits observed many years apart. For systems with only small eccentricities, e , however, observational errors make this very difficult. In an EB the times of minimum light are dependent on e and ω . The most basic observable is the time difference between a primary and successive secondary light minimum, which depends mainly on the quantity $e \cos \omega$ (e.g., Güdür 1978). The quantity $e \sin \omega$ is dependent mainly on the relative durations of primary and secondary eclipses so is in general less well constrained by observations (e.g., Popper & Etzel 1981).

The parameters on which photometric observations of apical motion in an EB depend are the rate of change of ω , $\dot{\omega}$, the value of ω at the reference time of minimum light, ω_0 , the eccentricity, e and the orbital inclination, i . The ephemeris curve takes

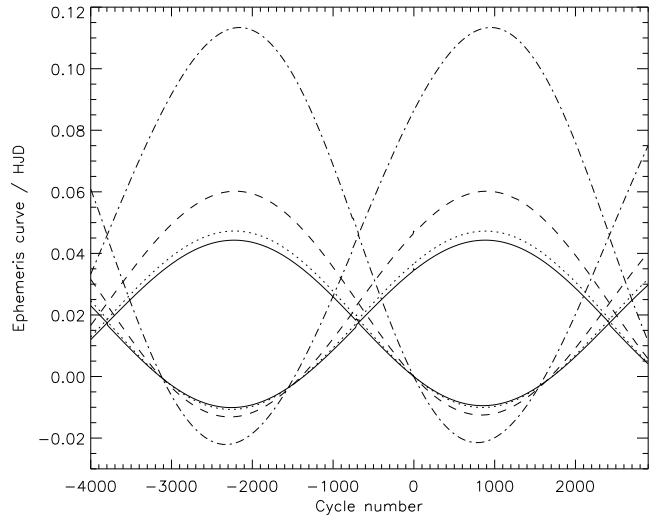


Figure 59. The effect of different values of the orbital inclination, i , on the ephemeris curve. The solid lines show the predicted times of primary and secondary eclipse for $i = 90^\circ$. Dotted lines are for 70° , dashed lines for 50° and dot-dash lines for 30° . This figure is based on the parameters of V453 Cygni (Sec. 14).

the form of a sinusoidal variation of the difference between the actual times of eclipse and the times of eclipse predicted using a linear ephemeris. The ephemeris curve does depend on i , but only very weakly for $i \gtrsim 70^\circ$ (the effect is shown in Fig. 59). As EBs generally have $i \gtrsim 80^\circ$, the exact value of i is unimportant, and this weak dependence makes it impossible to determine i from observations of apical motion. However, determinations of e and ω from the study of apical motion can be more accurate than direct determinations from the analysis of light curves or RV curves (Southworth, Maxted & Smalley 2004b).

Methods of deriving the apical motion parameters from observed times of minimum light depend on adjusting the parameters until they best match the observations. The traditional methods (e.g., Sterne 1939) provide easily-calculated approximations to the parameters, which are then optimised by the process of differential corrections or a similar technique. This method was taken to approximations involving the fifth power in eccentricity by Giménez & García-Pelayo (1983). More recently, Lacy (1992) has avoided the use of approximations altogether and provided an exact solution to the problem of deriving apical motion parameters from observations. Equations are formulated to predict exact times of eclipse given a set of parameters, and these parameters are adjusted towards the best fit using the Levenberg-Marquart nonlinear least-squares fitting algorithm MRQMIN (Press et al. 1992). Fig. 60 shows an example ephemeris curve fitted to observations of the times of minimum light of the dEB V523 Sagittarii, given as an example by Lacy (1992).

7.2.1 Relativistic apical motion

A general relativistic treatment of the gravitational forces in an EB shows that there is a contribution to the apical motion of

$$\dot{\omega}_{\text{GR}} = \frac{6\pi G}{c^2} \frac{1}{P} \frac{M_1 + M_2}{a(1-e)^2} \quad (75)$$

where G is the gravitational constant, c is the speed of light, P is the orbital period, a is the orbital semimajor axis and M_1 and M_2 are the masses of the component stars (Giménez 1985). If M_1 and M_2 are expressed in solar masses and P is expressed in days, this equation reduces to (Giménez 1985)

$$\dot{\omega}_{\text{GR}} = 5.45 \times 10^{-4} \frac{1}{1-e^2} \left(\frac{M_1 + M_2}{P} \right)^{\frac{2}{3}} \quad (76)$$

where $\dot{\omega}_{\text{GR}}$ is in units of degrees per orbital cycle. For dEBs with well-known apical motion periods, the general relativistic apical

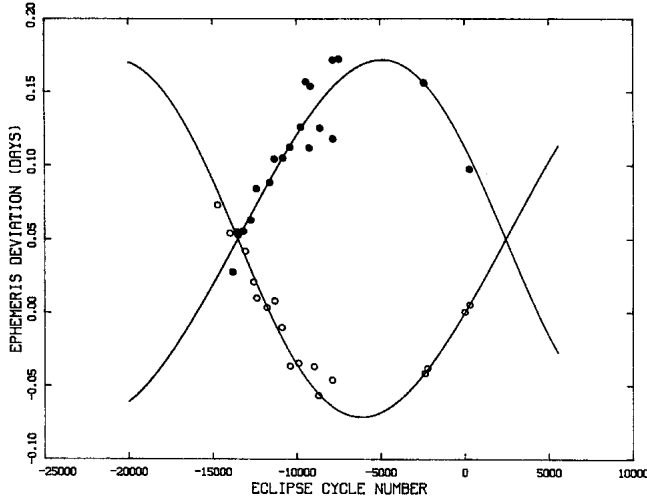


Figure 60. The best-fitting ephemeris curve for the dEB V523 Sagittarii. Observed times of minimum light are given by open circles (primary eclipses) and filled circles (secondary eclipses). Taken from Lacy (1992).

motion rate is in general about one order of magnitude smaller than the Newtonian rate.

Giménez (1985) has suggested a list of EBs which may be amenable to a test of general relativity. The method requires a determination of the total apsidal motion rate and subtraction of the Newtonian contribution using stellar model predictions. This is only reasonable if the general relativistic contribution is comparable in size to the Newtonian contribution, which occurs for only well-separated stars, or very eccentric orbits, so is difficult to observe. Giménez & Scalfirri (1982) applied this method to the dEB V889 Aquilae and determined a relativistic apsidal motion rate in full agreement with the theoretical predictions. Khaliullin (1985) undertook the same procedure, using V541 Cygni, and also found agreement with general relativity.

7.2.2 Comparison with theoretical models

Once an apsidal period has been derived, the internal structure constant $\log k_2$ can be calculated for comparison with the predictions of theoretical models. However, the two stars in a binary system do not in general have the same $\log k_2$, but the individual contributions to the overall apsidal motion rate are not known.

As discussed in Claret & Giménez (1993), the observed density concentration coefficient can be calculated from the apsidal motion period using the equation

$$\overline{k_2^{\text{obs}}} = \frac{1}{c_{21} + c_{22}} \frac{P}{U} \quad (77)$$

where the constants c_{2i} are weights which depend on the characteristics of each star ($i=1$ refers to the primary star and $i=2$ refers to the secondary). c_{2i} are given by the formulae

$$c_{2i} = \left[\left(\frac{\omega_i}{\omega_K} \right)^2 \left(1 + \frac{M_{3-i}}{M_i} \right) f(e) + 15 \frac{M_{3-i}}{M_i} g(e) \right] \left(\frac{R_i}{a} \right)^5 \quad (78)$$

$$f(e) = (1 - e^2)^{-2} \quad (79)$$

$$g(e) = \frac{8 + 12e^2 + e^4}{8} f(e)^{\frac{5}{2}} \quad (80)$$

where ω_i are the rotational velocities of the stars, ω_K are the synchronous (Keplerian) rotational velocities, R_i are the stellar radii and a is the orbital semimajor axis.

The weighted mean theoretical density concentration coefficient must be calculated from the individual theoretical density concentration coefficients using the equation

$$\overline{k_2^{\text{theo}}} = \frac{c_{21}k_{21} + c_{22}k_{22}}{c_{21} + c_{22}} \quad (81)$$

to find the weighted average coefficient which is directly comparable to the observed value.

Once the relativistic apsidal motion contribution, $\dot{\omega}_{\text{GR}}$, has been subtracted from k_2^{obs} , this value can then be compared directly with k_2^{theo} .

7.2.3 Comparison between observed density concentrations and theoretical models

Several dEBs which display apsidal motion have been studied to determine accurate absolute dimensions and apsidal periods. The majority of these were studied by the Copenhagen Group (for example Andersen et al. 1985, Clausen, Giménez & Scarfe 1986) and compared to the predictions of the Hejlesen (1980, 1987) stellar models. In general the theoretical values of $\log k_2$ were greater than observed, so the model stars were less centrally condensed than they should be (Young et al. 2001). More recent models (Claret 1995, 1997; Claret & Giménez 1995, 1998), incorporating convective core overshooting, newer opacity data (Stothers & Chin 1991; Rogers & Iglesias 1992) and effects of stellar rotation, are in much better agreement (Giménez & Claret 1992).

Benvenuto et al. (2002) determined the apsidal motion period of the high-mass non-eclipsing binary system HD 93205 and, considering the predictions of theoretical models, using this period to find the mass of the primary star to be $60 \pm 19 M_{\odot}$. This method allows the determination of absolute masses of binary stars which are not eclipsing, so is useful for stellar types which are rare in EBs (for example O stars), but is dependent on the predictions of theoretical models.

8 OPEN CLUSTERS

When a giant molecular cloud collapses to trigger an episode of star formation, many small parts of it separately contract and subsequently form stars. This creates a cluster of stars which were formed at the same time and from material of a uniform chemical composition. Many clusters in the Perseus spiral arm of our Galaxy have similar ages, suggesting that there was a triggering event which caused the collapse of many giant molecular clouds (PHELPS & JAMES 1994).

Stellar clusters are relatively easy to separate into three different morphological groups. Globular clusters generally contain between 10^5 and 10^7 metal-poor stars, and are very old. Open clusters contain between fifty and several thousand stars which are weakly gravitationally bound and have ages between zero and 10 Gyr. OB associations are collections of stars which formed at a similar time and in a similar place, but are too distant from each other to be gravitationally bound.

As the stars in an open cluster are all the same age, distance and chemical composition, the study of these objects can provide important insights into how stars, clusters and galaxies form and evolve. The usual method of studying these objects is to obtain absolute photometry of the cluster in several passbands, e.g., U , B and V . This allows each observed star to be plotted on colour-magnitude diagrams (CMDs) and colour-colour diagrams. The members of the cluster can then be compared to the radiative properties of nearby stars in order to determine the age and distance of the cluster and the amount of interstellar reddening which affects the light we receive from it.

The study of open clusters has several uses:-

- Critically test predictions of theoretical evolutionary models.
- Investigate the radial chemical abundance gradient of galaxies (e.g., Chen, Hou & Wang 2003).
- Find shape and dynamics of galaxies (Romeo et al. 1989).
- Set the distance scale in our Galaxy, which can be used to calibrate other distance indicators such as δ Cepheids (e.g., Sandage & Tammann 1969).
- As most stars are born in clusters, the study of clusters is important to the star formation history of galaxies.

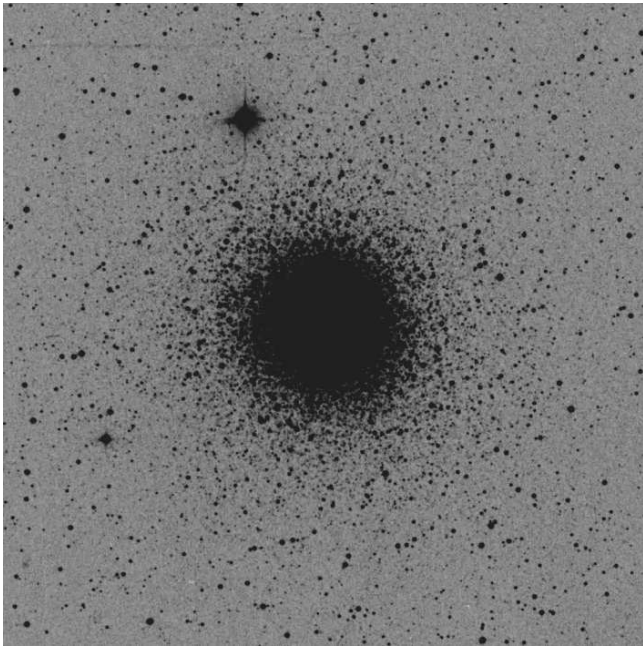


Figure 61. Photographic image of the old globular cluster M15 (NGC 7078) from the ESO Digital Sky Survey website (<http://archive.eso.org/dss/dss>).

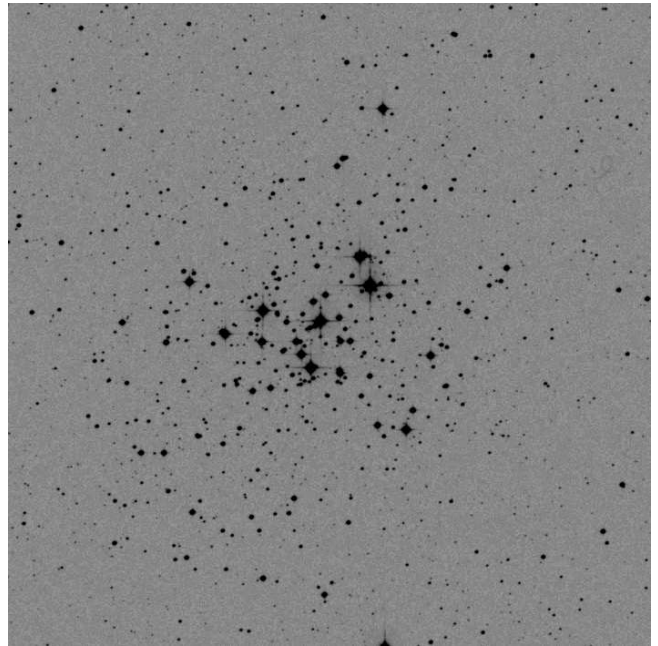


Figure 62. Photographic image of the young open cluster NGC 6231 from the ESO Digital Sky Survey website (<http://archive.eso.org/dss/dss>).

- Investigate the present-day and initial stellar mass functions (Meibom, Andersen & Nordström 2002).
- Provide a lower limit to the age of galaxies and of the Universe (Weiss & Schlattl 1995; Salaris, Weiss & Percival 2004).

There are somewhere over one thousand open clusters in our Galaxy (Balog et al. 2001) and some probably remain undiscovered due to a small size or large interstellar absorption. Large-scale studies and databases of open clusters and associations have been compiled by Mermilliod (1981), Lyngå (1987), Garmany & Stencel (1992), Phelps & Janes (1994), Dias et al. (2002), Chen, Hou & Wang (2003) and the WEBDA¹³ open cluster database maintained by J.-C. Mermilliod.

8.1 Photometric appearance of stellar clusters

Clusters appear as an area of increased stellar density on the sky. Globular clusters are generally very obvious (see Fig. 61) as they contain a large number of stars in a small angular area. Open clusters can be easy to spot (see Fig. 62) but some young and sparse clusters can be difficult to detect, particularly if there are many background and foreground stars. The stars in OB associations are very dispersed compared to open clusters; nearby associations can be extremely difficult to find (e.g., de Zeeuw et al. 1999).

The main way of extracting information from stellar clusters is to obtain photometry on a standard system for many stars in the cluster. The Johnson *UBV* system was introduced precisely for this (see Johnson 1957 and Sec. 12.1.1) but the Strömgren system is capable of providing more accurate results (e.g., Capilla & Fabregat 2002). Once the apparent magnitudes and photometric colours of many stars in a cluster have been measured they can be plotted on a CMD, which is the observational version of the HR diagram. Example CMDs are shown for a globular cluster (Fig. 63), open clusters which are old (Fig. 64), intermediate-age (Fig. 65) and young (Fig. 66), and an OB association (Fig. 67).

The position and shape of the MS of a cluster in its CMD depends on the cluster's distance, age, chemical composition, the evolutionary characteristics of the stars and the interstellar extinction between it and the Earth. These quantities can therefore, in principle, be inferred from the CMD of a cluster. The problem

with this is that many of these parameters are significantly correlated. Additional difficulties are caused by the presence of stars which are not cluster members. These field stars can be both foreground and background objects. Furthermore, unresolved binary stars will appear in the CMD as single stars which have apparent magnitudes up to 0.7 mag brighter than actual single stars of the same colour (for binaries composed of two identical stars), or redder colours if the primary component has a significantly higher T_{eff} than the secondary star. A binary sequence is noticeable in Fig. 65, sitting about 0.7 mag brighter than the single-star MS.

The properties of open clusters can be found from analysis of their CMDs if care is taken. Additional observational techniques which help this include:–

- Nonmember stars can be removed from the CMD by rejecting stars which would be in strange positions of the HR diagram if they were at the cluster distance. This is problematic if contamination by nonmember stars is high and the cluster sequences are difficult to define. One way round this problem is to observe a comparison field close to the cluster on the sky and remove analogues of the comparison stars from the cluster CMD. This approach is entirely statistical but works well for reasonably populous clusters (e.g., Kałużny & Udalski 1992).
- The effect of interstellar extinction (which makes stars appear both dimmer and redder) is different in CMDs involving different passbands, so a combined analysis of two or more different-passband CMDs of one cluster can allow reddening to be found with much greater accuracy (e.g., Chaboyer, Green & Liebert 1999). Colour-colour diagrams can be particularly useful for this (Figs. 69 and 70).
- Nearby clusters have a proper motion which is observable and may be quite different from the general proper motion of the field stars. All cluster member stars will have the same proper motion (allowing for observational errors and a small perspective adjustment) as they were formed from the one giant molecular cloud. Measurement of the proper motions of the stars in the field of the cluster, which requires imaging observations on a timescale of typically several decades, will allow co-moving stars to be detected and nonmember stars to be rejected (e.g., Dinescu et al. 1996). The proper motions of stars which are members of an open cluster all intersect at a ‘convergent point’ (Fig. 71). This point is where the stars are travelling from if the cluster is approaching

¹³ <http://obswww.unige.ch/webda/>

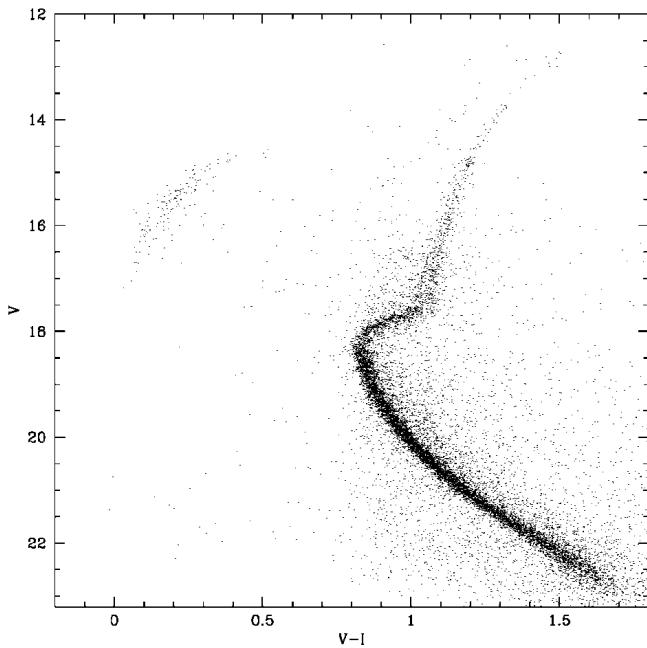


Figure 63. $V - (V-I)$ CMD for the globular cluster M12 (about 12 Gyr old). Taken from von Braun et al. (2002).

the Earth, or where they are travelling to if the cluster is leaving the Earth (Binney & Merrifield 1998, p. 40).

- If spectroscopy of many stars in the field of the cluster is obtained, the RV of the cluster can be found. All stars with a RV which is significantly different to that of the cluster are probable nonmembers; the typical velocity dispersion of an open cluster is only 1 km s^{-1} (Liu, Janes & Bania 1991); for globular clusters this value is typically 10 km s^{-1} (Ivanova et al. 2003). Additionally, binary members of the cluster will be rejected as nonmembers, so the final sample may contain only single cluster members. If several-epoch RV observations are made, binary cluster members can be differentiated from nonmember stars (e.g., Nordström, Andersen & Andersen 1997).

- Optical-wavelength stellar photometry does not allow a good estimate of the T_{eff} and colours of O and early-B stars as the majority of their emitted light is at UV wavelengths. This problem is illustrated by the fact that the bright part of the MS of the OB association LH 117 (which is in the LMC) is almost vertical in its CMD (Fig. 67). Therefore stars with significantly different distances are not detectable photometrically. However, classification-dispersion spectroscopy allows spectral types to be found for the stars. From these, spectroscopic parallaxes can be found and stars which are at significantly different distances from the cluster can be eliminated as nonmembers (e.g., Massey & Johnson 1993). The spectroscopy can also be used to find radial velocities and reject nonmembers in that way, too.

- The use of photometric calibrations (see Sec. 12) allows distances and reddenings to be found for individual cluster stars. This additional information can be used to avoid the strong correlations between these properties and the other astrophysical properties which influence the appearance of CMDs of stellar clusters. Problems may occur with this method if the stars used to define the calibration are significantly different from the stars in the cluster (Johnson 1957).

Some clusters exhibit gaps in their MS stellar distribution due to physical effects which affect the evolution of stars. There is a gap between approximate spectral types A7 and F0 due to the onset of atmospheric convection (Mermilliod 1976), which is called the Böhm-Vitense gap. Gaps can also occur near the MS turn-off due to rapid structural changes in stars near the TAMS (Bonifazi et al. 1990). These can provide interesting tests for theoretical stellar models.

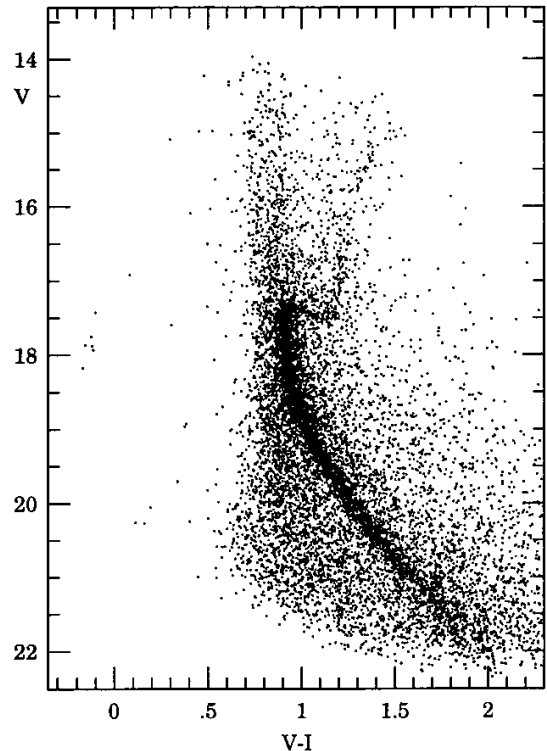


Figure 64. $V - (V-I)$ CMD for the old open cluster NGC 6791 (about 8 Gyr old). Taken from Kažuziņ & Rucinski (1995).

8.2 Analysis of the colour-magnitude diagrams of open clusters and OB associations

The appearance of the CMD of an open cluster depends on several astrophysical parameters:–

- The distance of the cluster.
- The chemical composition of the cluster.
- The cluster age. If the cluster contains stars of several ages (from an extended star formation history) then the stellar sequences in the CMD will be more scattered (Patience et al. 1998).
- The evolution of stars of different masses.
- The atmospheric properties of stars (which affect their photometric colours; Vergely et al. 2002).
- The reddening between the cluster and Earth.
- Differential reddening caused by intracluster matter causes increased scatter in CMDs.
- Binary and multiple stars in the cluster.
- Field stars – both foreground and background.
- Size of the observational errors.
- Intrinsic ‘cosmic’ scatter in the observed properties of stars due to rotation, magnetic fields and other physical properties.

Constraints can be placed on several of these parameters by comparing the morphology of the CMD to either empirically-derived ‘fiducial’ sequences of stars or to isochrones calculated using theoretical stellar models. The latter procedure is commonly adopted to study open clusters, and most current theoretical models have been converted into isochrones by researchers. An example isochronal fit to a CMD is shown in Fig. 68.

A change in distance causes a vertical shift in the position of the cluster sequence in the CMD. An increase in reddening causes a shift to the right and down so is correlated with distance – Reid (1998) states that an error of ΔE_{B-V} in reddening causes an error of $\approx 2\Delta E_{B-V}$ in distance modulus. A decrease in metal abundance or an increase in helium abundance causes a shift downwards in the CMD (Castellani et al. 2002), so these parameters are correlated with each other and with distance and reddening. The CMD shape of the MS turn-off of a cluster is very sensitive to age, but also to details of stellar evolution (par-

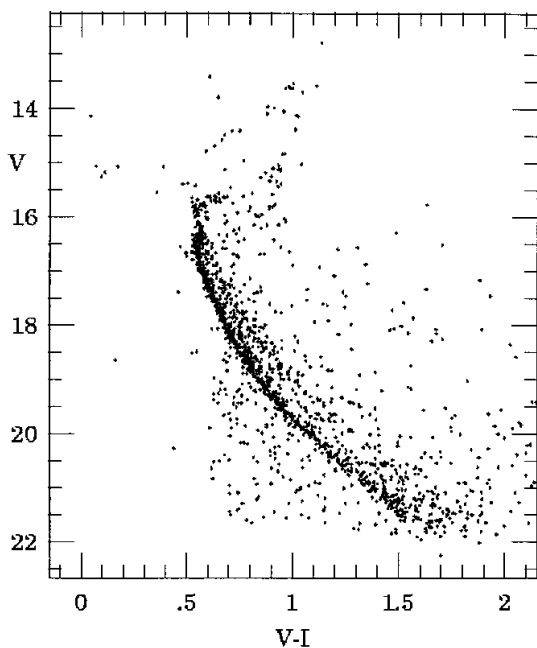


Figure 65. $V - (V-I)$ CMD for the intermediate-age metal-poor open cluster NGC 2243 (approximately 2 Gyr old). Taken from Kałuzny, Krzemiński & Mazur (1996).

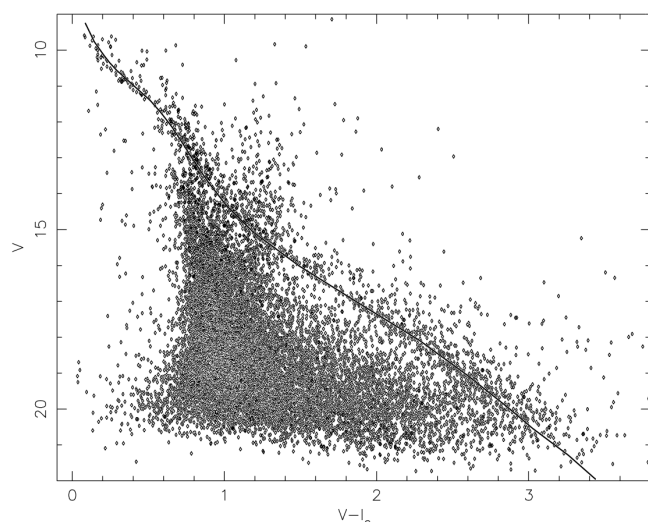


Figure 66. $V - (V-I)$ CMD for the young open cluster NGC 2516 (approximately 150 Myr old). The line is the theoretical ZAMS from the models of Siess, Dufour & Forestini (2000). Taken from Jeffries, Thurstun & Hambly (2001).

ticularly overshooting). The presence of binary stars also softens the curvature of the MS turn-off, decreasing the age derived from isochronal analysis (Raboud, Cramer & Bernasconi 1997). Overshooting increases the curvature, increasing the derived age (Bonifazi et al. 1990; Nordström, Andersen & Andersen 1997).

Attempts to derive the properties of open clusters have traditionally relied on fitting CMDs with isochrones by eye. This is statistically unacceptable (Taylor 2001) but remains a popular procedure due to the absence of a straightforward alternative. As the CMD morphology depends on many parameters which are correlated, most researchers assume reasonable defaults for some, for example tying helium abundance to metal abundance (as done in most theoretical models from which isochrones are derived) and assuming no age spread, differential reddening or theoretical uncertainties in the construction of isochrones. The position of the clump of red giant stars is a useful piece of extra information in intermediate-age clusters, although the theoretical uncertainty in

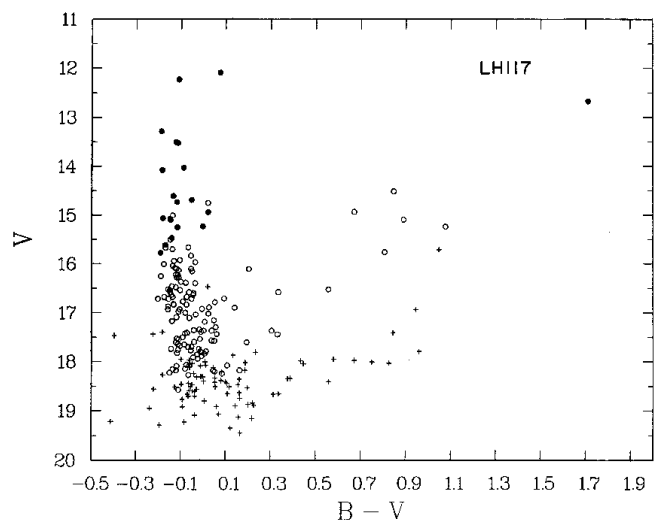


Figure 67. $V - (B-V)$ CMD for the young OB association LH 117 in the LMC. Taken from Massey et al. (1989).

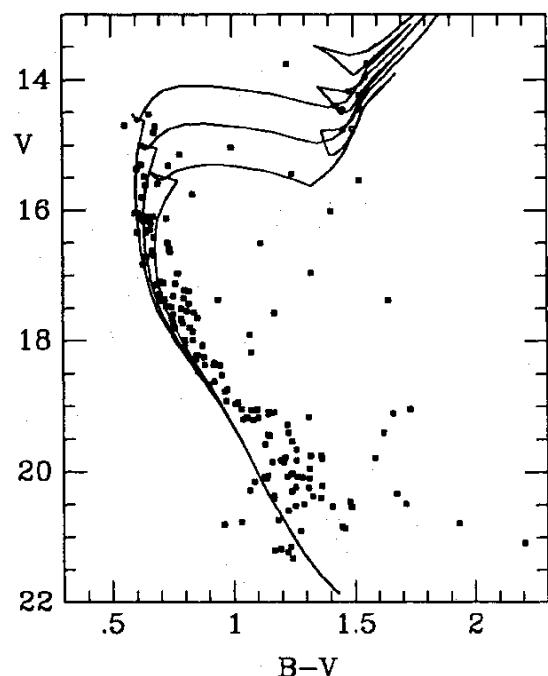


Figure 68. CMD of the old open cluster Berkeley 33 with best-fitting isochrones superimposed, for ages of 0.5, 0.7 and 1.0 Gyr. Taken from Mazur, Kaluzny & Krzeminski (1993).

its position is significant (Daniel et al. 1994; Romaniello et al. 2000). Simultaneous analysis of several CMDs or colour-colour diagrams allows the derivation of more accurate parameters by avoiding some correlations (Tosi et al. 2004).

The presence of overshooting has a significant effect on the MS turn-off shape of intermediate-age open clusters. Studies of such objects consistently find that a moderate amount of overshooting is required (e.g., Chiosi 1998; Nordström, Andersen & Andersen 1997; Woo et al. 2003).

Modifications to the study of CMDs have been successfully made by several researchers in order to avoid the fitting of isochrones by eye. The Padova group has pioneered the construction of synthetic CMDs for statistical comparison with observed ones (e.g., Carraro et al. 1993). Here a stellar modelling code is employed to predict how stars evolve. An initial mass function is chosen for a cluster, including binary and multiple stars, and the cluster is evolved to a desired age. The radiative properties of the cluster stars are evaluated, observational errors may be

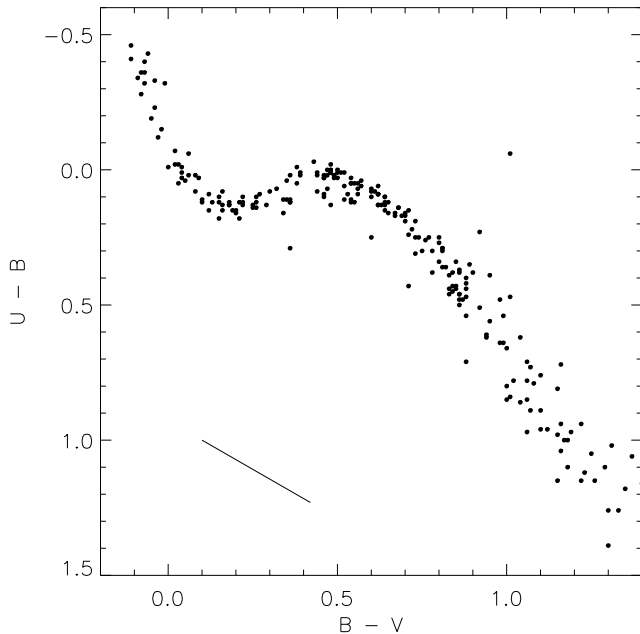


Figure 69. $(U-B) - (B-V)$ colour-colour diagram for the Pleiades open cluster. Constructed from data taken from Johnson & Mitchell (1957). The straight line shows the effect of an interstellar reddening of $A_V = 1$ mag.

added, and the resulting synthetic CMD is compared to observed CMDs. This comparison can be made using standard statistical techniques and has the added advantage that the *density* of stars in the CMDs are directly compared, unlike traditional techniques. Burke et al. (2004) have studied the open cluster NGC 1245 using straightforward χ^2 fitting of isochrones to the CMD of the cluster. One problem with this approach is that MS and evolved members of the cluster must be preselected to avoid analysing nonmembers in the field of the cluster. Wilson & Hurley (2003) have concentrated on fitting the ‘areal density’ of cluster CMDs to model predictions, using the approximate formulae fitted to the Cambridge evolutionary model predictions (Sec. 3.2.5).

8.3 Dynamical characteristics of open clusters

When open clusters form, their component stars have similar velocities but very different masses and so very disparate kinetic energies. Gravitational interactions cause the total kinetic energy of the stars to be distributed more evenly (equipartition of energy). Thus the more massive stars, and binary stars, will sink towards the centre of the gravitational potential (the cluster core) whereas the less massive stars will obtain larger velocities and may escape from the cluster entirely (Binney & Merrifield 1998, p. 387). This is called mass segregation and it occurs very early in the lives of clusters (Littlefair et al. 2003). Open clusters are composed of a core, which contains predominantly massive stars, and a corona, which is roughly five times larger and contains less massive stars (Nilakshi, Pandey & Mohan 2002).

Due to equipartition of energy, most clusters dissolve into a moving group and finally single stars orbiting the galaxy. Only more massive clusters will survive more than a few gigayears, so intermediate-age and old clusters tend to be very populous but lacking in low-mass stars (Friel 1995). The mass function of the stars in an open cluster is very different from the initial mass function of the cluster because of the loss of low-mass stars and the evolution of high-mass stars to stellar remnants. Open clusters are easily disrupted by encounters with the large gravitational potential of a galaxy, so are rare (Bergbusch, Vandenberg & Infante 1991) and are generally situated away from the discs of their parent galaxies (Salaris, Weiss & Percival 2003). They are all found at distances greater than 7.5 kpc from the centre of our galaxy,

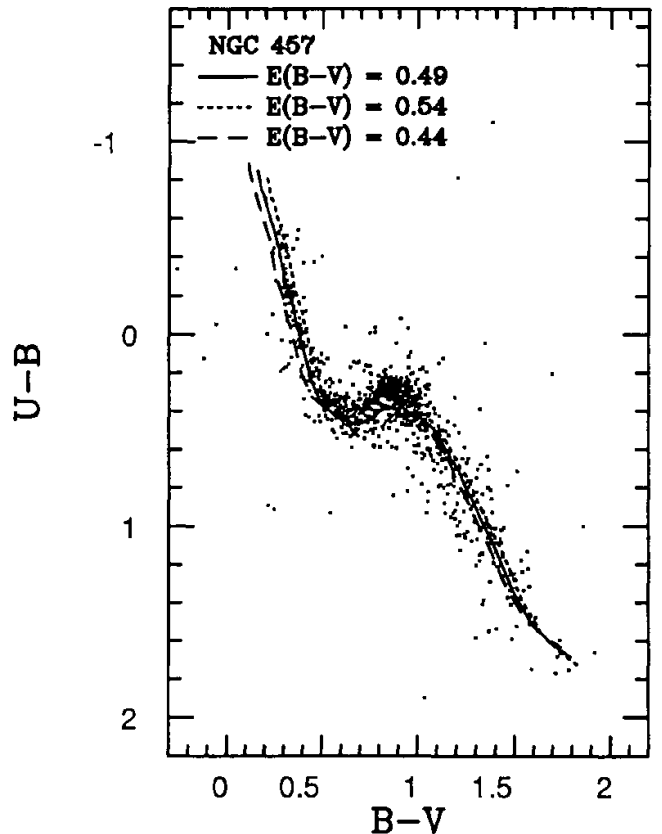


Figure 70. $(U-B) - (B-V)$ colour-colour diagram of the young open cluster NGC 457 with the empirical MS of Schmidt-Kaler (1982) superimposed. Taken from Phelps & Janes (1994).

where the probability of disruption by a giant molecular cloud is smaller (Friel 1995).

9 THE GALACTIC AND EXTRAGALACTIC DISTANCE SCALE

Knowledge of the distance scale of the Universe is one of the fundamental goals of astronomy and astrophysics. The distances in the Universe are large and varied so a number of different distance indicators have been developed. These are generally based on the concept of a ‘standard candle’ – an object with an observable apparent magnitude and specific properties which allow us to infer its absolute magnitude and so distance. The properties should have an easily quantified dependence on the chemical composition of the object as most external galaxies have a different chemical composition to the Milky Way (Allende Prieto 2001).

9.1 Parallaxes of individual stars

9.1.1 Trigonometrical parallax

As the Earth orbits the Sun every year, it is able to view extrasolar objects from positions separated by up to twice the distance from the Sun to the Earth (the Astronomical Unit). Measurement of the angle between the positions of a star, at two different points in the Earth’s orbit around the Sun, allows its distance to be determined using entirely geometrical calculations.

The Hipparcos space satellite¹⁴ was launched in 1989 by the European Space Agency¹⁵ to observe the trigonometrical parallaxes of nearby stars. The Hipparcos Catalogue (ESA 1997; Perryman et al. 1997) contains over 118 000 stars with accurate paral-

¹⁴ <http://www.esa.int/science/hipparcos>

¹⁵ <http://www.esa.int/>

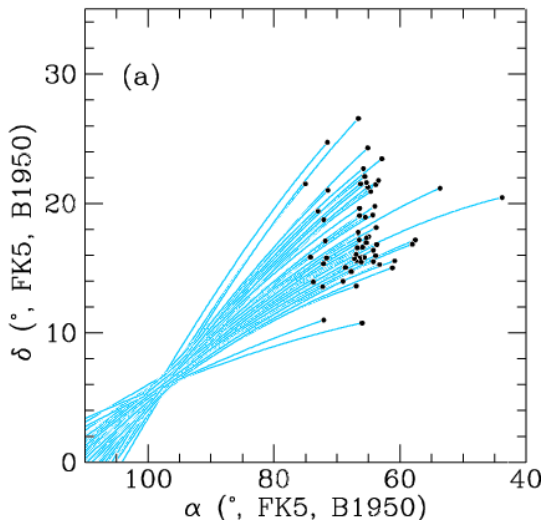


Figure 71. Convergent point analysis of Schwan (1991) for the Hyades open cluster. Taken from Perryman et al. (1998).

laxes, H_P -passband magnitudes (Sec. 12.1.5) and positions. The Tycho experiment on board the Hipparcos satellite observed accurate positions and B_T and V_T magnitudes (Sec. 12.1.5) for over one million stars (Høg et al. 1998, 2000). The magnitudes brighter than which Hipparcos and Tycho are complete are approximately 8.0 and 11.0 in V . The Hipparcos results supersede virtually all previous results and can be obtained online¹⁶ from the VizierR service operated by the CDS¹⁷ at Strasbourg.

The successor to the Hipparcos satellite will be the GAIA satellite¹⁸, which is being prepared by the European Space Agency (Lindgren & Perryman 1996) for a launch around the year 2010. GAIA is intended to obtain parallaxes to accuracies of about 10 microarcseconds for all stars brighter than $V \approx 18$. It will also obtain spectroscopic observations for about 10^8 stars brighter than $V \approx 17$ with a resolving power of 11 500 over the wavelength range 8480 to 8740 Å (Katz et al. 2004).

9.1.2 Spectroscopic and photometric parallax

The derivation of a spectroscopic or photometric parallax of a star requires a calibration between an observed stellar property and its distance. The spectroscopic parallax of a star is found by using relations between spectral type and absolute magnitude, so is generally inaccurate (see Sec. 1.1). Photometric parallax is found by using relations between the photometric characteristics of a star and its absolute magnitude. This is potentially more accurate than spectroscopic parallax, because the colours of a star are continuous quantities whereas its spectral type and luminosity class are discrete quantities, but is not sufficiently accurate to function as a primary distance indicator.

9.2 Distances to binary stars

9.2.1 Visual binaries

The distance to a visual binary star can be found using astrometric observations (to determine the angular size of the binary orbit) and RV observations (to determine the absolute size of the orbit) and is entirely geometrical in character. This is discussed in Sec. 5.2 and examples can be found in Torres, Stefanik & Latham (1997a, 1997b, 1997c). Stellar interferometers are well suited to providing astrometric observations of double stars (e.g., Pan, Shao & Kulkarni 2004; Zwahlen et al. 2004).

9.2.2 Eclipsing binaries

Methods of determining the distances to detached and semidetached EBs are discussed in Sec. 6.3. Once the physical parameters of an eclipsing system have been found, the distance to the system can be found in several ways using, e.g., bolometric corrections (Sec. 6.3.1) or surface brightness relations (Sec. 6.3.2). These methods have been used to find the distances to Galactic open clusters (e.g., Munari et al. 2004) and to nearby galaxies (Sec. 6.3.4), and are potentially able to stretch as far as the large spiral galaxies M31 and M33 (Kałużny et al. 1998; Ribas 2004).

A calibration for the absolute magnitudes of W Ursae Majoris stars has been derived by Rucinski (1994), who states that the scatter around the best fit is 0.5 mag so this is not an accurate distance indicator as yet.

9.3 Variable stars as standard candles

9.3.1 δ Cepheid variables

The type of intrinsically-variable stars which are most commonly used as a distance indicator are the δ Cepheids. This is a class of evolved star which pulsate with periods between about 1 and 50 days. They are giants of approximately 3 and 10 $M_\odot$ which pulsate due to a ‘bump’ in the opacity of metallic species as a function of temperature. The pulsation periods of δ Cepheids are related to their luminosities, which means a calibrated relation can be used to infer their intrinsic luminosities and so their distances. The period-luminosity relation is also known to depend on colour. It was calibrated by Sandage & Tammann (1969), using three δ Cepheids in the young open cluster NGC 7790:

$$M_{\langle V \rangle} = -3.425 \log P + 2.52(\langle B \rangle - \langle V \rangle) - 2.459 \quad (82)$$

$$M_{\langle B \rangle} = -3.425 \log P + 3.52(\langle B \rangle - \langle V \rangle) - 2.459 \quad (83)$$

where $M_{\langle V \rangle}$ and $M_{\langle B \rangle}$ are the mean absolute magnitudes in the B and V passbands, P is the period in days and $\langle B \rangle$ and $\langle V \rangle$ are the mean apparent B and V magnitudes.

Gieren, Barnes & Moffett (1993) used a surface brightness technique to calibrate the period-luminosity relation and found

$$M_V = -2.986(94) \log P - 1.371(95) \quad (84)$$

where the quoted uncertainties give the scatter about the relation, which was derived using 100 δ Cepheids.

The Hubble Space Telescope Key Project to determine the Hubble constant, H_0 , uses the relations (Freedman et al. 2001)

$$M_V = -2.760 \log P - 1.458 \quad (85)$$

$$M_I = -2.962 \log P - 1.942 \quad (86)$$

where the LMC is assumed to have a distance modulus of 18.50 mag. This results in $H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ kpc}^{-1}$.

This period-luminosity relation is known to also have some dependence on metallicity, which is important because external galaxies can have metallicities which are significantly different from those found in the solar neighbourhood. Any inadequacies in the compensation for this metallicity dependence will result in systematic errors in the distances to most other galaxies. Romaniello et al. (2005) have investigated this and find a dependence which is significantly different from zero and from a linear relation, where metal-rich δ Cepheids are brighter. This is in agreement with some theoretical δ Cepheid models but in strong disagreement with other models.

9.3.2 RR Lyrae variables

RR Lyrae are old, low-mass, metal-poor stars which pulsate with periods of the order of one day (Zeilik & Gregory 1998, p. 356). A period-luminosity relation derived for the K passband by Dall'Ora et al. (2004) is

$$\langle M_K \rangle = (-0.770 \pm 0.044) - 2.101 \log P + (0.231 \pm 0.012) \left[\frac{\text{Fe}}{\text{H}} \right] \quad (87)$$

¹⁶ <http://vizier.u-strasbg.fr/viz-bin/VizieR?source=I/239>

¹⁷ <http://cdsweb.u-strasbg.fr/>

¹⁸ <http://www.esa.int/science/gaia>

This has been used to find a distance modulus of $18.52 \pm 0.005 \pm 0.117$ mag to the LMC, where the quoted uncertainties are random and systematic, respectively. Clementini et al. (2003) found a slightly different dependence of

$$\frac{\Delta M_V}{\Delta[\text{Fe}/\text{H}]} = 0.214 \pm 0.047 \quad (88)$$

giving a distance modulus to the LMC of 18.45 ± 0.09 mag.

9.3.3 Type Ia supernovae

Type Ia supernovae have no hydrogen or helium lines in their spectra but show absorption of Si^+ (Binney & Merrifield 1998, p. 302). They are thought to be carbon-oxygen white dwarfs in close binary systems which accrete sufficient matter from their companion to collapse and then explode (Zeilik & Gregory 1998, p. 440). The absolute magnitude at peak brightness of a type Ia supernova is $M_B = -18.33 \pm 0.11$ mag (Zeilik & Gregory 1998, p. 440). These objects are extremely bright so are useful standard candles for distances up to a redshift of about 1 (Gal-Yam & Maoz 2004). However, supernovae in star-forming galaxies seem to be brighter than normal (van den Bergh 1994) and the maximum brightness may depend on metallicity (e.g., Shanks et al. 2002).

9.4 Distances to stellar clusters

The distances to open clusters, OB associations and globular clusters can be found from analysis of the morphology of the stellar distribution in CMDs (Sec. 8.2). Whilst this is a very useful distance indicator, the results are usually found using theoretical stellar evolutionary models, and uncertainties in reddening, chemical composition and age can cause distances derived this way to be inaccurate. However, this technique can be applied to clusters in external galaxies, as long as individual stars are resolved in the photometric observations, so can be used to find the distance to nearby galaxies such as the Magellanic Clouds.

The distances to resolved stellar populations can also be found by studying the properties of their red giant stars. This is because the absolute I magnitude of the clump of red giant stars (which are located in a relatively long-lived state) is only weakly dependent on age and chemical composition (Bellazzini, Ferraro & Pancino 2001). This absolute magnitude can be found from Hipparcos observations of nearby red giant stars and can then be used to find the distance to e.g., the LMC (Girardi et al. 1998; Romaniello et al. 2000). Red clump stars are the best-calibrated standard candle from Hipparcos (Alves et al. 2000).

9.5 The Galactic and extragalactic distance scale

The distance scale through our Galaxy is dependent mainly on the study of open clusters. In the 1960s and 1970s the distances were derived mainly by comparing the morphology of the CMD to empirical stellar sequences defined using nearby stars with trigonometrical parallaxes. When theoretical stellar models became reliable and generally accurate, their predictions were used to replace empirically defined stellar sequences with sequences which were available for arbitrary age and chemical composition. Trigonometrical parallaxes from the Hipparcos satellite have been used to define empirical stellar sequences, and some recent work has concentrated on converting the open cluster distance scale from theoretically-based to empirical (e.g., Percival, Salaris & Kilkenny 2003; Percival, Salaris & Groenewegen 2005).

The distances to the Magellanic Clouds are two of the most important quantities in astrophysics because they are in the overlap between the distance scales based on individual stars (discussed above) and those on galaxies and unresolved stellar populations (not discussed here). The SMC is actually larger than the LMC, but appears smaller because it is more distant. The LMC is therefore more useful as a distance calibrator because distance effects due to it having a finite size are less important (Feast 2003). Measurements of its distance modulus are converging on 18.50 ± 0.02 mag, partly because of the HST Key H_0 project

(Alves 2004; Freedman et al. 2001). Distance measurements have been made using the red giant clump (18.54 ± 0.10 mag, Sarajedini et al. 2003), δ Cepheid variables (18.55 ± 0.02 , Keller & Wood 2002), RR Lyrae variables ($18.52 \pm 0.005 \pm 0.117$ mag, Dall'Ora et al. 2004), photometry of the LMC cluster NGC 1866 (18.58 ± 0.08 , Groenewegen & Salaris 2003), analysis of the dust rings around the LMC supernova SN 1987A (18.46 ± 0.12 mag, Mitchell et al. 2002) and from the study of EBs.

Several studies of EBs in the LMC have recently been published (Sec. 6.3.4) and contain measurements of the distance to the LMC: HV 2274 (18.30 ± 0.07 mag, Guinan et al. 1998), HV 982 (18.51 ± 0.05 mag, Fitzpatrick et al. 2002; 18.63 ± 0.08 mag, Clausen et al. 2003), EROS 1044 (18.38 ± 0.08 mag, Ribas et al. 2002) and HV 5936 (18.18 ± 0.09 mag, Fitzpatrick et al. 2003). The weighted mean of these results, 18.43 ± 0.03 mag, is in reasonable agreement with other methods but the scatter is somewhat greater than the quoted uncertainties would suggest. Possible reasons for this include systematic errors which have not been explicitly quantified, a distance effect because the LMC is not an infinitely small object, or optimistic error analyses.

Finding the distance to the SMC is more problematic because of the larger size of this system. The use of EBs is an excellent way to avoid biases due to this problem, because an individual distance can be found for every EB studied and the problem averaged out. Harries, Hilditch & Howarth (2003; Hilditch, Harries & Howarth 2004, 2005) have studied fifty EBs in the SMC using the AAT/2dF multi-object spectrograph and OGLE-III photometric data (Sec. 6.3.4). The final distance modulus is $18.91 \pm 0.03 \pm 0.10$ mag (random and systematic uncertainties). This conclusively shows that EBs are capable of providing definitive distance measurements to stellar populations.

10 OBTAINING AND REDUCING ASTRONOMICAL DATA

10.1 Telescopes

Astronomical telescopes are used to gather light from celestial objects and concentrate it onto a light detector. Celestial objects are very remote so the photons which are detected by a telescope will be travelling in essentially parallel paths. This light is then focussed by the optics of a telescope and an image of the sky is formed in the telescope's focal plane.

The first recorded telescopic astronomical observations were made by Galileo Galilei in 1610 using the recently-invented Dutch eyeglass (Koestler 1989). Galileo did not invent the telescope, but refined the construction of refracting telescopes in order to study the Universe. Refracting telescopes consist of a large convex primary lens through which light shines onto a convex secondary lens placed further away than the focal length of the primary lens. The secondary collimates the light so it can be detected by the human eye, photographic plates, or other alternatives.

Refracting telescopes were constructed in ever-bigger sizes until the mid-twentieth century, the largest being the Yerkes refractor (completed in 1897) which has a primary lens with a diameter of 102 cm (Kaufmann 1994, p. 105). However, increasing the diameter of a primary lens, so as to detect more starlight, means that the refracting telescope becomes longer, less convenient and more expensive to manufacture. The lenses became very heavy so are difficult to support and cause telescope flexure. Refracting telescopes also suffer chromatic aberration (see below) and have very low efficiency at blue and UV wavelengths due to the transmission properties of glass.

Many of the problems with refracting telescopes can be solved by using mirrors instead of lenses to focus the incoming starlight. Mirrors are much lighter and easier to support so they require less engineering. They suffer no chromatic aberration, are cheaper to build and can be made much larger in diameter than glass lenses. All recently built optical telescopes are reflectors, primarily of the Cassegrain design in which a large curved primary mirror reflects incident starlight light onto a smaller secondary mirror, which reflects the light back through a hole in the centre of the primary mirror and focuses it onto the focal plane.

Some of the largest optical telescopes currently available to the astronomical community or being constructed or designed are:

- The two Keck telescopes at the Keck¹⁹ observatory (Hawaii) each of which have a 10 m diameter primary mirror.
- The Very Large Telescope (VLT²⁰) is operated by the European Southern Observatory (ESO²¹) in Chile and consists of four telescopes, each with an 8.2 m diameter primary mirror.
- The Hobby-Eberly Telescope (HET²²; McDonald Observatory, Texas) and the Southern African Large Telescope (SALT²³; Sutherland, South Africa) have segmented primary mirrors with a total diameter of 11 m. They are fixed in pointing altitude, which makes the structure easier to design and much easier to build. The HET is currently operational whilst the SALT is nearing the end of construction.
- Construction has started on the Giant Magellan Telescope (GMT²⁴), which is a single structure containing seven primary mirrors, each 8.4 m in diameter. It will probably be sited in Chile.
- The California Extremely Large Telescope (CELT²⁵) is being designed and is intended to have a 30 m primary mirror.
- Further into the future, the Overwhelmingly Large telescope (OWL²⁶) is being investigated by ESO. This project aims to build a telescope with a 100 m diameter primary mirror, but is such an

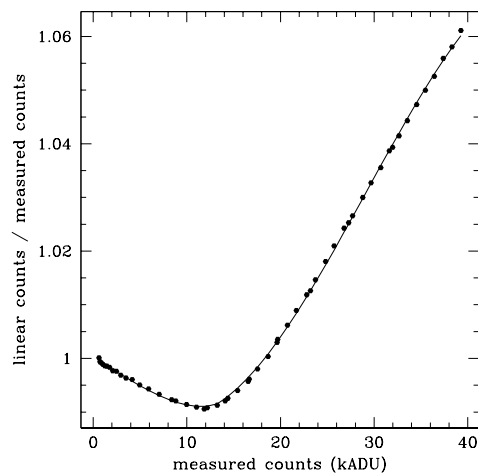


Figure 72. Example of the nonlinearity of the response of a CCD to incident light. The filled circles represent the observed ratio of actual electrons detected to that expected for a linear response, found from a sequence of dome flat fields with different exposure times. The curve is a fitted function used to correct data for the nonlinearity. Taken from Hidas et al. (2005).

ambitious concept that it has been nicknamed the WTT (Wishful Thinking Telescope) and even the FLT (quite Large Telescope).

10.1.1 Optical aberration

Several different phenomena adversely affect the quality of astronomical images and are caused by the properties of the optical elements of a telescope. It is not possible to remove these effects entirely, so the best procedure is to minimise the cumulative effects (Hilditch 1997).

Chromatic aberration occurs because the refractive properties of glass depend on wavelength. Glass lenses refract blue light more than red light, resulting in the focal plane of red light being in a slightly different place to that of blue light. Mirrors do not suffer from chromatic aberration.

Spherical aberration is caused by parallel light beams being focussed at different points depending on their distance from the optical axis when they enter the telescope.

Astigmatism is caused by an optical element having different focal lengths in the two dimensions normal to the optical axis. This means that the focal plane of transverse light beams is different to that for sagittal light beams.

Coma is where different light beams from an off-axis object hit an optical element at different distances from the expected focal plane, causing some light to be focussed earlier than other light. This gives a characteristic image shape which is reminiscent of a comet, from which the name comes.

Field curvature is where the focal plane of a telescope is not flat. This can be a major problem in Schmidt telescopes.

10.2 Charge-coupled devices

Charge-coupled devices (CCDs) are semiconducting chips containing two-dimensional arrays of silicon electrodes on one surface. Each electrode has a small positive charge. A photon which hits an electrode causes the production of an electron due to the photoelectric effect. The electrons which are produced are stored in a potential well below the electrode. When the electrons are counted after exposure to light, a map of the intensity of the incident light is obtained. CCDs in the focal plane of a telescope can therefore be used to make intensity images of the sky.

¹⁹ <http://www2.keck.hawaii.edu>

²⁰ <http://www.eso.org/projects/vlt/>

²¹ <http://www.eso.org/>

²² <http://www.astro.psu.edu/het/>

²³ <http://www.salt.ac.za/>

²⁴ <http://www.astro.lsa.umich.edu/magellan/>

²⁵ <http://celt.ucolick.org/>

²⁶ <http://www.eso.org/projects/owl/>

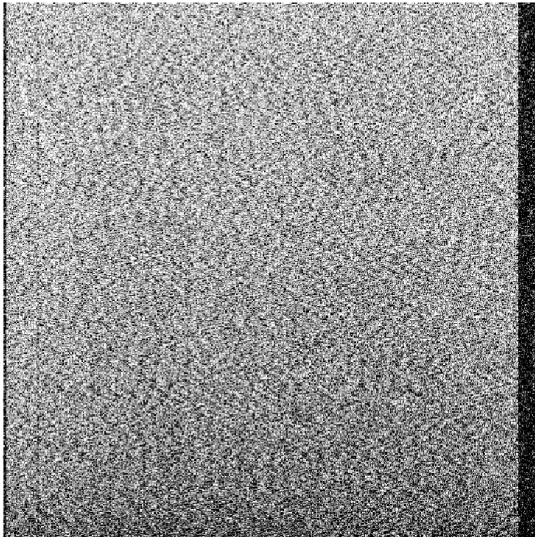


Figure 73. Example bias image, taken using the SITE2 CCD on the Jakobus Kapteyn Telescope (ING, La Palma). An overscan strip, which has slightly fewer counts per pixel, is visible.

10.2.1 Advantages and disadvantages of CCDs

CCDs have revolutionised astronomical photometry. They have several advantages and drawbacks. The advantages include:–

- The ‘quantum efficiency’ of CCDs is the (wavelength-dependent) detection efficiency of incident photons and can be up to 90%. This is a vast improvement compared to photographic plates, which were previously used to obtain astronomical images, which have quantum efficiencies of around 2% for blue light and are even worse at other wavelengths.
- CCDs respond (approximately) linearly to incident light. This is of fundamental importance for astronomical photometry, where apparent magnitudes are basic observable quantities.
- CCD pixels have a large dynamical range - they can store between zero and about 10^5 electrons.
- CCDs are efficient light detectors over a wide range of wavelengths ($0.4\ \mu\text{m}$ to beyond $1\ \mu\text{m}$).
- CCDs produce two-dimensional images, allowing photometry to be performed simultaneously on many stars. This increases the observing efficiency of telescopic observing and allows some systematic photometric errors to be avoided.
- CCDs do not consume any physical materials during use so can be run without human interaction. This is particularly important for astronomy using space satellites.

CCDs have some drawbacks:–

- They have read-out noise which lowers the signal-to-noise of astronomical observations, particularly faint sources.
- They can have a slightly non-linear response to light, although this is relatively straightforward to quantify (see Fig. 72).
- They can take of the order of one minute to read out after each exposure. This means that, when observing variable stars, more time can be spent on readout than on actual light collection. Faster read-out can be achieved using procedures which increase the read-out noise, or by reducing the area of the CCD which is used (‘windowing’).
- One CCD pixel can only store a certain number of electrons before charge overflows onto neighbouring pixels. Also, the majority of CCDs are operated as sixteen-bit devices so have a software limit of 65535 ADU (Astronomical Data Units). The gain of a CCD is the number of electrons required to generate one ADU and is dependent on the CCD controller. CCDs can have a significantly nonlinear response to high light levels
- Once a pixel is saturated, any further photoelectrons will ‘bleed’ into neighbouring pixels and make their data useless too.
- High-energy cosmic rays sometimes interact with a CCD

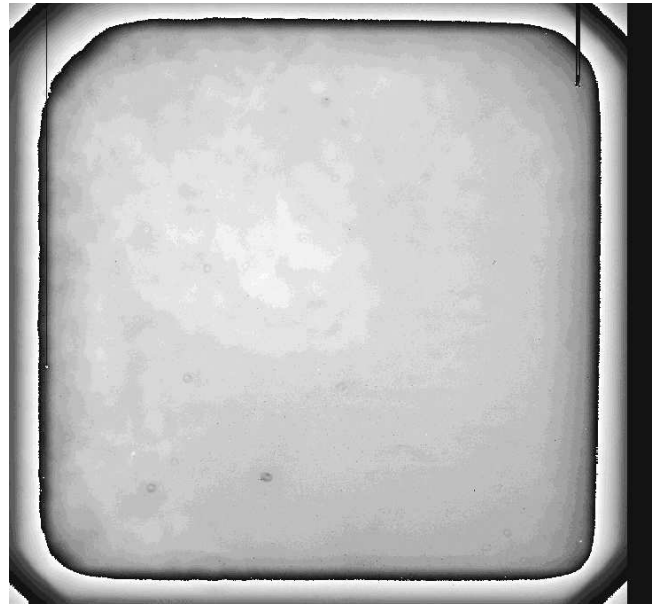


Figure 74. Example flat-field image, taken using the SITE2 CCD on the Jakobus Kapteyn Telescope (ING, La Palma) and a Johnson V passband. Overscan strips are visible at the top and on the right of the image. A ‘wrapped’ colour table has been adopted to make the structure of the image more obvious (the counts are highest towards the image centre and lowest at the corners).

pixel, producing a large number of photoelectrons which were not created by light coming from the intended source. This charge can then bleed into neighbouring pixels.

- Pixels, or even whole columns of pixels, can lose their sensitivity to light for a variety of reasons.

10.2.2 Reduction of CCD data

The images produced by CCD detectors contain some effects which must be removed before the images are analysed. These fall into the categories of debiasing and flat-fielding and can be mathematically represented by

$$D_{x,y} = \frac{R_{x,y} - B_{x,y}}{F_{x,y} - B_{x,y}} \quad (89)$$

where x, y are pixel indices, $R_{x,y}$ is a raw CCD image, $B_{x,y}$ is a bias image, $F_{x,y}$ is a flat-field and $D_{x,y}$ is the reduced image.

10.2.3 Debiasing CCD images

The electronics which read out CCDs cannot cope with negative counts from a potential well. A small safety voltage is therefore applied which causes a certain number of counts to be present even in pixels which detected no photons. This ‘bias’ must be subtracted from an image. The traditional technique for determining the bias is to take an exposure which lasts for zero seconds, so the only counts will be from the bias (and a negligible input from dark current; Fig. 73). The bias image can then be subtracted from science images.

The current generation of CCDs have well-behaved bias characteristics in which the bias voltage has a negligible difference for different pixels. In this case it is sufficient to subtract one bias value from every pixel on the detector rather than a bias image. This avoids adding any Poisson noise which is present in the bias image to the science image. This bias value can be found from regions where the CCD is ‘overscanned’, i.e., read out beyond the last pixels of each row of electrodes.

The best bias images are created from the median value for each pixel of several individual images. This reduces Poisson noise and avoids problems with cosmic rays. The average bias value



Figure 75. Example subsection of a science image. The target is the hPersei open cluster and problematic CCD columns are noticeable as vertical smears affecting some stars.

of an overscanned region is best found using median-calculation techniques for the same reasons.

10.2.4 Flat-fielding CCD images

Each pixel in a CCD has, in general, a slightly different size and efficiency as a result of the manufacturing process. This causes a variation in efficiency for different pixels, which must be corrected to remove this variation from the images used for science. Additional large-spatial-scale contributions to variation in efficiency come from the telescope optics and from grains of dust on the outer surface of the CCD detector (see Fig. 75).

A flat field can be observed by imaging a blank area of sky during evening or morning twilight. It is advisable to take several dithered flat fields and median-stack them to remove any contaminating light from visible stars and cosmic rays. As passbands cause some efficiency variation, flat fields must be taken separately for each one used during one observing night. Once a flat field has been obtained, science frames are divided by them to remove the variations in detection efficiency across the CCD.

10.2.5 Photometry from CCD images

The observed brightness of a star depends on its apparent magnitude, the amount of attenuation its light suffers when passing through the Earth's atmosphere, and on the efficiency of the observing equipment. The characteristics of the atmosphere above a telescope can change quite quickly but will be the same at the same time for stars which are close to each other on the sky. This means that whilst the total flux incident from different stars on one CCD image will depend on the observing conditions, the ratios of the fluxes of stars will be negligibly affected by the Earth's atmosphere. Differential CCD photometry is the determination of the relative brightnesses of several stars on one CCD image.

The Earth's atmosphere is in constant motion. Different parts of the atmosphere have a different temperature and so a different refractive index. This causes a slight transverse motion and dispersion of light from a point source. The dispersion is roughly 0.1 arcsec in good conditions. Over short periods of time (less than one second) the transverse motion is resolved and images can be made with 0.1 arcsec resolution. Devices which integrate the light received over longer exposures (e.g., CCDs and photographic plates) obtain images in which the transverse motion has dispersed the light over a larger area. The best observing sites in the world (e.g., La Silla and Paranal in Chile) suffer from a seeing of roughly 0.5 arcsec for many nights of the year. Other observing sites (e.g., Keele University, UK) have seeing which is almost always greater than 2 arcsec and often much worse.

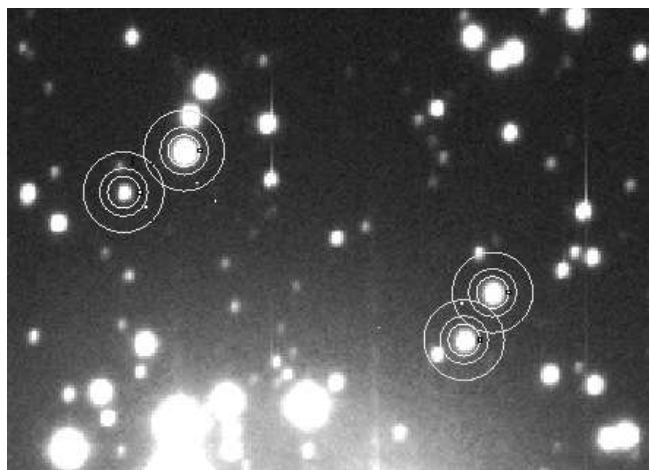


Figure 76. Software apertures placed over stars in Fig. 75. The inner aperture (6 pixels in radius) is used to calculate the flux of the star. The surrounding annulus (between the two outer circles which are 9 and 18 pixels in radius) defines the region used to assess the sky background flux. The plate scale is about 0.4 arcsec per pixel. The lowest star in the image with an aperture is V615 Per and the lower in the left-hand pair is V618 Per.

Nyquist's theorem states that no information is lost if a continuous function is sampled at twice the frequency of the highest-frequency component of the function (Press et al. 1992, p. 494). According to this theorem, the resolution elements of an observation (e.g., CCD pixels) should be smaller than half the size of the possible resolution of the image (e.g., a seeing of 1 arcsec) so images of stars on CCDs cover several pixels. In reality, the light from one star can be scattered by quite large angles (several degrees) due to atmospheric effects. CCD photometry must find ways of calculating the amount of light received from a star in which as much light as possible is counted (so large areas of a CCD are considered to detect the light from one star) without including too much noise, background light and light from other stars (so only small areas of a CCD should be counted). An example subset of a science frame is shown in Fig. 75.

10.2.6 Aperture photometry

This is a simple technique in which three concentric circular regions are defined around each star. This causes a complication because the regions are circular but pixels are square. When summing pixel counts, those pixels which are partially inside one region contribute the same proportion of their total counts as the proportion of their area which is inside the circular region. The annulus between the outer two circles is used to estimate the contribution of the sky background to the light detected in each pixel. It is best to adopt the median or mode of the pixel counts in order to avoid biasing the result due to a star or cosmic ray event. Once the sky background has been estimated, it can be subtracted from each pixel inside the inner circular region. The estimated number of counts detected from the star is then the total number of counts inside the inner circular aperture.

As discussed above, a compromise must be made between having large apertures (which receive more counts from a star) or small apertures (which suffer less from read-out noise and background light). It is usually wise to have relatively small apertures (see Fig. 76). Whilst this means that a significant amount of light from each star is ignored, this affects each star similarly so makes no differences to the ratios of fluxes of different stars on the CCD frame. It is important to ensure that the circular regions are centred very precisely on the star to ensure that the same proportion of total counts are being ignored for all stars. It is generally a good idea to have a somewhat larger sky region so the estimation of the sky background light is robust.

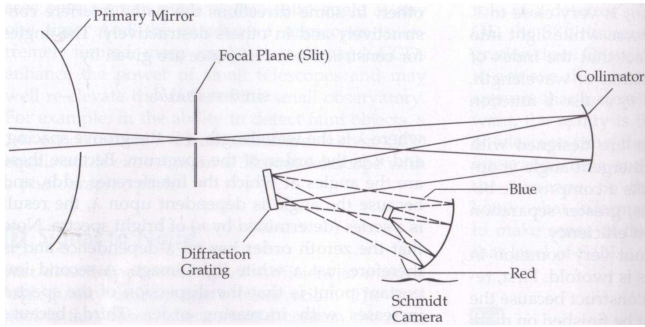


Figure 77. Diagrammatical representation of a grating spectrograph. Taken from Zeilik & Gregory (1998, p. 194).

10.2.7 PSF photometry

The two-dimensional structure of an image of a point source on a CCD is called the point spread function (PSF). For optical systems which suffer from only minor aberration, the PSF should be almost the same over a whole CCD image and will usually be similar to a two-dimensional Gaussian function. A PSF can be defined by averaging the image shapes of several bright stars on a CCD image and then fitted to the dimmer stars in that frame to determine the relative number of counts for each star. This photometry technique is much more effective in crowded fields, where the PSFs of several stars overlap, and can also be better for dim stars than aperture photometry.

10.2.8 Optimal photometry

Optimal extraction was introduced by Horne (1986) for the reduction of CCD spectroscopy and was generalised to CCD imaging photometry by Naylor (1998). It gives an approximately 10% increase in signal to noise over normal aperture photometry (Naylor 1998) and doesn't suffer from problems caused by poor estimations of stellar PSFs (Eaton, Draper & Allen 1999). Optimal photometry is inferior to PSF photometry in crowded fields and, because it gives high weight to the central few pixels, is sensitive to the exact placement of starlight on the CCD pixels.

The flux, F , within an aperture can be summed using

$$F = \sum_{x,y} W_{x,y} (D_{x,y} - S_{x,y}) \quad (90)$$

where x, y are pixel indices, $D_{x,y}$ and $S_{x,y}$ are the counts from the source and the sky and $W_{x,y}$ is the weight given to a pixel. For optimal extraction, an estimated stellar profile, $P_{x,y}^E$ is found from a bright star and normalised to unity. The weights for extraction of the optimal signal to noise ratio of the stellar counts are

$$W_{x,y} = \frac{P_{x,y}^E / V_{x,y}}{\sum_{x,y} (P_{x,y}^E)^2 / V_{x,y}} \quad (91)$$

where $V_{x,y}$ are the variances of the counts in the pixels. The overall variance of the measured counts is

$$\text{var}[F] = \sum_{x,y} W_{x,y}^2 V_{x,y} \quad (92)$$

Optimal photometry has been implemented in the Starlink software PHOTOM (Eaton, Draper & Allen 1999²⁷) and GAIA (Draper, Gray & Berry 2004²⁸), and in ARK²⁹, which is maintained by T. Naylor³⁰.

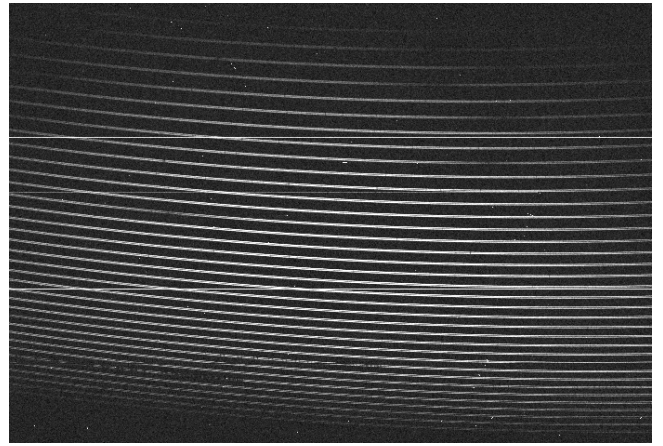


Figure 78. Example CCD image containing a portion of an échelle spectrum. This spectrum is of the dEB GV Carinae, and both the target and sky spectra (which are slightly separated) can be seen for each order.

10.3 Grating spectrographs

Grating spectrographs collimate light received from a telescope, and passed through a slit, onto a diffraction grating and then focus the light onto a detector (Fig. 77). CCDs are the best light detectors for grating spectrographs for the reasons stated above (Sec. 10.2.1). In particular, CCDs are two-dimensional so can have a spectral direction (normal to the image of the slit) and a spatial direction (along the slit). This allows the light from a star to be resolved in a spatial direction and the background light to be estimated from adjacent parts of the CCD.

If light from the focal plane of a telescope were fed directly to a spectrograph then the seeing disc of a star would cause a significant loss of spectral resolution. Any tracking errors with the telescope would also cause problems with the wavelength calibration, causing spurious shifts in RVs derived from the spectra. For these reasons the light is passed through a slit in order to increase spectral resolution and wavelength-calibration reliability. Additional 'instrumental' broadening comes from dispersion caused by the telescope and spectrograph optics and sets a limit on the resolution of a spectrograph. The instrumental broadening can be assessed by fitting Gaussian functions to the emission lines in the spectra of arc lamps taken to provide a wavelength calibration. The slit should generally be made sufficiently narrow so that the atmospheric broadening is of a similar size to the instrumental broadening. The slit width is generally about one arcsec and projects onto about two pixels on the CCD (in the wavelength-dispersion direction). Wider slits will allow more light through but will blur spectra and make RVs derived from them less precise. A spectrograph should be set up so that one resolution element projects onto at least two pixels of the CCD detector (Nyquist's theorem).

The dispersion of a spectrograph is defined to be the number of Ångströms per millimetre at the detector. This is often taken to mean Ångströms per pixel for CCD detectors. The sampling of a spectrograph is the number of pixels per resolution element. The resolution of spectra observed using a spectrograph, $\Delta\lambda$, is the number of Ångströms per resolution element (which should be at least two pixels). The resolving power of a spectrograph is

$$R = \frac{\Delta\lambda}{\lambda} \quad (93)$$

where λ is the wavelength of an observation.

10.3.1 Reduction of CCD grating spectra

CCD images from spectrographs (Fig. 79) must be debiased and flat-fielded in a similar fashion to photometric data. Flat fields are obtained by exposing an image whilst the spectrograph slit is illuminated by a tungsten lamp, which produces a continu-

²⁷ <http://www.starlink.rl.ac.uk/star/docs/sun45.htx/sun45.html>

²⁸ <http://www.starlink.rl.ac.uk/star/docs/sun214.htx/sun214.html>

²⁹ <http://www.astro.ex.ac.uk/people/timn/Photometry/description.html>

³⁰ <http://www.astro.ex.ac.uk/people/timn/>

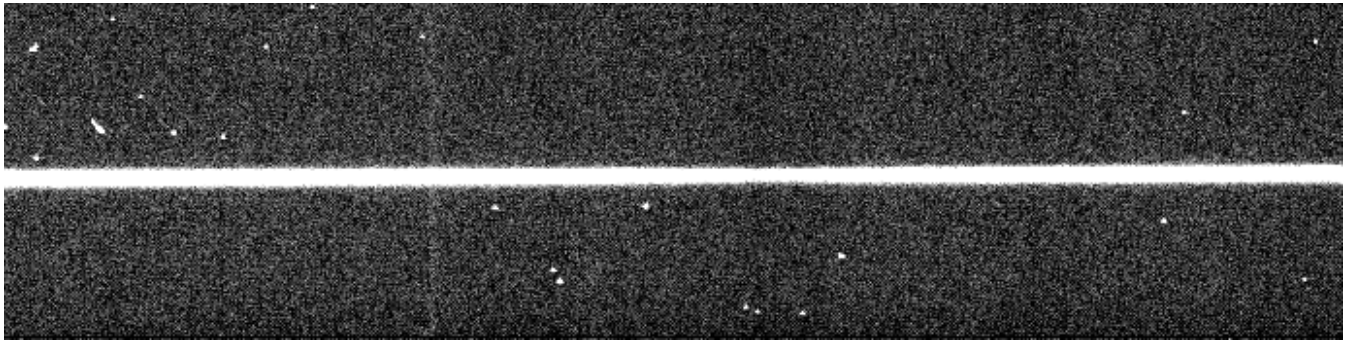


Figure 79. Example CCD image containing a portion of spectrum from a grating spectrograph. The spectrum is of V615 Per (Sec. 14) and the image is centred around the $H\gamma$ line (4340 Å).

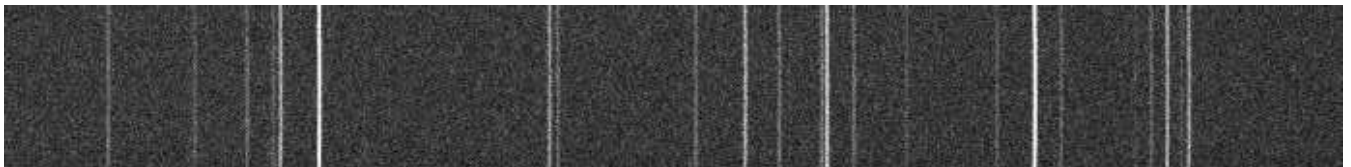


Figure 80. CuAr arc lamp spectrum showing the emission lines used to wavelength-calibrate the spectrum shown in Fig. 79.

ous spectrum. The wavelength dependence of the intensity of the tungsten lamp should be removed by fitting and subtracting a one-dimensional polynomial, leaving only the small-spatial-scale variations in pixel efficiency.

A wavelength calibration must also be applied to the observed spectra. This is particularly important when the spectra are going to be analysed to find RVs as any inaccuracy in the wavelength calibration causes an error of the same size in the derived RVs. Emission-line spectra (Fig. 80) are taken by illuminating the spectrograph slit with the light from an arc lamp (such as copper-argon or copper-neon). The rest wavelengths of the emission lines are known from laboratory studies and allow a wavelength to be associated with each pixel. Spectrographs and telescopes flex slightly when they are moved to point at different areas of the sky, so arc spectra should be taken immediately before and after each science exposure whilst the telescope is still pointing at the science target. Once the emission lines have been analysed, the wavelength scale they give can be applied to the science spectrum.

The extraction of a one-dimensional spectrum from a two-dimensional image can be done using aperture techniques or optimal extraction techniques in similar ways to CCD photometry (sections 10.2.6 and 10.2.8). With grating spectrographs the sky background light can be estimated from portions of the CCD image which are close to the stellar spectrum and receive light through the spectrograph slit. One-dimensional ‘apertures’ are defined to enclose the area containing the stellar spectrum and the background light for each CCD image column. Extraction then proceeds in the same way as for photometry (Horne 1986; Marsh 1989). The optimal extraction of grating spectroscopy has been implemented by T. Marsh³¹ (Marsh 1989) in the software PAMELA and MOLLY.

10.4 Échelle spectrographs

Échelle spectrographs use an échelle to disperse incoming light in wavelength. Échelles produce highly-dispersed light split into many ‘orders’, where each order contains perhaps 100 Å of the spectrum. These orders are then passed through an element which provides a small wavelength dispersion in the second spatial dimension (‘cross-dispersion’), so separates the different orders from each other before they are focussed onto the light detector. Échelle

spectrographs are often fed using optical fibres, to increase their thermal and mechanical stability by allowing them to be floor-mounted rather than bolted to the telescope, and a separate sky background spectrum must be provided using a different optical fibre to the science target but passed through the same spectrograph optics. An example CCD image from an échelle spectrograph is shown in Fig. 78. The reduction of échelle data is very complex and will not be discussed here.

10.5 Observational procedure for studying dEBs

The acquisition of data for the study of dEBs has some complexity due to the need to observe at the correct orbital phases.

10.5.1 CCD photometry

The observation of light curves for dEBs requires complete coverage of the light variation through both primary and secondary eclipses, plus regular observations outside eclipse to provide the reference light level and constrain effects such as reflection. The minimum requirements for a light curve to be definitive are discussed in Sec. 13.2.

Using a telescope and CCD imager is a good way to obtain light curves of a dEB. During eclipses the dEB must be monitored continually by repeatedly imaging it and a comparison star. Differential photometry can then be performed on the images to obtain the light curve. It is advisable to observe light curves in several passbands to provide independent photometric datasets. This can be done by cycling continually through several passbands whilst observing but will obviously decrease the amount of data contained in each light curve. A balance must therefore be struck between obtaining several light curves and ensuring that each has sufficient data to be useful. The best approach depends strongly on the length and depth of the eclipses of the dEB, its brightness and the passbands being used, on the amount of telescope time available, and on the observational efficiency achievable with the telescope and imager.

10.5.2 Grating spectroscopy

Obtaining grating spectroscopy of dEBs is more interesting and time-efficient than observing light curves. The requirements for a definitive spectroscopic orbit are discussed in Sec. 11.4, but

³¹ <http://www.warwick.ac.uk/staff/T.R.Marsh/index.html>

mainly comprise regular observations throughout the orbital period of a dEB. As continual monitoring is not required, spectra can be obtained to determine the orbits of several dEBs at once. One observing run can therefore yield definitive orbits for many dEBs. The observing run must be long enough to cover most of the orbital phases of each dEB for good spectroscopic orbits to be obtained.

When acquiring spectroscopic observations of a dEB, it is a good idea to observe a spectrum when the RV separation of the two stars is minimal. This spectrum can be useful as a template spectrum when determining RVs by cross-correlation. It is also good practice to observe one spectrum with a very high signal to noise and a large RV separation between the two stars. This spectrum can then be analysed using spectral synthesis techniques to find more accurate T_{eff} and rotational velocities for the stars.

11 SPECTROSCOPIC ORBITS

11.1 Equations of spectroscopic orbits

A full derivation of the equations of motion of binary stars in an elliptical orbit is lengthy and readily available from other sources (e.g., Hilditch 2001, p. 38). Therefore I shall quote the resulting equations which are of use to the study of spectroscopic binary stars. For these stars, the RVs of one or both components are observed at certain times, allowing the derivation of the mass function (for single-lined spectroscopic binaries) or the individual masses and overall stellar separation (for double-lined spectroscopic binaries with a known or assumed orbital inclination).

Radial velocity (RV) as a function of time is given by:

$$V_r = K[\cos(\theta + \omega) + e \cos \omega] + V_\gamma \quad (94)$$

where θ is the orbital phase in radians, ω is the longitude of periastron of the orbit, e is the orbital eccentricity, V_γ is the systemic velocity and the velocity semi-amplitude K is

$$K = \frac{2\pi a \sin i}{P\sqrt{1-e^2}} \quad (95)$$

where a is the orbital semimajor axis, i is the orbital inclination and P is the orbital period.

From the definition of K the minimum masses of the stars are

$$M_{1,2} \sin^3 i = \frac{1}{2\pi G} (1-e^2)^{\frac{3}{2}} (K_1 + K_2)^2 K_{2,1} P \quad (96)$$

where G is the gravitational constant, and

$$a_{1,2} \sin i = \frac{\sqrt{1-e^2}}{2\pi} K_{1,2} P \quad (97)$$

$$a \sin i = a_1 \sin i + a_2 \sin i \quad (98)$$

Using the usual astrophysical units of solar masses, period in days and velocities in km s^{-1} , we obtain:

$$M_{1,2} \sin^3 i = 1.036149 \times 10^{-7} (1-e^2)^{\frac{3}{2}} (K_1 + K_2)^2 K_{2,1} P \quad (99)$$

where the value of the numerical constant has been recommended by the International Astronomical Union (Torres & Ribas 2002). Note that Andersen (1997) gives a different value of 1.036055×10^{-7} . We also get

$$a \sin i = 1.3751 \times 10^4 \frac{\sqrt{1-e^2}}{2\pi} (K_1 + K_2) P \quad (100)$$

where the projected separation, $a \sin i$, is in kilometres.

In the case of single-lined spectroscopic binaries we can calculate the mass function

$$f(M) = \frac{1}{2\pi G} (1-e^2)^{\frac{3}{2}} K^3 P = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} \quad (101)$$

where the factor $\frac{1}{2\pi G}$ has the numerical value of 1.036149×10^{-7} as used in eq. 99. The significance of the mass function is that it provides an estimation of the mass of the secondary component of a single-lined spectroscopic binary.

11.2 The fundamental concept of radial velocity

The classical definition of RV is the component of the velocity of a star along the line of sight of the observer (e.g., Kaufmann 1994; Zeilik & Gregory 1998). Whilst this definition has the advantage of being simple, the observed spectroscopic RV of a star is somewhat different to its actual motion through space due to several physical effects. This has prompted the International Astronomical Union³² to re-examine the fundamental concept of RV and provide a more precise definition (Lindgren & Dravins 2003).

There are several physical effects which cause observed spectroscopic RVs to differ from the actual RVs of celestial bodies (Lindgren & Dravins 2003):-

- Gravitational redshift is the increase in wavelength of photons caused by their escape from the gravitational potential of the star which emitted them. The term also encompasses the slight blueshift due to the photons falling into the gravitational potential well of the Sun and the Earth before being detected by observers. The gravitational redshift effect is of the order of 1 km s^{-1} for MS stars, increasing to 30 km s^{-1} for white dwarfs. It is usually unimportant because it affects all similar stars in a similar way, and is constant over long periods of time for individual stars. The velocity change due to gravitational redshift is given by

$$V_{\text{grav}} = \frac{GM}{rc} \quad (102)$$

where G is the gravitational constant, M is the mass of the emitting body, r is the distance the photon is emitted from the centre of the body and c is the speed of light.

- Convective blueshift is the decrease in wavelength caused by convective motions on the surfaces of stars of types F and later. These convective motions cause stellar surfaces to be divided into columns of rising and falling gas, visible as the granulation effect on the surface of our Sun. The rising and falling components occupy roughly equal areas of a stellar surface but the convective velocities cause spectral lines to be blueshifted from rising columns and redshifted from falling columns. As the rising material is hotter, it is brighter, so it contributes more to the stellar flux, so the overall effect is a blueshift. This shift is of the order of 1 km s^{-1} for F stars, falling to 200 m s^{-1} for K stars. The magnitude of the effect is greater at shorter wavelengths but, again, is usually unimportant as its effects cause a constant RV offset for a specific star.

- The rotation of stars causes spectral line profiles to become asymmetric (Gray & Toner 1985).

The above effects have recently become more important due to improvements in instrumentation, so a precision of 1 m s^{-1} is possible on bright stars, and due to the development of the concept of the astrometric RV. The analysis of this effect can provide accurate individual RVs of a group of stars with accurate trigonometric parallaxes and the same motion in space. Astrometric RVs are not determined spectroscopically so are not subject to the difficulties and limitations given above (see Dravins, Lindgren & Madsen 1999 and subsequent works).

The total effect of convective blueshift and gravitational redshift was investigated by Pourbaix et al. (2002) for the components of the nearby visual binary α Centauri. The estimated difference between the two components, $215 \pm 8 \text{ m s}^{-1}$, is much smaller than that predicted by hydrodynamical model atmosphere calculations. This technique may provide a valuable constraint on theoretical model atmospheres in the future.

11.3 RV determination from observed spectra

There are two major difficulties in determining double-lined spectroscopic orbits from observations.

The first problem is that the spectral lines of the secondary star, which is usually dimmer than the primary star, are diluted by the continuum emission of the primary star. It can be impossible to find signatures of the secondary component in spectra if the light ratio is very small. For a given mass ratio, the light ratio

³² <http://www.iau.org/>

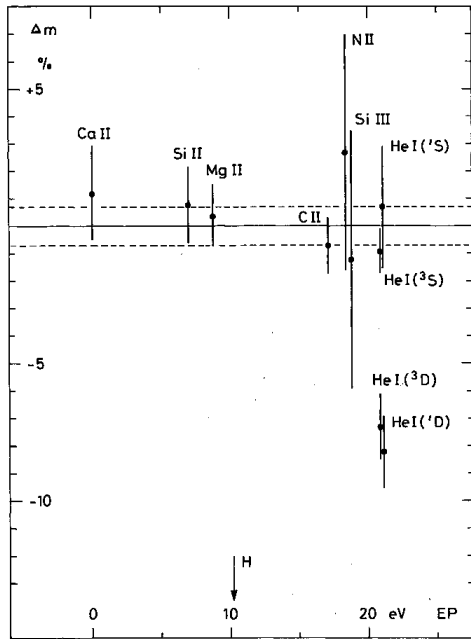


Figure 82. Percentage deviation of the masses of CV Velorum derived using the lines of individual ions, plotted against excitation potential. The reference masses are averages of the values for several of these ions. Taken from Andersen (1975a).

in the IR is usually much closer to unity than the light ratio in the optical (Mazeh et al. 1995) because cooler stars are redder. Whilst spectra intended for RV work are usually targeted towards the blue, observations in the IR can be very useful in detecting and measuring the spectral lines of secondary stars. If the luminosity, L , and mass, M , of a star are related by the expression $L \propto M^x$ (Mazeh et al. 1995) then for the B and V passbands the quantity x is approximately 9.8 and 8.3, respectively. These values are dependent on stellar mass, age and metallicity but illustrate the problem well.

The second problem is that the spectral lines of one star can be distorted by the presence of spectral lines due to a second star. This blending can cause the centres of the lines to be apparently displaced towards each other, lowering the masses derived from the spectra. This affects hydrogen and diffuse helium lines most strongly as they are much wider than metallic lines. Whilst the measurement of individual spectral lines can be badly affected by this, more recent techniques for determining radial velocities from composite spectra are much more reliable. This will be covered in more detail below.

11.3.1 RV determination from individual spectral lines

The traditional method of the determination of RVs from observed spectra involves the measurement of the wavelength centres of individual spectral lines, which are then compared with rest wavelengths found in either the laboratory or in high-resolution, high signal-to-noise stellar spectra. This method is ideally suited to the analysis of photographic plate spectra, where the plates are placed inside one of several different types of machine for interactive measurement of spectral line positions. Due to the small number of sharp (metallic) spectral lines exhibited by many early-type stars, this method is often competitive with more recent techniques of RV analysis of these stars, and has the advantages of simplicity and robustness.

One problem with the measurement of individual spectral lines is that the line centres may be displaced in wavelength by interference from other nearby lines – the blending effect (Petrie & Andrews 1966). If the interfering lines are from the same star then the blending effect will be constant and therefore easily dealt with. If, however, the interfering lines are from another star, in the case of composite spectra, the effects of blending can be very strong

Table 6. Selected spectral lines indicated in the literature to be good for the determination of RVs in early-type stars. Only the earliest reference is given for each line.

Species	Wavelength (Å)	Reference
Si II	3853	Andersen (1975a)
Si II	3856	Andersen (1975a)
Si II	3862	Andersen (1975a)
He I (^3S)	3867	Andersen (1975a)
Fe I	3878.5	Andersen (1975b)
C II	3919	Andersen (1975a)
C II	3920	Andersen (1975a)
Ca II	3933	Andersen (1975a)
N II	3995	Andersen (1975a)
Fe I	4071.7	Andersen (1975b)
Si III	4089	Burkholder et al. (1997)
Si III	4116	Burkholder et al. (1997)
He I (^3S)	4120	Andersen (1975a)
Si II	4128.0	Popper (1982)
Si II	4130.9	Popper (1982)
Fe I	4143.6	Andersen (1975b)
He I (^1S)	4169	Andersen (1975a)
Si IV	4212.4	Hensberge et al. (2000)
Sr II	4215.7	Andersen (1975b)
C II	4267	Andersen (1975a)
Fe II	4351.7	Andersen (1975b)
He I (^1S)	4437	Andersen (1975a)
Mg II	4481	Andersen (1975a)
Ti II	4501.3	Andersen (1975b)
Fe II	4508.3	Andersen (1975b)
Si III	4552	Andersen (1975a)
Si III	4567	Popper & Guinan (1998)
Ti II	4572.0	Andersen (1975b)
Si III	4574	Popper & Guinan (1998)
O II	4591.0	Hensberge et al. (2000)
O II	4596.2	Hensberge et al. (2000)
Fe II	4583.8	Andersen (1975b)
N III	4634	Burkholder et al. (1997)
N III	4641	Burkholder et al. (1997)
C II	4650	Burkholder et al. (1997)
Si IV	4654.3	Hensberge et al. (2000)
O II	4661.6	Hensberge et al. (2000)
He I (^3S)	4713	Andersen (1975a)
Si II	6347.1	Zwahlen et al. (2004)
Si II	6371.4	Zwahlen et al. (2004)

and difficult to quantify. Hilditch (1973) suggests that spectral lines should be used for RV determination only if the flux returns to the continuum level on both sides of the line. Andersen et al. (1987) found, during a study of V1143 Cygni using spectral lines measured from photographic plate spectra, that line blending can lower the derived RV difference in a double-lined spectrum without distorting the shape of the spectroscopic orbit, so blending cannot necessarily be detected by analysing the residuals of a spectroscopic orbital fit. Andersen (1991) suggests that spectra of a high signal to noise ratio should be obtained so radial velocities can be measured from the (weak) metal lines rather than the (strong) helium or hydrogen lines.

Several researchers have investigated the best spectral lines for measurement of RVs and have generally found that hydrogen and helium lines should be avoided wherever possible. During the study of the EB PV Puppis (spectral type A8 V, $T_{\text{eff}} = 6920$ K), Vaz & Andersen (1984) found that the velocity semiamplitudes derived from analysis of hydrogen lines were 72% of those derived using sharp metal lines. Andersen (1975a) noted that the helium lines in the spectrum of CV Velorum (spectral type B2.5 V, $T_{\text{eff}} = 18300$ K), gave velocity semiamplitudes 8% smaller than those derived from sharp metallic lines.

Andersen (1975a) studied many blue spectra of CV Vel and suggested several spectral lines which are good for the determina-

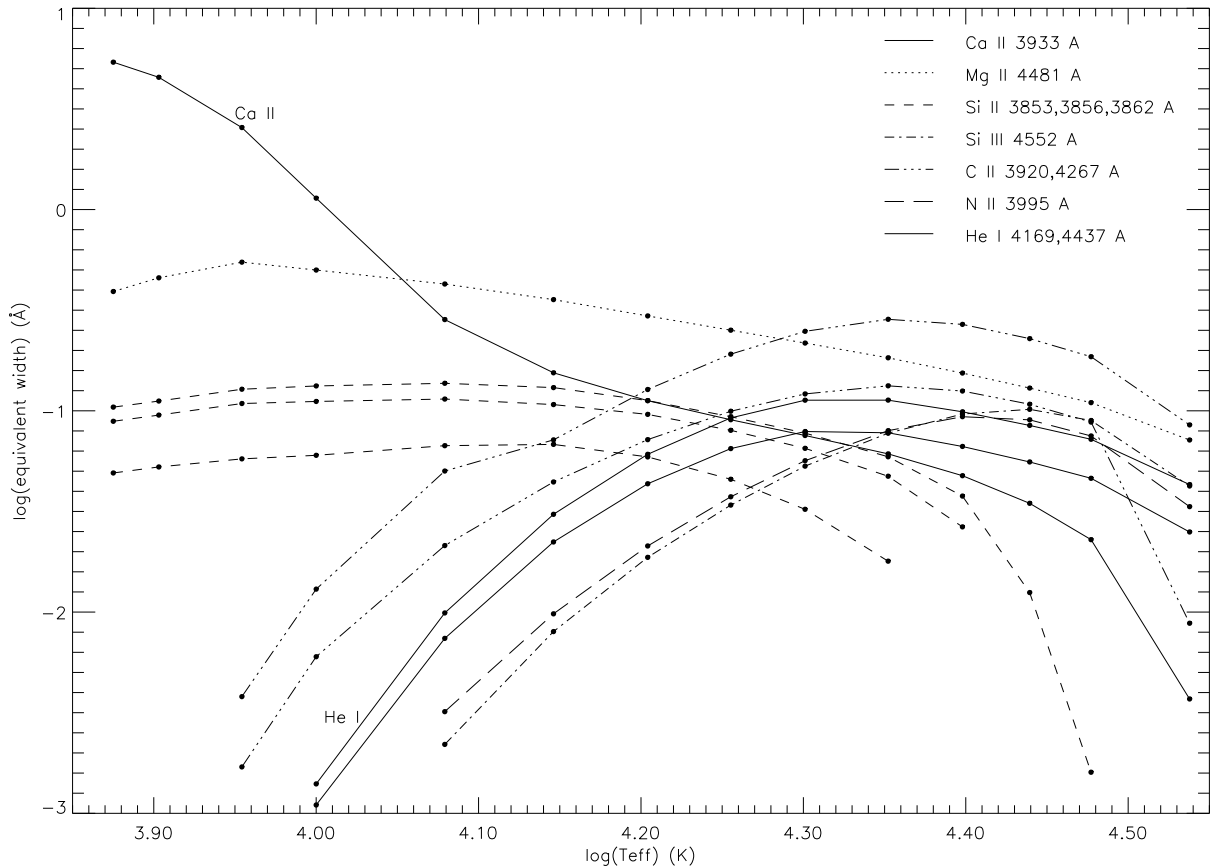


Figure 81. Variation of the equivalent widths, with T_{eff} , of the spectral lines given by Andersen (1975a) as good for deriving RVs of early-type EBs, with particular reference to CV Velorum ($\log T_{\text{eff}} = 4.26$ K). The data were generated using UCLSYN (Sec. 4.4.2).

tion of RVs in composite spectra. He noted that it was important to avoid hydrogen lines and diffuse helium lines (at wavelengths of 3819, 4009, 4026, 4143, 4388, 4471 Å) but that sharp helium lines at 3867, 4120, 4169, 4437, 4713 Å were reliable. Fig. 82 shows the masses derived for CV Vel from different spectral lines against the final adopted values. Mg II 4481 Å is the most reliable line despite it being a close triplet. Fig. 81 shows the equivalent widths of the spectral lines selected as good by Andersen for CV Vel, against T_{eff} . Note that the Mg II 4481 Å line is strong over a wide range of T_{eff} s, making it the best individual line for derivation of RVs in early-type stars (e.g., Popper 1980). For spectral types later than mid-A, there is a profusion of spectral lines and the main problem faced in RV determination is the identification of lines which are not blended with neighbouring lines. For stars of spectral types mid-B to late-O, there are several useful helium lines and a large number of weak, sharp O II lines in the blue spectral region. For stars earlier than late-O, very few lines are visible at optical wavelengths (the high level of ionisation means there are many lines in the UV), partially due to the generally fast rotation (Popper & Hill 1991), and often only helium lines are reliable providers of RV information. Table 6 gives several spectral lines, selected from the literature, which are considered to be reliable sources of RV information.

11.3.2 Radial velocity determination using one-dimensional cross-correlation techniques

The cross-correlation technique can be used to determine the RV shift of a star, or several stars if the observed spectra are composite, by comparison with a template spectrum. First introduced by Simkin (1974), the method was further developed by Tonry & Davis (1979). The cross-correlation function is

$$C_{f,g}(s) = \frac{\sum_n f(n)g(n-s)}{N\sigma_f\sigma_g} \quad (103)$$

where $f(n)$ is the observed spectrum, $g(n)$ is the template spectrum, s is a shift in velocity, $g(n-s)$ is a velocity-shifted template spectrum, N is the number of points in each spectrum, and the root-mean-squared values of the spectra are given by

$$\sigma_f^2 = \frac{1}{N} \sum_n f(n)^2 \quad (104)$$

$$\sigma_g^2 = \frac{1}{N} \sum_n g(n)^2 \quad (105)$$

The velocity shift between the observed and template spectra is estimated from the location, $\hat{s}$, of the maximum of the cross-correlation function $C_{f,g}$. The method of cross-correlation effectively involves the comparison between the observed spectrum and a velocity-shifted template spectrum for a range of velocity shifts, the derived RV difference being where the two spectra have best agreement.

In choosing the template spectrum it is important, as is clear from eq. 103, that it matches the observed spectrum as closely as possible. A close match is useful when studying single-lined spectra, but can be vital when analysing composite spectra. In this case, the spectral lines of each star will cause a local maximum in the cross-correlation function. If the maxima are well-separated in velocity, this causes no significant problem, but if the RV separation of the two stars is significantly less than the sum of their spectral line broadenings then the individual maxima in the cross-correlation function will become blended in a very similar way to individual spectral lines.

Template spectra can be observed spectra of standard stars or synthetic spectra calculated using stellar atmosphere models. The advantage of using observed spectra is that the researcher is utilizing only observational data, and so avoiding the use of any theoretical calculations. The disadvantages are that it takes telescope time to obtain template spectra, and the available templates may not be a very good match to the spectrum being

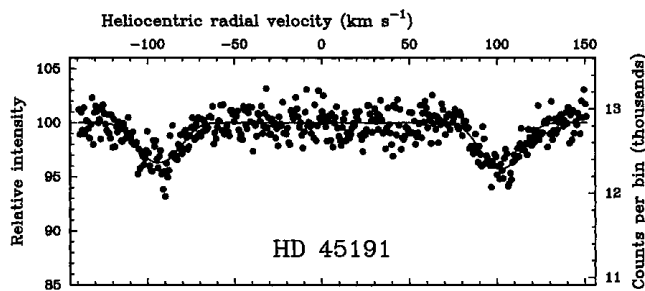


Figure 83. An example cross-correlation function observed with the Cambridge 1.5 m telescope and CORAVEL photoelectric spectrograph. Each dip corresponds to the RV of one star in this double-lined system. Taken from Griffin (2001).

analysed. An alternative possibility is to use synthetic spectra as templates. Whilst this means that careful steps must be taken to minimise the dependence of the result on theoretical calculations, it has the advantage that synthetic spectra are more readily available and are free of observational noise. Having no observational noise, the results will be more precise, and the synthetic spectrum can be carefully adjusted to best match the observed spectrum just by use of a desktop computer. However, systematic biases may occur if the synthetic spectrum has missing lines, or similar difficulties. Such problems are negligible for the analysis of relatively well-understood stars such as mid-B to G dwarfs.

The light ratios of double-lined binary systems can be found by comparison of the areas under the maxima of the cross-correlation function (e.g., Howarth et al. 1997) because these areas are approximately constant for different rotational velocities, but differences between the intrinsic stellar spectra can affect the area under the maxima of the cross-correlation function.

11.3.3 Direct observation of cross-correlation functions

An alternative to using cross-correlation in the analysis of observed spectra is to obtain cross-correlation functions directly at the telescope. This method was suggested by Griffin (1967) and the resulting CORAVEL spectrographs are or have been available at several telescopes. CORAVEL spectrographs have the usual spectroscopic elements but the light detector is a simple photoelectric photometer. A spectral mask is located in the focal plane, oriented along the direction of dispersion. This mask is a physical representation of an observed spectrum – the Cambridge CORAVEL spectrograph uses a mask based on the spectrum of Arcturus (which is a giant with a spectral type of K1) – where light is allowed through slits placed at the centres of spectral lines. The mask is shifted along the direction of dispersion and the photometer records the amount of light which passes through as a function of the corresponding velocity offset. At the physical shift corresponding to the RV of the observed star, less light is allowed through the mask as the spectral lines align with the slits in the mask. The resulting dip is fitted with a Gaussian function to find the actual RV of the star. A K1 giant was chosen as a template because this has a large number of spectral lines so can be reliably applied to a wide range of spectral types. The systematic errors due to template mismatch tend to cancel out so are negligible for all stars with spectral types between mid-F and mid-M.

Griffin (Griffin & Emerson 1975, and subsequent papers in Griffin's series in the *Observatory Magazine*) is managing the longest-running RV determination project in the UK, allowing him to concentrate on long-period stars which show very little orbital motion and therefore require analysis over long periods of time with the same observing equipment. An example double-lined CORAVEL cross-correlation function, from paper No. 160 of Griffin's series, is shown in Fig. 83. Note that only stars with spectral types relatively similar to the physical mask (F, G and K stars for a mask based on Arcturus) can be reliably observed with CORAVEL instruments. Andersen et al. (1987) studied the F-type dEB V1143 Cygni using both photographic and photoelec-

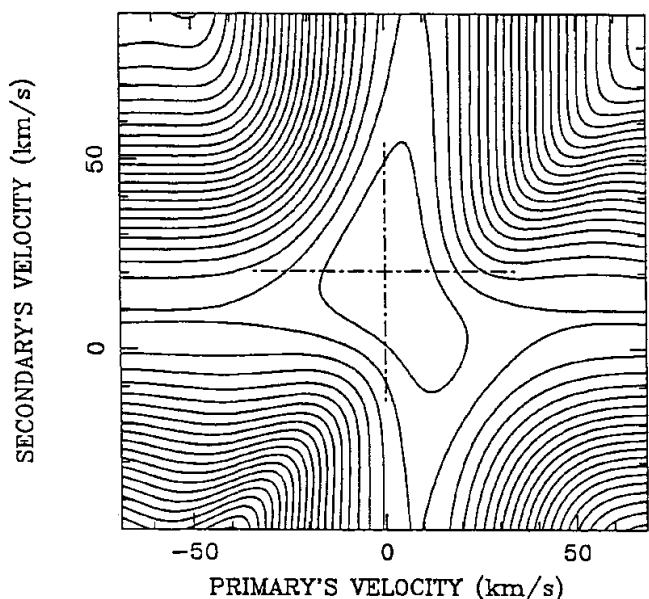


Figure 84. An example contour plot of the two-dimensional cross-correlation function around the global correlation maximum. The dashed lines are parallel to the axes and go through the maximum correlation value. Taken from Zucker & Mazeh (1994).

tric spectroscopy. Separate analyses gave identical results, but the CORAVEL data required much less telescope and reduction time. Therefore, for certain types of stars, the use of CORAVEL is preferable to photographic techniques. In recent works (e.g., Griffin 2004) the accuracy of the derived RVs has reached 0.25 km s^{-1} per observation for cool giant stars.

11.3.4 Radial velocity determination using two-dimensional cross-correlation techniques

The main shortcoming of the technique of cross-correlation in the determination of stellar RVs is that the cross-correlation function in composite spectra contains contributions from several stars, which may interfere with each other and bias the derived RVs. Zucker & Mazeh (1994) and Mazeh et al. (1995) extended cross-correlation to explicitly allow for contaminating spectral lines from a second star. They called this two-dimensional cross-correlation algorithm TODCOR. The cross-correlation function is

$$R_{f,g_1,g_2}(s_1, s_2, \alpha) = \frac{\sum_n f(n)[g_1(n-s_1) + \alpha g_2(n-s_2)]}{N \sigma_f \sigma_g(s_1, s_2)} \quad (106)$$

where $g_1(n)$ and $g_2(n)$ are the template spectra, s_1 and s_2 are velocity shifts, α is the intensity ratio of the two stars, which can be evaluated analytically, and

$$\sigma_g(s_1, s_2)^2 = \frac{1}{N} \sum_n [g_1(n-s_1) + \alpha g_2(n-s_2)]^2 \quad (107)$$

This method effectively involves the simultaneous comparison between the observed spectrum and two template spectra, over a range of velocity shifts for each template spectrum. R_{f,g_1,g_2} is a two-dimensional function where the global maximum gives the RV shifts of both stars. Blending is much less important because two template spectra are fitted simultaneously, so lines which would otherwise contaminate the RV determination of the other star are explicitly dealt with (Latham et al. 1996). An example cross-correlation function is shown in Fig. 84.

The comments in the previous section on the choice of template spectra are equally valid for two-dimensional cross-correlation, but one important advantage of TODCOR is that the template spectra do not have to be the same – in fact it is helpful if they are not – so each template can be a close match to one of the two stars. This was not possible with one-dimensional cross-

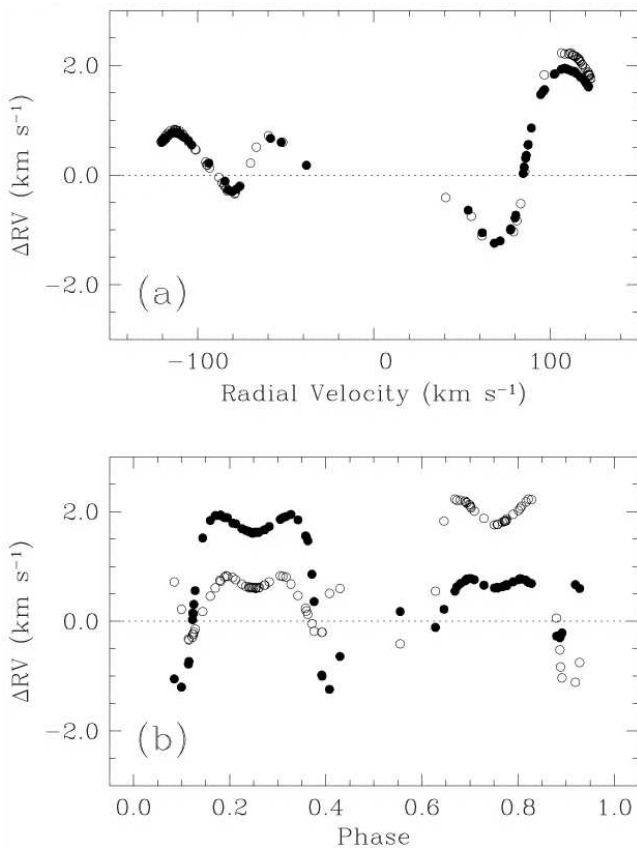


Figure 85. Systematic errors of the RVs derived by using TODCOR to analyse the M dwarf dEB YY Geminorum. The systematic error is shown as a function of RV and of orbital phase. Open circles refer to the primary star and filled circles to the secondary star. Taken from Torres & Ribas (2002).

correlation where one template had to fit the spectra of all the stars in the spectrum.

One problem with this technique concerns the edges of the spectra. As the observed and template spectra are required to be the same length for cross-correlation, but a velocity shift is usually imposed, parts of one spectrum extend beyond the end of the other spectra. These parts do not contribute to the correlation function so can lower the overall correlation value, biasing the derived RVs. A simple compensation method is to taper the ends of each spectrum, but whilst this smooths out the bias it cannot remove it entirely. An alternative method is to explicitly assess the systematic RV error by analysing synthetic spectra with known RVs and observational noise added. An example graph of systematic errors, which were removed from the individual velocities, is given in Fig. 85.

Zucker, Torres & Mazeh (1995) extended TODCOR to triple-lined spectra where the correlation function is

$$R_{f,g_1,g_2,g_3}(s_1, s_2, s_3, \alpha, \beta) = \frac{\sum_n f(n)[g_1(n - s_1) + \alpha g_2(n - s_2) + \beta g_3(n - s_3)]}{N \sigma_f \sigma_g(s_1, s_2, s_3)} \quad (108)$$

This is effectively a three-dimensional function where three template spectra are simultaneously correlated against one observed spectrum. As such, it is quite expensive in terms of computational time, and extensions to four or more templates would be prohibitively expensive. However, the stellar intensity ratios α and β can still be evaluated entirely analytically.

Zucker et al. (2003) have applied TODCOR to multi-order échelle spectroscopic observations. In this case, cross-correlation over the whole spectrum is problematic because of the gaps between individual orders, so orders were cross-correlated individually and the resulting functions combined, using the maximum-likelihood technique of Zucker (2003), to produce one function.

11.3.5 RV determination using spectral disentangling

The spectral disentangling technique can be used to find the individual spectra of a double-lined binary star from several observed spectra. The algorithm requires a set of observed spectra together with the RVs of both stars for each spectrum and outputs estimated individual disentangled spectra with a calculated residual of the fit. The RVs can be determined by minimising the residual value, either directly or by fitting a spectroscopic orbit. The algorithm was introduced by Simon & Sturm (1994) and applied to the high-mass EBs DH Cephei (Sturm & Simon 1994) and Y Cygni (Simon et al. 1994). The method was intended to help in the derivation of RVs when the spectral lines of one star were badly blended with those of the other star, and to create individual spectra which were suitable for spectroscopic analysis in the same way as single-lined spectra.

Hynes & Maxted (1998) investigated spectral disentangling and found that the quality of the results was dependent mainly on the total exposure time of the observed spectra, although Simon & Sturm (1994) suggest the minimum useful signal-to-noise ratio is 10. Hynes & Maxted were unable to find a robust method of estimating the errors in the derived RVs because the disentangling process is not strictly equivalent to least-squares minimisation. It is still not clear if disentangling can provide robust errors (P. F. L. Maxted, private communication), but Ilić (2003) has pioneered the estimation of uncertainties by fitting spectroscopic orbits to observed spectra by disentangling. The code FDBINARY (Ilić 2003) calculates the best-fitting spectroscopic orbits for several data subsets where each subset contains $N - 1$ observed spectra, where N is the total number of spectra. This gives $N - 1$ estimations of the spectroscopic parameters, which can then be subjected to straightforward error analysis. This method has been used by Zwahlen et al. (2004) to determine a spectroscopic orbit in a double-lined binary system exhibiting severe blending of spectral lines.

An alternative approach to the use of singular value decomposition of matrix equations by Simon & Sturm (1994) is to use Fourier techniques as implemented in KOREL (Hadrava 1995). KOREL has been used in several studies, for example Hensberge, Pavlovski & Verschueren (2000).

A simple approach to the determination of individual spectra from double-lined observed spectra is piecewise reconstruction of individual spectral lines (Ferluga et al. 1997). This method uses the fact that in early-type stars with few spectral lines, RV shifts due to orbital motion will move part or all of a secondary line to a wavelength where the primary spectrum is entirely continuum. This allows the shape of part or all of the line from both stars to be determined, and through iteration the whole shapes of the lines can be found using only two spectra. However, this method is much less advanced than spectral disentangling and is very sensitive to observational noise beyond the second iteration, so has not been pursued further.

11.3.6 RV determination using Doppler tomography

Doppler tomography is a method of separating the spectra of multiple stars. It is used in medical software to analyse images of humans, from different angles, to determine the three-dimensional structure of the interior of the body. In the analysis of composite spectra, the main principle is that all the observed composite spectra of one binary star can be considered to be “images” of two individual spectra (which are next to each other but slightly separated) viewed from slightly different angles (Bagnuolo & Gies 1991). The Doppler tomography algorithm requires estimated spectra and individual RVs of the stars in each spectrum. It then iteratively refines the RVs and estimated spectra by least squares until they best fit the observations. It is usually modified to fit individual spectra and a spectroscopic orbit, which is used to predict the RVs of stars in different observed spectra. The estimated spectra are usually spectra observed when the two stars have the same RV so their spectrum appears single-lined, but the algorithm is able to take flat continuum as input without affecting the final results (Bagnuolo, Gies & Wiggs 1992).

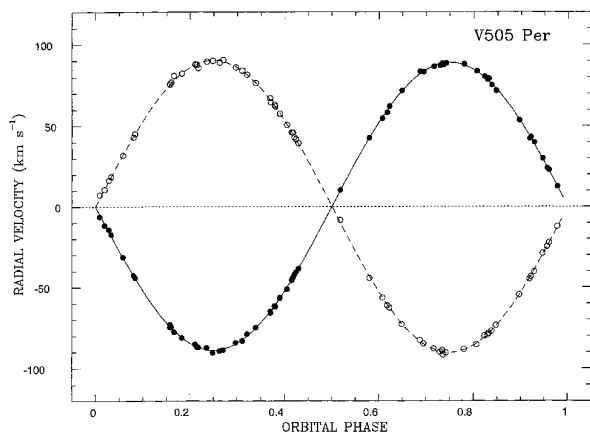


Figure 86. Example of a definitive spectroscopic orbit, for the dEB V505 Persei. Radial velocities were derived using one-dimensional cross-correlation of synthetic spectra against CCD spectra observed using an échelle spectrograph. Velocities for the primary star are shown by filled circles, and for the secondary star are shown using open circles. Taken from Marschall et al. (1997).

Torres & Ribas (2002) used Doppler tomography to investigate the starspots present on the components of the M dwarf dEB YY Geminorum.

11.4 Determining spectroscopic orbits

It is clear from the above discussion that determination of the gravitational masses of dEBs requires measurement of only the velocity semiamplitudes and the orbital inclination (Popper 1967). Under the assumption of a circular orbit, these quantities can be found using only four RVs measured from two spectra (e.g., Wilson 1941), but accurate and robust results require at least 25 RVs with individual uncertainties of 1 km s^{-1} (Andersen 1991). However, several complications exist:–

- The measured systemic velocities for the two stars may differ. This is an observational effect caused by (Popper & Hill 1991):–

- assumption that the orbit is circular when it has a small eccentricity,
- spectral line profile differences between the two stars,
- blending effects, where the spectral lines of one star cause the spectral line centres of the other star to shift slightly, particularly if the rotational velocities of the two stars are different (Popper 1974),
- small-number statistics,
- stellar winds or gas streams modifying the spectral line profiles (the Barr effect; Barr 1908; Howarth 1993),
- the use of different spectral lines or regions for determination of the RVs of the two stars.

- The Rossiter effect causes asymmetric spectral line profiles, shifting the observed velocity centre away from the actual RV of the star. As most spectral line profiles depend mainly on rotational broadening, different parts of a star contribute to different parts of a spectral line. Therefore if one side of a star is not observed, for example during partial phases of eclipses, part of the spectral line profile is not present in observations, shifting the measured RV value. This effect was first noticed by Rossiter (1924) and an example RV curve is shown in Fig. 87. The Rossiter effect can be allowed for by solving spectroscopic and photometric observations simultaneously using, for example, the Wilson-Devinney code (Sec. 13.1.4). In this case the information it holds on the sizes of the two stars can also be accessed.

- When the exposure time of a spectroscopic observation becomes more than a few percent of the orbital period of the spectroscopic binary under study, the changes in RV of the two stars during the observation become important (Andersen 1975b). This

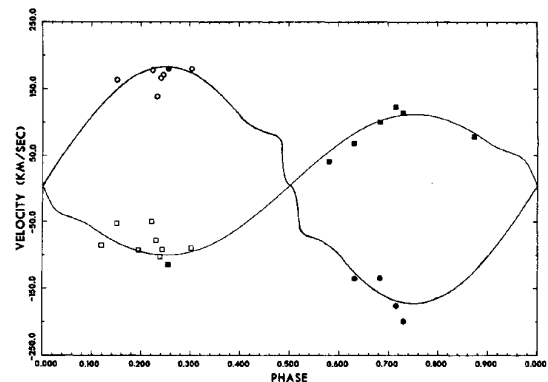


Figure 87. Example of a spectroscopic binary orbit which is not definitive. Radial velocities of the dEB DS Andromedae were obtained by analysing photographic spectra using one-dimensional cross-correlation techniques and the solution was calculated using the Wilson-Devinney code (Sec. 13.1.4 so the predicted Rossiter effect is shown. Taken from Schiller & Milone (1988).

orbital smearing can be corrected by adjusting each wavelength shift by (Lacy 1982)

$$\Delta\lambda = \frac{2\pi\lambda K}{c} \frac{t_{\text{exp}}}{P} \cos\theta \quad (109)$$

where t_{exp} is the exposure time in the same units as the period and θ is the orbital phase in radians. This shift must be applied to individual observations after a preliminary orbit has been calculated. An example of its use is in the study of the dEB CM Lacertae by Popper (1968). CM Lac has an orbital period of 1.6 days but exposure times of 150 minutes (6.5% of the period) were used for the spectroscopic observations.

- For RV work where the precision of an individual observation approaches 100 m s^{-1} , a level now routinely being passed by spectroscopic searches for extrasolar planets (e.g., Butler et al. 1996), relativistic effects due to the position and motion of the Earth and Sun must be allowed for (Griffin et al. 1985).

- Spectroscopic orbital solutions often indicate an uncertainty, σ_e , in the orbital eccentricity, e , which is of the same order as the value itself. In this case the researcher must decide whether the orbit is circular, and the small eccentricity is a spurious effect caused by observational uncertainty, or that the orbit really is eccentric. Arias et al. (2002) note that if $e/\sigma_e > 3.83$ then eccentricity is significant at the 5% level. Several studies have been devoted to the reanalysis of eccentric orbits which were previously assumed circular (e.g., Wilson 1970), and of circular orbits for which a spurious eccentricity was previously found (e.g., Lucy & Sweeney 1971). In the absence of consensus (as indicated by the last two references) it is up to the researcher to decide which procedure is appropriate for each analysis (see Bassett 1978).

- Fast apsidal motion (Sec. 7.2) can cause the orientation of an elliptical orbit to change during a spectroscopic observing campaign. Whilst this can be incorporated into any analysis, the effect should be negligible in the vast majority of cases.

- The spectroscopic binary may be part of a hierarchical triple star system. This can cause a variation in the systemic velocity of the binary. The presence of the third star can be detected by observation of its spectral lines, light travel time effects (for an EB) or by the systemic velocity variation of the close binary.

- Reflection between the components of a close binary will tend to draw the light-centres of the two discs together and reduce the observed RV difference. This effect is significant for MS EBs only if the fractional sum of the radii ($r_A + r_B = \frac{R_A + R_B}{a}$) is greater than 0.4 (Andersen 1975a), or when there is a large difference in luminosity between the two stars.

- The Struve-Sahade effect is that the secondary star tends to exhibit stronger lines when approaching the observer (Struve 1944; Penny, Gies & Bagnuolo 1999). It may result from interaction between the winds of the two stars (Arias et al. 2002).

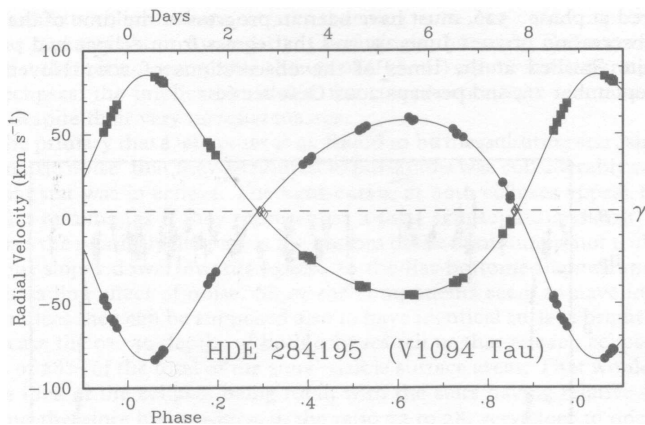


Figure 88. Example of a definitive spectroscopic orbit, for the eccentric dEB V1094 Tauri. Radial velocities were observed using the Cambridge CORAVEL instrument. Squares and circles represent observed primary and secondary velocities, respectively. Taken from Griffin (2003).

Examples of spectroscopic orbits calculated from RV observations are given in Figs. 86, 87 and 88.

11.4.1 SBOP – Spectroscopic Binary Orbit Program

SBOP was written by P. B. Etzel³³ (2004) and is a modification of an earlier code by Wolfe, Horak & Storer (1967). The code fits single-lined or double-lined spectroscopic orbits to the observed RVs of a spectroscopic binary using one of several optimisation schemes based on differential corrections.

11.5 Determination of rotational velocity

The total broadening of metallic spectral lines can easily be measured using a Gaussian function (e.g., Abt, Levato & Grosso 2002). An alternative is to measure broadening from the cross-correlation function of the spectrum against a template, but this must be calibrated on stars with known rotational velocities, or using synthetic template spectra. However, broadening values determined from consideration of cross-correlation functions are better than those from individual spectral lines as they include contributions from all the lines and so are more precise (increased signal to noise) and accurate (they avoid any difficulties associated with individual spectral lines) (Hilditch 2001, p. 79).

The broadening due to the rotational velocity of the star may be smaller than the total broadening. Additional broadening comes from microturbulence and macroturbulence, which are in principle separable from rotational broadening but in reality are highly degenerate. For most types of star the additional broadening is known to be negligible, from the study of dEBs which are rotationally synchronized, but for O stars and evolved B stars the contribution from macroturbulence can be much larger than the contribution from rotation (Trundle et al. 2004).

Popper (2000) used the measured rotational velocities for four late-type dEBs, and an assumption of synchronous rotation, to predict the stellar radii. The relevant equation is

$$V_{\text{synch}} = 50.58 \frac{R}{R_{\odot}} \frac{\text{days}}{P} \quad \text{km s}^{-1} \quad (110)$$

where R is the stellar radius and P is the orbital period (Abt, Levato & Grosso 2002). This analysis is also possible in slightly eccentric orbits under the (slightly more optimistic) assumption of pseudosynchronous rotation (rotational velocity equal to the orbital velocity at periastron). In this case the periastron rotational frequencies of the stars, ω_{peri} are related to the mean

Table 7. Colour indices of the Sun found from calibration.

References: (1) Zombeck (1990); (2) Alonso, Arribas & Martínez-Roger (1996); (3) Edvardsson et al. (1993).

$T_{\text{eff}}(\text{K})$	5770	1
$\log g(\text{cm s}^{-2})$	4.4377	1
$U - B$	0.16 ± 0.03	2
$B - V$	0.62 ± 0.02	2
$V - R$	0.53 ± 0.02	2
$V - I$	0.85 ± 0.02	2
$V - J$	1.13 ± 0.02	2
$V - H$	1.40 ± 0.02	2
$V - K$	1.48 ± 0.02	2
$b - y$	0.406 ± 0.004	3
β	2.601 ± 0.015	2

orbital frequency of the orbit, $\bar{\omega}_{\text{orbit}}$ by (Griffin, Carquillat & Ginetet 2003):

$$\omega_{\text{peri}} = \frac{(1+e)^2}{(1-e^2)^{-3/2}} \bar{\omega}_{\text{orbit}} \quad (111)$$

12 PHOTOMETRY

Photometry is the most fundamental of all observational tools used in astronomy (Crawford 1994). Its main function is to allow us to find out what exists in our Galaxy and Universe. The second function it performs is that of connection. Once bright objects are discovered, they can be classified by how much light we receive from them at different wavelengths. This classification relies on comparing the object being studied to objects with similar photometric characteristics for which much more is known.

An example of this procedure involves the determination of stellar parameters from photometry, using calibrations based on stars for which these parameters are independently known. Using interferometric techniques, observers have determined the apparent angular diameters of some stars which are close to Earth (Sec. 1.3.1). Allied with their distances, measured empirically from their trigonometrical parallaxes (Sec. 9.1.1), and spectrophotometric observations, the luminosities and T_{eff} s of these stars can be found entirely empirically. This allows researchers to estimate T_{eff} s and luminosities of other stars from comparison of their photometric indices with the indices of stars of known properties. Other properties, such as metal abundance and surface gravity, can also be found using calibrations reliant on stars with fundamental determinations of these quantities.

12.1 Photometric systems

The first good photometric passband systems used wide-band filters, to maximise the amount of detected light whilst still not being badly affected by chromatic effects such as atmospheric transmission. Broad-band passband systems, however, must be very well constructed to provide accurate and precise information about stars, and so often are not able to do so. This has led to the construction of intermediate-band systems, such as the Strömgren *uvby* and Geneva *UBB₁B₂VV₁G* passbands, which are much better suited to the classification of most types of stars than the broad-band *UBVR₁JKLMN* passband systems. Broad-band Johnson-style passband systems are currently the most popular with observers, but intermediate-band systems have an important place in many research programmes and can be surprisingly successful at estimating stellar parameters.

The Asiago Database of Photometric Systems³⁴ (Moro & Munari 2000) lists detailed information on the passbands and

³³ <http://mintaka.sdsu.edu/faculty/etzel/>

³⁴ Also available on the internet at

<http://ulisse.pd.astro.it/Astro/ADPS/>

Table 8. Central wavelengths and bandwidths of broad-band passbands. Taken from Moro & Munari (2000).

Passband	Central wavelength (μm)	FWHM (μm)
<i>U</i>	0.36	0.04
<i>B</i>	0.44	0.10
<i>V</i>	0.55	0.08
<i>R</i> (Johnson)	0.70	0.21
<i>I</i> (Johnson)	0.90	0.22
<i>R</i> (Cousins)	0.67	0.15
<i>I</i> (Cousins)	0.81	0.11
<i>R</i> (Eggen)	0.635	0.18
<i>I</i> (Eggen)	0.79	0.15
<i>R</i> (Kron)	0.65	0.23
<i>I</i> (Kron)	0.825	0.15
<i>J</i>	1.25	0.3
<i>H</i>	1.62	0.2
<i>K</i>	2.2	0.6
<i>L</i>	3.4	0.9
<i>M</i>	5.0	1.1
<i>N</i>	10.2	6.0

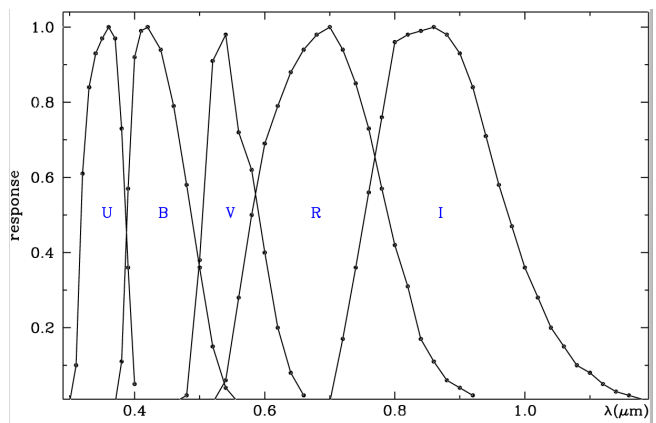
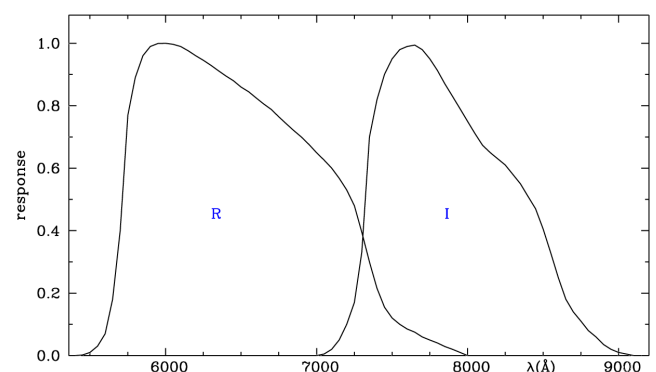
other characteristics of 167 optical, UV and IR photometric systems, starting with the *UVBGR* system of Stebbins & Whitford (1943) and ending with the suggested passbands of the GAIA satellite (Sec. 9.1.1), along with brief descriptions of 34 more systems. The colour indices of the Sun are given in Table 7.

Intermediate band systems have many intrinsic advantages. Firstly, they are defined mainly by their filters because the change in sensitivity of a light detector over 200 Å is usually negligible. The narrower filters can also be carefully targeted to measure the effects of individual features in the spectra of certain stars, resulting in easier and more accurate calibrations. Intermediate-band systems tend to be better defined than broad-band systems, and perhaps used by researchers who have more idea of what they are doing. Using intermediate-band systems rather than broad-band systems is only advantageous if 1% photometric accuracy is achieved (Bessell 1979). It is also important to have passbands very close to the original definition, as unusual stars (e.g., white dwarfs, Population II stars, carbon stars) can have extreme spectral energy distributions (Bessell 1995).

Mermilliod & Paunzen (2003) have studied the interagreement between different sets of photometry and photometric systems in the WEBDA open cluster database³⁵. They conclude that the best photometry, in terms of agreement between different datasets, is photoelectric photometry in the Strömgren system and then the Johnson system (other intermediate-band systems were not considered). Intriguingly, CCD photometry is not as good as photoelectric photometry for both Strömgren and Johnson, despite CCDs being better suited to photometry (R. Jeffries, 2005, private communication). This does suggest that the difficulties associated with photoelectric photometry – where only one star can be observed at once – means that particularly robust methods have been developed for reduction of their data. Another difficulty is that different pixels on a CCD detector are used to observe light from different stars, whereas the same detector area is used for all stars when using a photoelectric photometer, so CCD accuracy can be limited by flat-fielding errors.

12.1.1 Broad-band photometric systems

The most commonly used photometric system is *UBV* (UV, blue, visual) developed by Johnson & Morgan (1953) to aid in the classification of stars (Hilditch 2001, p. 186). The original system was defined using glass filters and photoelectric photometers. This system was subsequently extended to redder wavelengths with the

**Figure 89.** The response functions of the Johnson *UBVRI* pass-band system plotted against wavelength (Moro & Munari 2000).**Figure 90.** The response functions of the Cousins *RI* passband system plotted against wavelength (Moro & Munari 2000).

R_JI_J (Johnson red, Johnson IR) filters when more advanced photometers were developed. Alternative *RI* passbands have been defined by Cousins (1980), Kron & Smith (1951) and Eggen (1965). Bessell (1979) suggested that the Cousins passbands are the best broad-band red-light system, and provided transformation equations between the different passband systems. Fig. 89 shows the response functions of the Johnson *UBVRI* passbands and Fig. 90 shows the Cousins *RI* passband responses. The *U–B* index is significantly dependent on the response functions of the light detector used, and some discrepant observations have given it a reputation for unreliability (Bessell 1995).

As more sensitive observations have become possible (with the construction of larger telescope apertures and efficient CCDs), the standard stars on which broad-band photometric systems are based have become too bright to be observed in many situations. To solve this problem, Landolt (1983, 1992) has provided a set of fields which contain dimmer standard stars. These fields are situated around the celestial equator so have also solved another previous problem with many photometric systems; that they are valid for only one of the celestial hemispheres.

The *UBVRI* system has been extended to IR wavelengths by Johnson (1966) with the passbands designated *JKLMN*, which are targeted at wavelength ranges where water vapour in the Earth's atmosphere does not attenuate photons significantly. *JHKL* standard stars were published by Elias et al. (1982) and Bessell & Brett (1988) have revisited the *JKLMN* system by Johnson and several alternative IR broad-band passband systems (e.g., Glass 1973; Elias et al. 1982; Jones & Hyland 1982), and defined a homogenized system. Fig. 91 shows the response functions of the Johnson *JKLMN* passbands and Table 8 gives the central wavelengths of all the broad-band passbands discussed above. The *J–K* index is sensitive to metallicity, but most IR indices vary little for MS stars (Pinsonneault et al. 2003). The

³⁵ Available on the internet at <http://obswww.unige.ch/webda/>

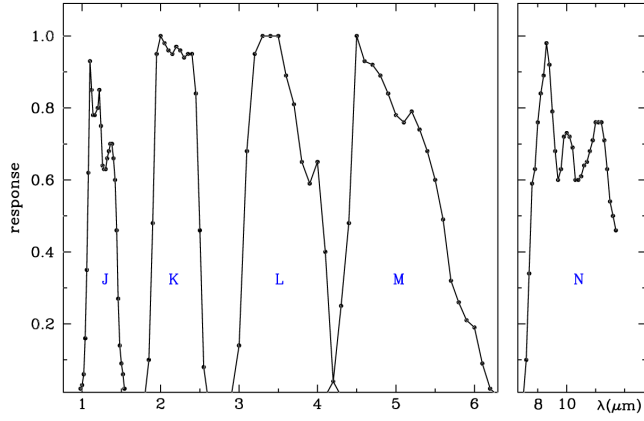


Figure 91. The response functions of the Johnson *JKLMN* passband system versus wavelength. (Moro & Munari 2000).

K passband is very insensitive to surface gravity and metallicity (Johnson 1966).

12.1.2 Broad-band photometric calibrations

UBVRI photometry is not the best way to get individual stellar parameters, but the large light throughput of the filters causes them to remain popular with researchers. *B–V* is sensitive to T_{eff} whereas *U–B* is sensitive to T_{eff} and surface gravity (Phelps & Janes 1994). The *B* passband is also known to be sensitive to metallicity via flux redistribution due to line blanketing (Alonso, Arribas & Martínez-Roger 1996). However, for F, G and K stars *V–I* is a good metallicity-independent T_{eff} indicator, and *R–I* is useful for later-type stars (Alonso, Arribas & Martínez-Roger 1996).

The photometric index *Q* was introduced by Johnson & Morgan (1953) to provide a reddening-free estimator of T_{eff} :

$$Q = (U-B) - \frac{E_{U-B}}{E_{B-V}}(B-V) = (U-B) - 0.72(B-V) \quad (112)$$

where E_{X-Y} is the interstellar reddening effect in the colour index *X–Y*. The *Q* index can also be used to deredden colours using (Johnson 1958):

$$(B-V)_0 = 0.332Q \quad (113)$$

The ratio $\frac{E_{U-B}}{E_{B-V}}$ is empirically determined and depends on the properties of the interstellar matter which causes reddening (e.g., Reimann 1989). Barnes, Evans & Moffett (1978) investigated *UBVRI* reddening using interferometrically measured angular diameters and found the relations

$$E_{U-B} = 0.75E_{B-V} \quad (114)$$

$$E_{V-R} = 0.75E_{B-V} \quad (115)$$

$$E_{R-I} = 0.76E_{B-V} \quad (116)$$

Moro & Munari give the total extinction in the *UBVRIJKL* bands to be

$$A_U = 4.4E_{B-V} \quad (117)$$

$$A_B = 4.1E_{B-V} \quad (118)$$

$$A_V = 3.1E_{B-V} \quad (119)$$

$$A_R = 2.3E_{B-V} \quad (120)$$

$$A_I = 1.5E_{B-V} \quad (121)$$

$$A_J = 0.87E_{B-V} \quad (122)$$

$$A_K = 0.38E_{B-V} \quad (123)$$

$$A_L = 0.16E_{B-V} \quad (124)$$

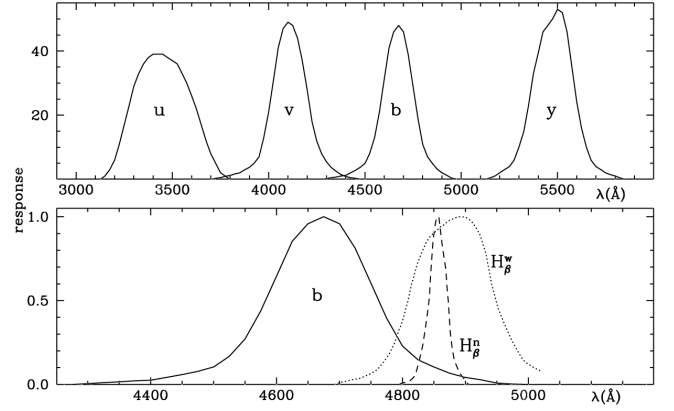


Figure 92. The response functions of the Strömgren passband system plotted against wavelength (Moro & Munari 2000).

Table 9. Central wavelengths and spectral widths for the Strömgren-Crawford *uvbyβ* photometric system. Data taken from Strömgren (1963) and Crawford & Mander (1966).

Passband	Central wavelength (Å)	FWHM (Å)
<i>u</i>	3500	300
<i>v</i>	4110	190
<i>b</i>	4670	180
<i>y</i>	5470	220
Hβ wide	4861	150
Hβ narrow	4861	30

where A_V is the total interstellar extinction in the *V* band.

Q is a useful T_{eff} indicator for hot stars, but the value of *Q* for MS stars with masses greater than $30 M_{\odot}$ is almost constant. Therefore higher-mass stars must be studied using spectroscopy (Massey & Johnson 1993). Massey, Waterhouse & DeGioia-Eastwood (2000) found theoretical relations between T_{eff} and *Q*, using Kurucz model atmospheres, for stars of luminosity classes I, III and V, respectively:

$$\log T_{\text{eff I}} = -0.9894 - 22.76738Q - 33.09637Q^2 - 16.19307Q^3 \quad (125)$$

$$\log T_{\text{eff III}} = 5.2618 - 3.42004Q - 2.93489Q^2 \quad (126)$$

$$\log T_{\text{eff V}} = 4.2622 - 0.64525Q - 1.09174Q^2 \quad (127)$$

12.1.3 Strömgren photometry

The Strömgren *uvby* photometric system was defined by Strömgren (1963, 1966), and is designed to be used for the simultaneous determination of the parameters of early-type stars and the amount of interstellar reddening affecting their light. The Hβ index was defined independently by Crawford (1958) and Crawford & Mander (1966) and complements the *uvby* passbands very well. Fig. 92 shows the response functions of the *uvbyβ* passbands and Table 9 gives the central wavelengths and spectral widths.

The main drawback of using the *uvbyβ* system is that the passbands allow much less light through than broad-band passbands; the original *uvby* passbands had *peak* transmission efficiencies of only about 50% (Crawford & Barnes 1970). The advantage is that the passbands are good at measuring particular features in early-type stellar spectra. The *u* passband measures flux density bluewards of the Balmer discontinuity, but does not extend to wavelengths short enough to be affected by water vapour in the Earth's atmosphere (Hilditch 2001, p. 192). The *v* passband is targeted at a part of the spectrum where iron lines are abundant so is sensitive to metallicity. The *b* and *y* passbands are intended to measure continuum flux and are sufficiently red to not be subject to line blanketing effects. The *y* passband has a very similar central wavelength to the Johnson *V* passband and is closely

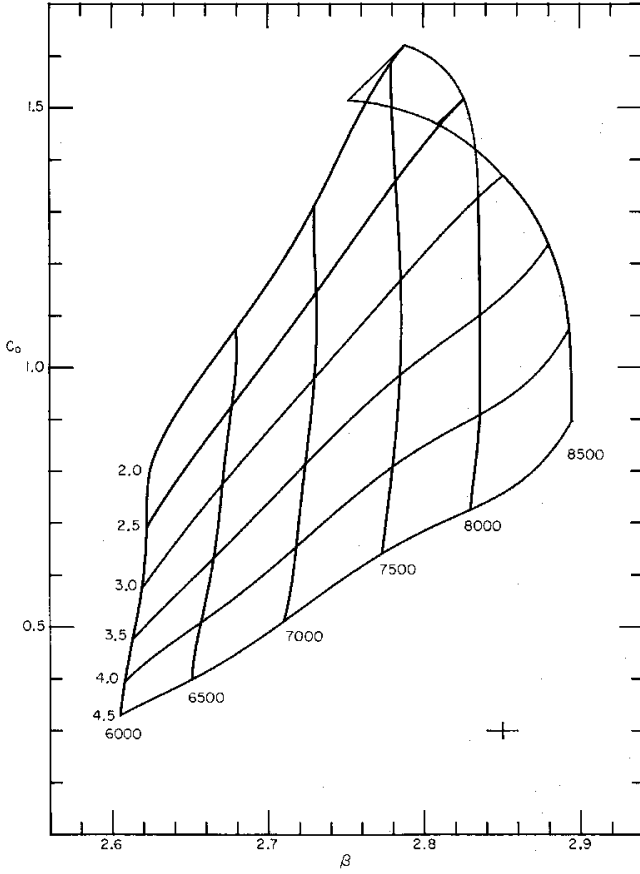


Figure 93. Calibration of T_{eff} (figures in thousands) and $\log g$ (figures less than 5.0), in terms of Strömgren photometric indices, for stars with $T_{\text{eff}} \leq 8500$ K. Moon & Dworetzky (1985).

comparable. The β index, the ratio of intensities in the $H\beta$ wide and $H\beta$ narrow passbands, is useful because it is not affected by reddening and so provides an unambiguous measurement of the strength of the $H\beta$ line in stars.

The main Strömgren indices are the Balmer discontinuity index (c_1) and the metal-line index (m_1):

$$c_1 = (u-v) - (v-b) \quad (128)$$

$$m_1 = (v-b) - (b-y) \quad (129)$$

(Strömgren 1966), and the $b-y$ index is also commonly used. The dereddened indices are denoted by a subscripted 0, and c_0 and m_0 are given by (Strömgren 1966) as:

$$c_0 = c_1 - 0.20E_{b-y} \quad (130)$$

$$m_0 = m_1 + 0.18E_{b-y} \quad (131)$$

c_0 is sensitive to surface gravity through its dependence on the Balmer discontinuity shape, but also has a slight sensitivity to rotational velocity (Crawford & Perry 1976; Gray, Napier & Winkler 2001). m_0 is sensitive to metal abundance and line blanketing effects but also is affected by convection in cool stars and by microturbulence (Smalley & Kupka 1997). $b-y$ is in general sensitive to T_{eff} , and β is in general sensitive to luminosity. However, the sensitivities of the different indices change significantly over T_{eff} , and different types of stars must be studied using different indices. The β index is also slightly affected by an interstellar absorption band at $4890 \pm 35 \text{ \AA}$ (Nissen 1976), has a minor dependence on rotation due to the narrow passband being only $30 \text{ \AA}$ wide (Crawford & Perry 1976; Relyea & Kurucz 1978), and is also affected by systemic velocities above about 200 km s^{-1} .

12.1.4 Strömgren photometric calibrations

The calibration of Strömgren (1966) is split into five groups:—

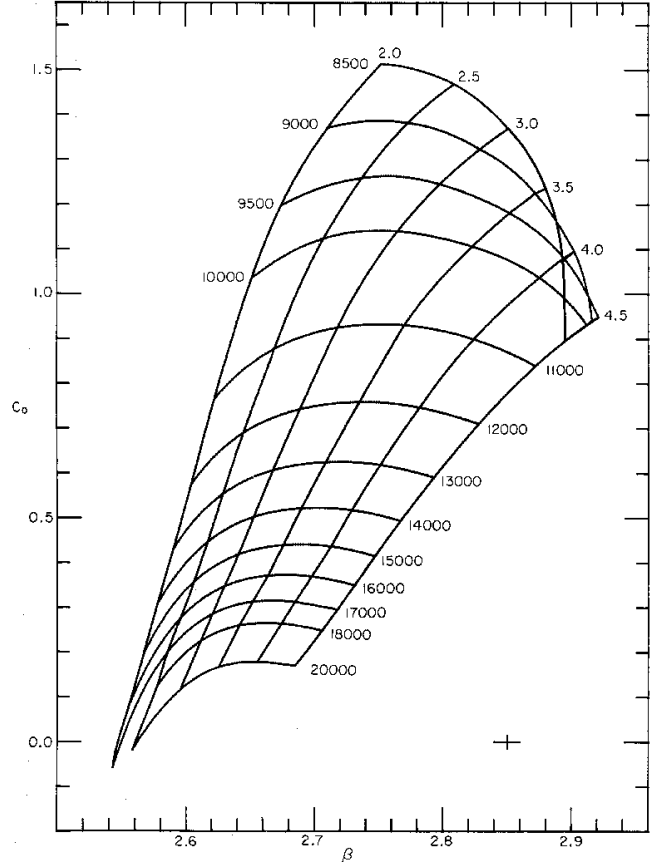


Figure 94. Calibration of T_{eff} (figures in thousands) and $\log g$ (figures less than 5.0), in terms of Strömgren photometric indices, for stars with $T_{\text{eff}} \geq 8500$ K. Moon & Dworetzky (1985).

(i) For stars earlier than B9, c_0 and $(u-b)_0$ are excellent T_{eff} indicators and for a given T_{eff} the Balmer line strength gives the surface gravity and absolute visual magnitude, M_V .

(ii) For A0–A3 stars, which is where the Balmer line reaches its maximum strength, two indices are defined:

$$a_0 = (b-y) + 0.18[(u-b) - 1^m36] \quad (132)$$

$$r^* = (\beta + 2^m565) + 0.35c_0 \quad (133)$$

(with corrections in the equation for r^* given by Moon & Dworetzky 1984). The index a_0 is a good indicator of T_{eff} and is practically independent of surface gravity, whereas for a given a_0 , r^* is a good indicator of surface gravity.

(iii) For A4–F0 stars, T_{eff} is indicated by β , and c_0 gives surface gravity and M_V . The index m_0 indicates whether the star is chemically peculiar.

(iv) For F1–F9 stars, T_{eff} and surface gravity are given by β and c_0 , and the metallicity, $\left[\frac{\text{Fe}}{\text{H}}\right]$, can be determined to an accuracy of 0.1 dex using m_0 .

(v) For G0–G5 stars, the β index ceases to be useful due to the number of contaminating metal lines around the $H\beta$ line. It is suggested that the indices c_0 , m_0 and $b-y$ are good for parameter determination, but the calibration was not constructed.

Crawford (1975, 1978, 1979, 1980) provided a detailed and careful calibration of the physical parameters of early-type stars, using $uvby\beta$ photometry obtained for about twelve nearby open clusters and some nearby stars. Crawford did not use information from spectral classifications, space motions, previous calibrations or theoretical calculations. Crawford (1975) investigated the F type stars. He gives relations for the reddening between the $uvby\beta$ photometric indices:

$$E_{b-y} \approx 0.73E_{B-V} \quad (134)$$

$$E_{m1} \approx -0.3E_{b-y} \quad (135)$$

$$E_{c1} \approx 0.2E_{b-y} \quad (136)$$

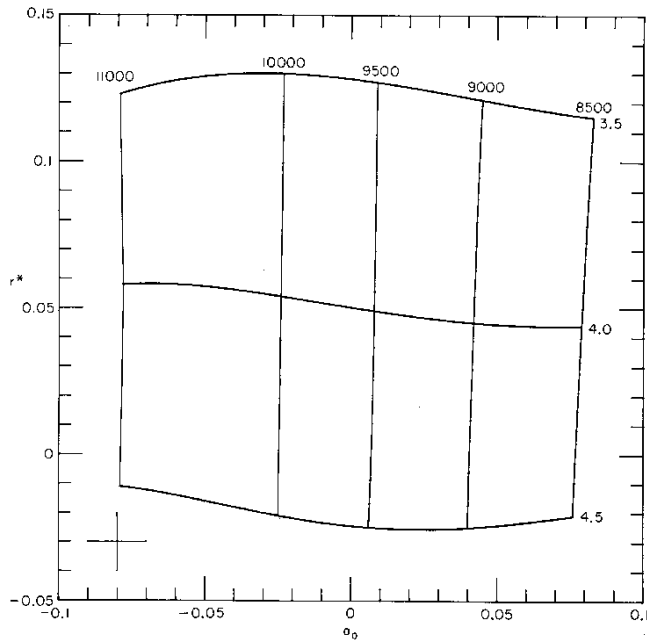


Figure 95. Alternative calibration of T_{eff} (figures in thousands) and $\log g$ (figures less than 5.0), in terms of Strömgren photometric indices, for stars with $8500 \geq T_{\text{eff}} \geq 11000$ K. Taken from Moon & Dworetsky (1985).

$$A_V = 3.2E_{B-V} \approx 4.3E_{b-y} \quad (137)$$

The calibration is tabulated and is valid for F2–G0 stars of luminosity classes III–V; in particular it is intended for stars with $2^{\text{m}}590 < \beta < 2^{\text{m}}720$. B stars with β in this range can be detected by their blue colour or lower m_0 values. F stars have significant line blanketing effects due to the profusion of metal lines in the blue part of the spectrum. The blanketing parameter is

$$\delta m_1 = m_1(\text{standard}) - m_1(\text{observed}) \quad (138)$$

and is a good indication of the metal abundances of A and F stars. Crawford (1978) investigated the B stars, the resulting calibration being valid for stars with $c_0 < 1.0$. Crawford (1979, 1980) calibrated the A stars, defined as those in between the previous two calibration validity ranges.

Olsen (1984) published a preliminary calibration of *uvby* photometry for G and K dwarfs, using the indices $b-y$, m_1 and c_1 . Distances were calculated using the method of trigonometrical parallax, and metal abundances were found using high-resolution spectroscopy. However, the calibration is affected by variation in values of an unknown “fourth parameter”, which may be helium abundance. A large number of observations have subsequently been published (Olsen 1994a, 1994b) but the final calibration has not yet appeared. Olsen (1988) has constructed a calibration for dereddening *uvby* photometry of F stars.

Moon & Dworetsky (1985) produced a calibration to find the T_{eff} s and surface gravities of B2–G0 stars. Their method was to determine the main functional form of the relationship using synthetic *uvby* values found from Kurucz model atmospheres (Relyea & Kurucz 1978). The synthetic *uvby* values were adjusted to bring them into agreement with observational data and the resulting calibration plotted as diagrammatical grids. These grids are shown in Fig. 93 for stars with $T_{\text{eff}} \leq 8500$ K, in Fig. 94 for stars with $T_{\text{eff}} \geq 8500$ K, and the “problem stars” with spectral types A0–A3 are dealt with in Fig. 95 using the a_0 and r^* indices given by Strömgren (1966), but defined here by

$$a_0 = 1.36(b-y)_0 + 0.36m_0 + 0.18c_0 - 0^{\text{m}}2448 \quad (139)$$

$$r^* = 0.35c_1 - 0.07(b-y) - \beta + 2^{\text{m}}565 \quad (140)$$

where subscripted zeros refer to dereddened values. The Moon & Dworetsky calibration has been transformed into a convenient FORTRAN program (called TEFFLOGG) by Moon (1985a). A FOR-

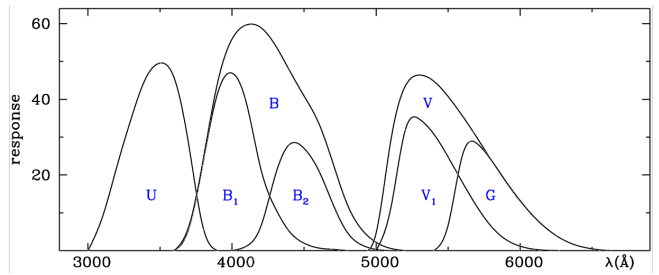


Figure 96. The response functions of the Geneva passband system plotted against wavelength (Moro & Munari 2000).

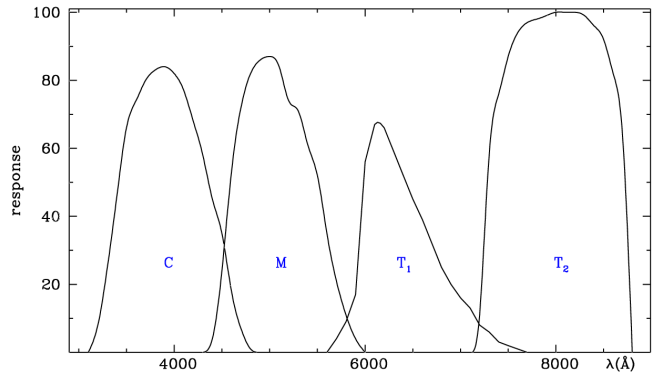


Figure 97. The response functions of the Washington passband system plotted against wavelength (Moro & Munari 2000).

TRAN program for dereddening Strömgren photometry and then applying several calibrations, called UCBYBETA, has been written by Moon. Dworetsky & Moon (1986) extended their calibration to Am stars, and adjusted the calibration of surface gravities to include a slight dependence on metallicity.

A calibration similar to Moon & Dworetsky (1985) has been provided by Balona (1984) for early-type stars, and updated by Balona (1994). Balona also gives a calibration of bolometric correction in terms of $\theta = \frac{5040}{T_{\text{eff}}(\text{K})}$:

$$M_{\text{bol}} - M_V = -5.5637 + 18.9446\theta - 19.8827\theta^2 + 6.1303\theta^3 \quad (141)$$

Schuster & Nissen (1989) calibrated the reddening, E_{b-y} , and metallicity, $\left[\frac{\text{Fe}}{\text{H}}\right]$, for metal-poor F and G stars, from the m_0 , c_0 and β indices. This calibration is intended for the study of the local Pop II stellar population.

Napiwotzki, Schönberner & Wenske (1993) investigated several calibrations for determination of T_{eff} and surface gravity for B, A and F stars. Their calibrating stars were those with good T_{eff} determinations selected from the literature, for which they also obtained spectra of hydrogen lines and derived surface gravities from fitting the $\text{H}\gamma$ profile with theoretical profiles. They recommended that the Moon & Dworetsky (1985) calibration be used, with a minor correction in the surface gravity calibration of

$$\log g = \log g_{\text{MoonDworetsky}} - 2.9406 + 0.7224 \log T_{\text{eff}} \quad (142)$$

Smalley (1993) determined the metal abundance $\left[\frac{\text{M}}{\text{H}}\right]$ for 28 A stars from medium-resolution spectra and used these to provide a calibration of $\left[\frac{\text{M}}{\text{H}}\right]$ in terms of the δm_0 index. This index is the difference between the expected m_0 value for the ZAMS and the actual observed m_0 value:

$$\delta m_0 = m_0 \text{ ZAMS} - m_0 \text{ STAR} \quad (143)$$

Ribas et al. (1997) used empirical data for MS dEBs to provide a calibration of stellar mass and radius (and so surface gravity) using Strömgren photometric indices. The intention was to use one index sensitive to T_{eff} and one sensitive to evolutionary status, and the stars were as usual split into early-type, intermediate, and late-type. The claimed accuracy is 5–8% in mass,

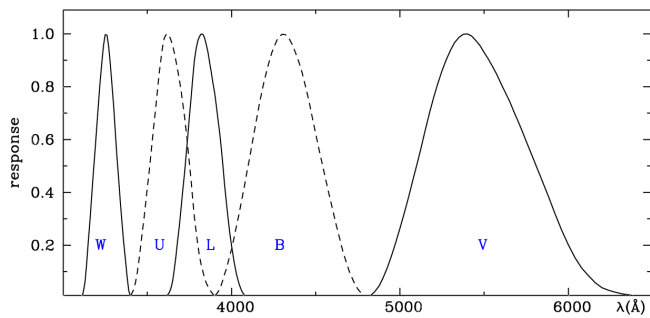


Figure 98. The response functions of the Walraven passband system plotted against wavelength (Moro & Munari 2000).

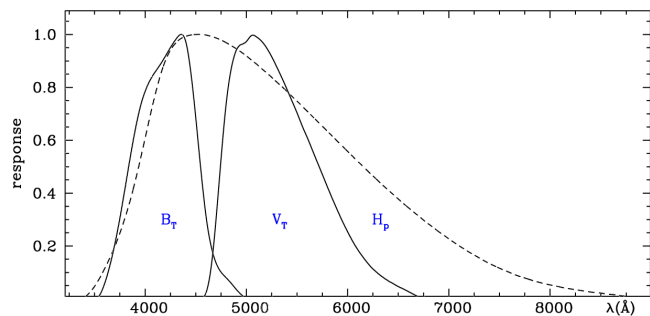


Figure 99. The response functions of the Hipparcos passband system plotted against wavelength (Moro & Munari 2000).

10–15% in radius and 0.08–0.10 dex in $\log g$ for MS stars with T_{eff} s between 7000 K and 20 000 K, but metal abundance is important for late-type stars.

Martell & Laughlin (2002) presented Strömgren calibrations for T_{eff} and $\left[\frac{\text{Fe}}{\text{H}}\right]$ for F, G and K dwarfs based on data taken from the compilation of metal abundances from high-resolution spectroscopy by Cayrel de Strobel, Soubiran & Ralite (2001) and the Strömgren photometry catalogue of Hauck & Mermilliod (1998). The calibration is valid for $0.288 < b-y < 0.571$ (which roughly corresponds to $4750 < T_{\text{eff}} < 6600$ K) and includes a “planeticity” indicator. This aspect of the calibration is designed to predict the probability of stars hosting an extrasolar planet, but the authors conclude that it simply reflects the larger metal abundances of the stars which are known to have planets. Martell & Smith (2004) updated this calibration and investigated if there was any dependence on X-ray luminosity. Haywood (2002) has investigated the metallicity of F, G and K stars and has produced a calibration for $\left[\frac{\text{Fe}}{\text{H}}\right]$, which uses m_1 and $b-y$, and is valid for $0.22 < b-y < 0.59$.

12.1.5 Other photometric systems

There exist well over one hundred different photometric systems (Moro & Munari 2000), of which many are variations on the broad-band Johnson-Cousins *UVBRI* and Johnson *JKLM* systems. Whilst the majority of these systems are no longer used, or observations through their passbands are always transformed to more commonly used systems, there are a number of photometric systems which are well-designed, actively maintained and of particular interest. A few will be discussed below.

The Geneva system (Golay 1966) consists of seven passbands, designated *UBB₁B₂VV₁G*, of which *U*, *B* and *V* are broad-band passbands and the remainder are intermediate-band passbands. The passbands are shown in Fig. 96. The system has been well treated by researchers so published observations are very homogeneous and reliable. The Geneva system is particularly good at detecting variable and peculiar stars from their colour indices alone (Waelkens et al. 1990). A calibration for T_{eff} and surface gravity of B stars has been provided by North & Nicolet (1990) and updated by Kunzli et al. (1997).

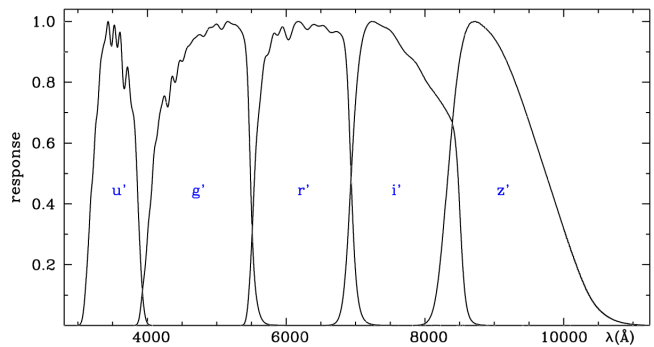


Figure 100. The response functions of the Sloan Digital Sky Survey passband system versus wavelength (Moro & Munari 2000).

The Washington photometric system (Canterna 1976) is a broadband system designed to measure metallicities of red giants from iron-peak elements and the abundance of (CN + CH) (Daniel et al. 1994). The passbands are *CMT₁T₂* at 3900, 5000, 6100 and 8100 Å (see Fig. 97). The index *C-T₁* is very sensitive to metallicity so the Washington system is becoming more popular with researchers.

The Walraven system (Walraven & Walraven 1960) is intended to provide a photometric method for the determination of luminosity and spectral type of O and B stars and supergiants. It consists of the intermediate-width passbands *WULB* and the broad-band passband *V* (see Fig. 98). The Walraven system underwent a major revision in 1980 and has also since changed its characteristics slightly (van Genderen 1986).

The Hipparcos satellite obtained observations of over one million stars during the years 1989 to 1993. The main experiment, which used the broad-band *H_P* passband, was to measure the distances to stars with $V \lesssim 8$ by the technique of trigonometrical parallax. The Tycho experiment on the Hipparcos satellite measured the brightnesses of stars with $V \lesssim 11.5$, at over one hundred epochs, through the passbands *B_T* and *V_T*. The Hipparcos passband throughputs are plotted in Fig. 99.

The Sloan Digital Sky Survey (SDSS) photometric system (Fukugita et al. 1996) was designed to be used as a survey passband system, but to avoid some strong telluric lines. Consequently, the passbands have a very wide wavelength coverage (of the order of 1300 Å except for the *u'* passband) but no significant overlap with each other. The *u'g'r'i'z'* passband throughputs are plotted in Fig. 100. The SDSS passbands have proved to be very useful for classifying stars (Izević et al. 2003) and are expected to become very popular with researchers in the future.

13 LIGHT CURVE ANALYSIS OF DETACHED ECLIPSING BINARY STARS

The variation of the apparent brightness of an EB depends on the geometry of the system (which is generally taken to also include the direction it is viewed from), the variation of T_{eff} over the surfaces of the stars, the rotational velocities of the stars, and the characteristics of the mutual orbit of the two stars. Additional complications can arise from contaminating light, usually coming from a third star orbiting the EB, but possibly due to an entirely unrelated foreground or background star along the line of sight. Third light can also be contributed by gas streams or colliding winds produced by the components of the EB.

The analysis of the light variations during and outside eclipse is a relatively complex procedure due to the number of different effects which cause the light variation. The first useful method, also referred to as rectification, was introduced by Russell (1912a, 1912b) and first applied to the EBs Z Draconis and RT Persei (Russell & Shapley 1914). This method, based on calculations by hand, was extensively refined by researchers including Russell, Merrill and Kopal, who took it as far as could reasonably be achieved without the aid of computers (Wilson 1994).

In the late 1960s it was noticed that the increased sophistication of computers allowed the analysis of EB light curves without many of the restrictions imposed by use of the Russell method. This led to three computer-based models for the simulation of EB light curves (in increasing order of sophistication): EBOP, WINK and the Wilson-Devinney code (WD). The initial releases of EBOP and WD were able to fit a model to observed data using the differential corrections minimisation algorithm. WINK was not, but this feature was always intended to be implemented and was afterwards quickly made available. EBOP and WINK approximate the surfaces of stars using geometrical shapes so are only applicable to stars which are detached and so close to the shapes used. WD is based on the Roche equipotential model so is able to represent semidetached and contact binary stars, a fundamental advance on previous methods for the analysis of the light curves of these types of variable star.

13.1 Models for the simulation of EB light curves

Quantities are derived from the light curves of EBs by defining a model and adjusting the parameters of the model towards the best fit. The evaluation of the total brightness of an EB, as a function of orbital phase, is achieved by summing the light emitted by all parts of the surface which are visible to the observer, usually achieved by numerical integration calculations.

The simplest model of a dEB – uniformly-illuminated spheres moving in a circular or eccentric orbit – is analytically exactly solvable, but the inclusion of effects such as limb darkening and asphericity cause the analytical integration equations to become intractable. The models discussed below split the surface of each star into many small elements. The evaluation of the total light of the system then requires the summation of the light from each element which is visible to the observer, and the light emitted by each element depends on its area (elements are not of uniform area because the stars are not undistorted spheres). Limb darkening, gravity brightening and the reflection effect also affect the brightness of an element

The reflection effect arises because each star intercepts light emitted by its companion. This causes the sides of the star facing the companion to be hotter and brighter. Whilst effects such as limb darkening and gravity brightening are fairly easy to incorporate into a light curve model, a detailed treatment of the reflection effect – such as contained in WD – is complex and extremely expensive in terms of calculation time. All models therefore incorporate some simplification of this effect.

The choice of the parameters used to define the model – and to adjust towards the best solution – can be very important. Light curves depend on a large number of parameters which are significantly correlated. At best this means that many iterative adjustments are required to reach the least-square solution and, at worst, minor observational errors can cause large changes in the derived parameters. Possibly the most worrying aspect of this is that the formal errors of the fit can become hugely optimistic in the presence of large parameter correlations and so lose all their significance. The estimation of uncertainties is dealt with below.

The procedure for solving a light curve is to choose an appropriate model and estimate a set of parameters for which the model gives a light curve as similar as possible to the observed data. The model is then iteratively refined to find the best-fitting least-squares solution parameter values.

13.1.1 Rectification

This procedure is based on calculations by hand and was introduced by Russell (1912a). The parameters of this model are:–

- r_A and r_B , the radii of the primary and secondary star expressed in fractions of the semimajor axis of the relative orbit of the two stars, a , i.e., $r_A = \frac{R_A}{a}$
- e , the orbital eccentricity
- ω , the orbital longitude of periastron
- L_A and L_B , the amounts of light emitted by the two stars
- i , the orbital inclination

- P and T_0 , the orbital ephemeris.

Note that a cannot be determined from the light curve, so that spectroscopy is needed to calculate the absolute sizes of the stars. L_A and L_B are usually defined to be in units of the total light of the system so $L_A + L_B = 1$ – at this point contaminating (‘third’) light, L_3 , is not included in the model. Russell (1912b) extended the model to include eccentric orbits. Russell & Shapley (1912a, 1912b) incorporated a treatment of the limb darkening effect.

The method of rectification dealt with the complications of ellipticity and the reflection effect by fitting cosine waves to the light variation outside eclipse, then removing the functions from all observations. The rectified light curve is then assumed to pertain to spherical stars in circular orbits and displaying no reflection effect, an assumption central to this procedure but questionable even in uncomplicated eclipsing systems. The rectified light curve was then analysed to determine the sizes of the stars and their light ratio by graphical methods. The coefficients which give the size of the fitted cosine waves (called the rectification coefficients) are useful indicators of the sizes of the ellipticity and reflection effect in eclipsing systems (Popper 1981).

The method of rectification has been extensively refined by Kopal (1946, 1950, 1959) and by Russell & Merrill (1959). More recent adjustments have been made by Kitamura (1967) and by Lavrov (1993) but the method of rectification is now regarded as thoroughly outdated and somewhat untrustworthy.

13.1.2 EBOP – Eclipsing Binary Orbit Program

EBOP was written by Dr. P. B. Etzel for his Master’s thesis and used to analyse light curves of the dEB WW Aurigae (Etzel 1975). Based on the simple Nelson-Davis-Etzel (NDE) model (Nelson & Davis 1972, and modifications by Etzel 1980), its main advantage is that it involves far fewer calculations than the WINK and WD models so is much faster to run on a computer. Details can also be found in Popper & Etzel (1981) and in Etzel (1981, 1993).

The geometric shape chosen to represent stars in the NDE model is the biaxial approximation of a triaxial ellipsoid (the two minor axes are the same length), although a quantity called oblateness is misleadingly assessed after the method of Binnendijk (1974) (Etzel, private communication). The three axes of the triaxial ellipsoid, a_3 , b_3 and c_3 , are given by

$$a_3 = r_A \left[1 + \frac{1}{6}(1 + 7q)r_A^3 \right] \quad (144)$$

$$b_3 = r_A \left[1 + \frac{1}{6}(1 - 2q)r_A^3 \right] \quad (145)$$

$$c_3 = r_A \left[1 - \frac{1}{6}(2 + 5q)r_A^3 \right] \quad (146)$$

where q is the mass ratio. To calculate the equivalent quantities for the secondary star, replace q with $\frac{1}{q}$. Setting $b_2 = c_3$ and adopting oblateness $\epsilon = 1 - \frac{b_3}{a_3}$ (Binnendijk 1974) gives the axes, a_2 and b_2 , of a biaxial spheroid

$$b_2 = r(1 - \epsilon)^{1/3} \quad (147)$$

$$a_2 = \frac{b_2}{1 - \epsilon} = \frac{r}{(1 - \epsilon)^{2/3}} \quad (148)$$

Note that the radii given by EBOP relate to a sphere of the same volume as the biaxial spheroid.

For partially-eclipsing systems with large oblatenesses, the orbital inclination can be underestimated because of the biaxial ellipsoids adopted to approximate stars. For V478 Cyg, which has $\langle \epsilon \rangle = 0.029$, the inclination is underestimated by 0.48° , several times its standard error (Popper & Etzel 1981). This effect was confirmed to exist by Andersen, Clausen & Giménez (1993).

The main philosophy of the EBOP code is to base the model, and the least-square fitting to observations, on parameters which are most closely related to the shape of light curves, and which are correlated as little as possible. This means that the adjustable parameters are

- r_A

- $k = \frac{r_B}{r_A}$, the ratio of the stellar radii (where r_B is the radius of the secondary star)
- $J = \frac{J_B}{J_A}$, the surface brightness ratio where J_A and J_B are the central surface brightnesses of the primary and secondary star respectively
- L_3
- i
- q
- u_A and u_B , the linear limb darkening coefficient for each star
- β_A and β_B , the gravity brightening exponent for each star
- $e \sin \omega$
- $e \cos \omega$

The orbital period, P , and reference time of primary eclipse minimum, T_0 , are also required but must be fixed during least-squares fitting by differential corrections. Another two parameters, the outside-eclipse light of the system and the phase difference between the midpoint of primary eclipse and phase zero, are also needed to place the light curve properly in parameter space.

The quantities $e \sin \omega$ and $e \cos \omega$, rather than e and ω , have been chosen as model parameters because they tend to be better determined when the orbit is only slightly eccentric (Etzel 1993). To a first approximation, $e \cos \omega$ depends on the phase of midpoint of secondary eclipse and $e \sin \omega$ depends on the relative durations of the eclipses. More formally, and ignoring terms in eccentricity to powers greater than one,

$$e \cos \omega \approx \frac{\pi(\phi_{\text{Min II}} - 0.5)}{1 + \text{cosec}^2 i} \quad (149)$$

where $\phi_{\text{Min II}}$ is the phase difference between secondary minimum and the immediately preceding primary minimum (Güdü 1978). Zakirov (2001) gives the ratio of the durations of secondary and primary eclipses to be

$$\frac{\delta\phi(\text{Min II})}{\delta\phi(\text{Min I})} = \frac{1 + e \sin \omega}{1 - e \sin \omega} \quad (150)$$

$e \sin \omega$ is generally less well-determined than $e \cos \omega$, although the opposite situation exists in the calculation of spectroscopic orbits (Sec. 11).

Limb darkening is incorporated in EBOP using the linear law (Sec. 1.4.1, eq. 29) – the simple nature of the NDE models means that more complex limb darkening laws are of limited importance. However, their inclusion is advantageous and has been implemented by A. Giménez and J. Díaz-Cordoves. Their revised version of EBOP also has a slightly improved geometrical basis and the ability to allow for apsidal motion (7.2), and was used by Giménez & Quintana (1992) in a study of the eccentric dEB V477 Cygni.

The reflection effect in EBOP is dealt with in a very simple bolometric manner based on Binnendijk (1960) and is usually calculated from the geometry of the system being analysed. This approximation becomes less accurate when the T_{eff} s of the two stars are very different or vary significantly over the stellar surfaces, but in any case it is not recommended to use EBOP for systems with a significant reflection effect (Etzel 1980).

The proximity effects (reflection and asphericity) are not included in the calculation of the light lost during eclipse, so only well-detached systems, where the change in proximity effects throughout eclipse is negligible, can be studied.

Popper & Etzel (1981) find that the NDE model and the EBOP code are trustworthy for stars with oblateness $\epsilon < 0.04$. Beyond this point, biaxial ellipsoids are unable to satisfactorily approximate the shape of the distorted star. North & Zahn (2004b) studied dEBs in the Magellanic Clouds using EBOP and WD. They found that for average fractional radii of 0.25 and 0.3, the radii derived using EBOP were 1% and 5% different, respectively, to the radii found using WD. These studies provide good estimates of the limits of applicability of EBOP. A study of the LMC dEB HV 2274 by Watson et al. (1992) found that the differences between an EBOP and WINK solution were minor for this system, for which $r_A + r_B \approx 0.5$.

13.1.3 WINK

The WINK light curve model was written by D. B. Wood (Wood 1971a, 1972) and made available to the astronomical community (Wood 1973b). It is a geometrical model which approximates the surfaces of stars with triaxial ellipsoids (equations 144, 145 and 146 in previous section) and incorporates the model parameters:–

- P and T_0
- a
- $e \cos \omega$
- $e \sin \omega$
- i
- q
- a_A, b_A, c_A , the fractional semiaxes of the primary star
- a_B, b_B, c_B , the fractional semiaxes of the secondary star
- $\bar{I}_A, \bar{I}_B$, the central surface brightnesses at quadrature
- L_3
- u_A, u_B
- β_A, β_B
- w_A, w_B , the reflection coefficient (albedo) for each star

The model is fitted to observations using the method of differential corrections.

The six stellar semiaxes are actually replaced in the model by the dimensionless quantities $r_A, k, \epsilon_A, \epsilon_B, \zeta_A$ and ζ_B where

$$a_A = r_A a \quad (151)$$

$$a_B = k r_A a \quad (152)$$

$$b_A = \epsilon_A r_A a \quad (153)$$

$$b_B = \epsilon_B k r_A a \quad (154)$$

$$c_A = (1 + \zeta_A) \epsilon_A^2 r_A a \quad (155)$$

$$c_B = (1 + \zeta_B) \epsilon_B^2 k r_A a \quad (156)$$

Here the r_A is the fractional major semiaxis of the primary star and k is effectively the ratio of the radii. The ϵ are the ellipticities in the ab (orbital) plane and the ζ measure how much the ellipticities in the bc plane differ from those in the ab plane.

Limb darkening is included using the linear law (Sec. 1.4.1), as with EBOP. Gravity brightening is defined in a similar manner with the equation

$$\bar{I} = I_0 \left[1 - \beta + \beta \left(\frac{r}{r_0} \right) \right] \quad (157)$$

where I_0 is the central surface brightness, r is the local radius and r_0 is the radius of the central point of the stellar disc. This definition means that β is approximately four times the quantity referred to as β_1 in Sec. 1.5.

The initial treatment of reflection (Wood 1971a) was to calculate the amount of light incident at a point on the surface and to reradiate some fraction w of it. A more advanced treatment was incorporated by Wood (1973a) but was shown to still be inadequate for high-precision work. Synchronous rotation was initially assumed but the effects of rotation were subsequently added. WINK corrects the stellar radii for the expansion effects due to rotation (Clausen et al. 2003) whereas WD does not.

13.1.4 The Wilson-Devinney (WD) code

The WD code is probably the most commonly used light curve analysis code, partly due to its much greater sophistication compared to EBOP and WINK. Rather than modelling the discs of stars using geometrical shapes, the components of a binary system are modelled in three dimensions using the Roche prescription of equipotential surfaces. This is implemented by defining points on the surface of the star, distributed approximately uniformly in a spherical coordinate system. The number of points is of the order of one thousand per star, although the WD code allows the user to choose the approximate amount.

Adoption of the Roche model for calculating the shapes and

sizes of the stars being studied allows a very realistic approximation of the actual stellar shapes, and the WD code can accurately model not only semidetached but also contact binary systems. The radii of the stars are given by one value of the potential per star for detached and semidetached binaries, or one value of potential for the whole system in the case of contact binaries. Once this model has been implemented, it needs only minor adjustment for different stellar shapes. The model is fitted to observations using the method of differential corrections.

The model parameters are:–

- P and T_0
- e and ω
- i and q
- F_A, F_B , rotational velocities of the stars
- Φ_A, Φ_B , the gravitational potentials of the stars
- $T_{\text{eff}A}, T_{\text{eff}B}$
- L_A, L_B , the light contributions of the stars
- $u_{1,A}, u_{1,B}, u_{2,A}, u_{2,B}$, the wavelength-dependent limb darkening coefficient(s) for each star
- $u_{\text{bolo},1,A}, u_{\text{bolo},1,B}, u_{\text{bolo},2,A}, u_{\text{bolo},2,B}$, the bolometric limb darkening coefficient(s) for each star
- β_A, β_B , the gravity brightening coefficient for each star
- w_A, w_B , the reflection coefficient (albedo) for each star
- λ , the effective wavelength of the observations

There are a large number of additional control characters to choose between several solution options, and some other capabilities have also been implemented in more recent versions.

The stellar radii are calculated by WD for four different points on the surface of the star: at the pole, towards the companion star, and on the equator at 90° and 180° from the line joining the centres of the two stars.

The T_{eff} of one of the stars must be fixed at a previously known value as light curves do not contain enough information to directly fit for both T_{eff} s. Calculations involving T_{eff} s and the reflection effect can be performed using black-body physics or using the predictions of model atmospheres (Leung & Wilson 1977). The model atmospheres of Carbon & Gingerich (1969) are provided with the WD code but more advanced Kurucz predictions have been added by Milone and co-workers (Kallrath et al. 1998) and used by several researchers. It is possible to link the luminosities (which here refer to the light contribution in the light curve under analysis, not the astrophysical definition of luminosity) to the T_{eff} s of the stars (mode 0 in the WD code) but this is not advisable due to the inadequacies of the black-body or model-atmosphere calculations required (Wilson et al. 1972). Groenewegen & Salaris (2001) found that for the LMC close binary HV 2274 the use of different model atmosphere did not significantly affect the derived radii but changed the T_{eff} ratio by 1.6%.

Wilson & Devinney (1973) modified the WD code, using considerations of symmetry to reduce the total number of calculations by approximately a factor of eight.

It is notable that the WD code has no provision for performing more than one iteration without human intervention, although the output files contain all the data needed for the researcher to apply the needed corrections to the parameters of the model. Wilson (1998) and Wilson & van Hamme (2004) clearly state that this apparent shortcoming has been deliberately included to force researchers to pay careful attention to matters of convergence, and to the success of the WD model as a whole. Wilson & Woodward (1983) state that some researchers have been iterating until parameter corrections are small, whereas iteration must continue until corrections are negligible so as to get good error estimates.

Wilson & Devinney (1973) also modified the WD code to allow the simultaneous solution of several light curves. In this case the geometrical parameters such as potential and orbital inclination are common to all light curves, but each set of data has its own values of the wavelength-dependent parameters such as limb darkening coefficients.

Wilson & Biermann (1976) modified the WD code to increase the reliability of the differential-corrections optimisation procedure. Having noted that convergence becomes difficult when many

parameters have large correlations (rather than two parameters being very highly correlated, for which a solution can usually be reached with reliability), they introduced the method of multiple subsets. Here the adjustable parameters are ordered into several sets and an iteration is undertaken for each set with only the parameters in that set being adjusted. Wilson (1983) revisited this method to highlight its existence.

Wilson & Caldwell (1978) added the ability to fit for small amounts of light-attenuating circumstellar matter, in this case a thick circumstellar ring.

Wilson (1979) extended the capabilities of the WD code to include the simultaneous solution of light curves and of RV curves. The advantages of this approach have been covered in detail by van Hamme & Wilson (1984). The main advantage is that common parameters such as the mass ratio (which can be well determined by the light curves of close and contact binaries) have one unique value, although it could be argued that the inconsistent values occasionally found by separate analysis suggest the existence of subtle physical effects and inadequacies of the method of analysis, and should therefore be noted and investigated. The extra information contained in subtle physical effects, such as the Rossiter effect (Sec. 11.4), can most easily be accessed by a simultaneous photometric and spectroscopic solution. Note that it is important to get the observational errors correct for the two different types of data, and that when this is done it is found that the photometric data are generally more important due to the larger number of observations in a light curve compared to an RV curve. Wilson & Sofia (1976) have investigated the proximity effects on spectroscopic orbital solutions of close binaries.

The original treatment of reflection was elaborated upon by Wilson et al. (1972) but criticised by Wood (1973a). Wilson (1990) added a more detailed treatment of the reflection effect which is able to consider multiple reflections too. However, the detailed treatment of reflection had to be achieved by considering the light incident from each surface element on one star to each surface element on the other star, so can be very expensive in terms of computing time when analysing eccentric systems. This is because the reflection effect in eccentric EBs is dependent on orbital phase, so must be calculated once for every datapoint.

The 1993 version of the Wilson-Devinney code (generally referred to as WD93) is much faster than previous versions (Wilson 1998). Other advantages include the consideration of apsidal motion and a constant period change to the code, and the ability to fit for the parameters of several starspots. Whilst starspots were included in previous versions (defined by a position, area and relative surface brightness), their parameters could not be adjusted prior to WD93. The latest (WD2003) version of the program is capable of fitting many starspots simultaneously whereas WD93 and WD98 were only able to adjust two per iteration.

Whilst the original WD code adopted the linear limb darkening law, the WD93 code (and later versions) is able to use the logarithmic and square-root laws too (Sec. 1.4.1). Whilst WD can fit for the linear coefficients of the limb darkening laws, it is not able to optimise the values of the nonlinear coefficients; these must be fixed at appropriate values.

The programming style of the WD code has been discussed extensively by Wilson (1993).

13.1.5 Comparison between light curve codes

It is preferable to analyse a light curve with more than one light curve analysis code, to check that the models are reliable and there are no programming bugs. Some codes may have other advantages, such as speed. For example, EBOP is over twenty times faster than WINK and WD because of its simplicity (Popper & Etzel 1981), although care has been taken to make WD quicker (Wilson 1998). It is a necessary but *not* sufficient condition that the parameters of an observed system are well known if two analysis codes agree on its parameters (Linnell 1984).

The Copenhagen research group usually analyses light curves using EBOP and WINK, or WINK and WD. The results have always been essentially identical (e.g., Andersen, Clausen & Nordström 1984, 1990a, who used EBOP and WINK to analyse the dEBs

VV Pyxidis and V1031 Orionis) except for a slight disagreement in the value of orbital inclination (Andersen, Clausen & Giménez 1993), which has been discussed in Sec. 13.1.2. Popper (1980) also notes that EBOP and WINK agree very well.

13.1.6 Other light curve fitting codes

Quite a few computer-based light curve analysis tools have been written and used by researchers.

Linnell (1984) introduced a physical model based on numerical integration between points on a surface. This model is sophisticated enough for the analysis of contact binaries (Linnell 1986) and has been equipped with a simplex least-squares fitting routine (Kallrath & Linnell 1987), but has not been widely used. It is more complex than the WD code (Kallrath & Linnell 1987).

G. Hill has constructed the light curve model LIGHT, followed by LIGHT2 (Hill & Hutchings 1970; Hill 1979).

P. Hadrava has written FOTEL (Hadrava 1990, 1995), which models stars using triaxial ellipsoids and has the ability to make simultaneous photometric and spectroscopic solutions.

Further light curve analysis codes are mentioned in Linnell (1984) and Wilson (1994). Lorentz, Mayer & Drechsel (1998) use the code MORO (Drechsel et al. 1995) to study the dEB SZ Camelopardalis. This code is based on WD but accounts for the change in radius caused by the radiation pressure incident on a close binary component from its companion.

13.1.7 Least-squares fitting algorithms

The fitting of a model to an observed light curve involves many parameters, some of which are quite correlated. An algorithm is required to navigate from a point in parameter space to the point where the best fit occurs. It is instructive to visualise this problem in the form of a χ^2 surface in two dimensions, although it must be remembered there are usually significantly far more dimensions to worry about and these cannot be easily visualised by the human brain. The χ^2 surface is high at its edges and low towards the middle, where the best fit is found. Added to this large-scale form are many valleys, bumps and dips, caused by the parameters correlations and observational errors.

All least-squares fitting algorithms navigate in steps (iterations) from the starting parameter values towards the best fit. There are, however, several problems. Large local gradients in the χ^2 surface can give a bad idea of the overall surface and cause excessive adjustments to be made to parameter values. Often this will result in values diverging to infinity and causing the solution to break down. If two parameters are strongly correlated, they will cause a deep valley in the χ^2 surface which can cause a large number of iterations until a good fit is found. The most worrying possibility, though, is that there are small dips in the χ^2 surface which can catch solutions on their way to the global minimum. These local minima can be difficult to detect and often give plausible results. In many cases it is difficult to be confident that a global, and not local, minimum has been reached, and also whether or not this difficulty is actually important. Global search algorithms are not difficult to construct but are impractically expensive in terms of computer time.

EBOP, WINK and WD are all capable of adjusting the parameters of their models to find the least-squares best fit to an observed light curve. They use the process of differential corrections (see Wyse 1939; Irwin 1947) to adjust the parameters from the starting estimates to the final solution. This method estimates parameter adjustments from the local gradient of the χ^2 surface. It requires reasonably good initial conditions because it is capable of both divergence and of settling in local minima (it is a local minimisation algorithm). It is capable of giving formal errors on the final fitted parameter values.

The simplex algorithm (see Press et al. 1992, p. 402, who have implemented the Nelder-Mead simplex algorithm in the sub-routine AMOEBA) has been implemented in the WD code by Kallrath & Linnell (1987). As used by these authors it has some characteristics of a global search algorithm; it is certainly incapable of divergence but is still able to get trapped in local minima.

One advantage is that it uses only χ^2 values, not the gradient of the χ^2 surface, so does not require the calculation of partial derivatives. This can cause it to be faster than the differential corrections process, but it may often require more iterations so will be slower.

The Levenberg-Marquardt method (Press et al. 1992, p. 678, who have implemented the method in the MRQMIN algorithm) is probably the most popular fitting algorithm at present. It was suggested by Levenberg (1944) and by Marquardt (1963), and utilises two minimisation algorithms simultaneously, one algorithm being slow and robust, the other fast and less reliable. The former method is used far from minimum, with a continuous switch towards the latter method close to the minimum. MRQMIN also has a provision for calculating formal errors of the fit (but see Sec. 13.3). MRQMIN is still a local search algorithm and is technically capable of diverging.

There are many more least-squares fitting algorithms, such as singular value decomposition (Press et al. 1992, p. 670), simulated annealing and genetic algorithms (Ford 2003) available, but the three methods detailed above are quite adequate for fitting light curve models to observed data (Wilson 1994).

13.2 Solving light curves

Firstly a decent set of observations must be obtained. There are several requirements for a set of light curves to be definitive:–

- Good light curves in two or more passbands are needed (Andersen 1991), although I would suggest that data in three passbands has become the minimum requirement, preferably in intermediate band photometric systems such as Strömgren (Sec. 12.1.3). Separately analysing three or more light curves means that the mean and standard deviation of the resulting values can be calculated for each parameter.
- Both eclipses must be covered without any gaps in the phased data greater than a tenth of the total eclipse duration.
- The eclipses must contain at least one hundred datapoints with low observational errors. If limb darkening is to be studied then each observation must have an error of 0.005 mag or less (Popper 1984, 2000) for simple systems. More complicated dEBs will require better data. North, Studer & Kunzli (1997) suggest that meaningful results for limb darkening require five hundred points per eclipse, although this may be a little conservative.
- Sufficient data must be available outside the eclipses to give an accurate reference brightness, to cover any outside-eclipse variation such as ellipticity and reflection effects, and to be sure that no significant complications could exist without being noticed. A minimum requirement is perhaps twenty accurate and well-spaced datapoints between each eclipse for an uncomplicated dEB.
- There are no significant night errors. If they are present then the results of analysing the light curves, which depend on observational errors being random, could be systematically wrong (Popper & Etzel 1981).

It appears that secondary eclipses are more sensitive to the variation of model parameters than primary eclipses (Popper 1986) although it is not clear why this should be so. The effect will be smaller for dEBs composed of similar stars than for those with very dissimilar components.

It is a good idea to have two different sets of observations obtained at different times and with different equipment, and possibly with different observers (Popper 1981). This can highlight difficulties such as night errors or data reduction errors. Also, if there are no problems, the uncertainties on the parameters will be reduced as there are more data available.

Three example sets of light curves are given in Figs. 101, 102 and 103. The light curves of the dEB GG Lupi in Fig. 101 were observed using a four-channel photoelectric photometer observing simultaneously in the Strömgren u , v , b and y passbands. Fig. 102 shows an example of a light curve which is not definitive and may have been compromised by difficult observing conditions. Fig. 103 shows the most complete light curve obtained (until the year 2002), consisting of 5759 robotic-telescope observations of the dEB WW Camelopardalis through a V passband.

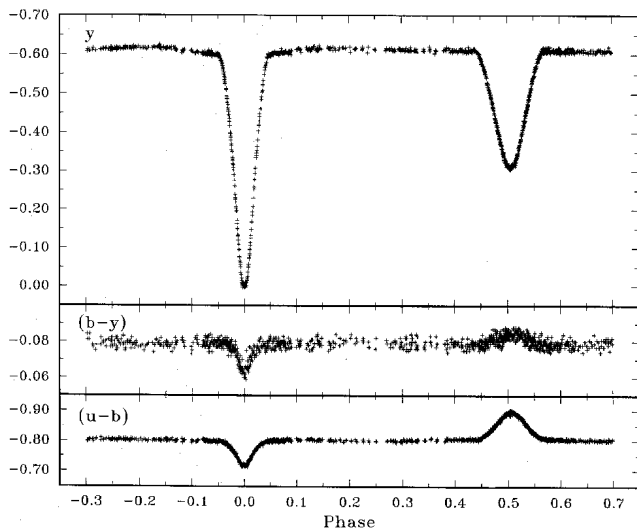


Figure 101. Example of a definitive light curve of a dEB. These data, of GG Lupi, were taken using a four-channel photoelectric photometer observing simultaneously in the Strömgren *uvby* passbands. Here the *y*-band light curve is plotted and data from other passbands have been used to construct colour curves. Taken from Andersen, Clausen & Giménez (1993).

13.2.1 Calculation of the orbital ephemeris

The first quantities to calculate are the orbital period and reference time of minimum (unless these quantities are going to be included in the overall fit using e.g., the WD code). For most dEBs it is entirely satisfactory to assemble times of minimum light, adopt a cycle number for each, and fit the data with a straight line. Many times of minima are available from the literature, particularly from the Information Bulletin of Variable Stars³⁶, and the only point to be careful about is the quality of the data used and the method of determination.

Times of minima must be obtained from the observational data which are about to be analysed by least-squares. The traditional method for doing so was outlined by Kwee & van Woerden (1956). This requires the observational data to be resampled to constant time intervals. For a trial time of minimum (halfway between two resampled datapoints), one branch of the eclipse is reflected onto the other and the agreement is quantified. This is repeated for times midway between the preceding and proceeding pairs of datapoints and the amount of agreement is calculated. A parabola is then fitted to the three measures of agreement and the time of minimum found from the minimum of the parabola.

If the minima are asymmetric (due to the shape of the orbit) then the method of Kwee & van Woerden (1956) should be replaced by a parabola fitted directly to the data around the time of minimum (Giménez 1985). Alternatively, if the minima are symmetrical but not total, a Gaussian fit is usually quite acceptable and the uncertainty of the result is then easier to estimate.

Once times of minima have been found, a reference time is chosen. The particular choice is not important but it is best to choose an accurate time of primary minimum near the middle of the times covered by the data, as this will give lower uncertainties in the resulting reference time, T_0 . In the study of EBs the primary minimum is defined to be deeper than the secondary minimum, so in general refers to a transit of the star of lower surface brightness across the disc of the star of higher surface brightness. Then an approximate orbital period should be used to calculate how many orbits have occurred between each time of minimum and the reference time. This cycle number will be an integer for primary eclipses and an integer plus 0.5 for a secondary eclipse. A straight line can then be fitted, using standard techniques, to

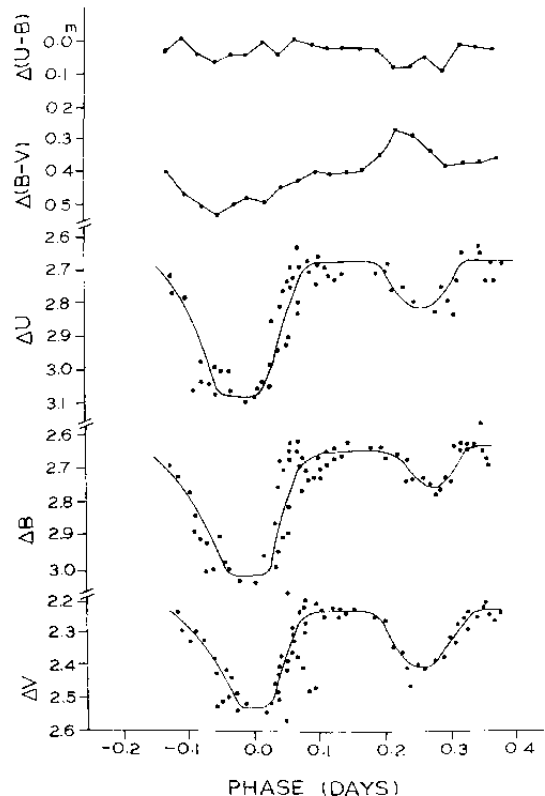


Figure 102. Example of a set of light curves which are not definitive. Taken from Srivastava & Sinha (1985).

the cycle numbers (ordinate) and time of minima (abscissa). The period and reference time are the parameters of the straight line.

The technique expounded in the last paragraph runs into problems when the EB has an eccentric orbit. In this case the secondary minima will not in general occur halfway between the adjacent primary minima and the times of primary and secondary eclipse should be analysed separately. This, though, runs into trouble if apsidal motion is present (Sec. 7.2), as this effect causes the periods found from the primary and secondary minima to be different. In this case a full apsidal motion analysis is needed.

13.2.2 Initial conditions

Once the data have been assembled it is important to estimate a realistic set of initial parameter values to input into the least-squares fitting routine. Several parameters can be adopted directly from theory or previous observation. Theoretical limb darkening coefficients have been tabulated by many authors (Sec. 1.4.1) and gravity brightening exponents have expected values (Sec. 1.5). The mass ratio can usually be fixed to a value available from a spectroscopic study of the dEB. The quantities just mentioned have only a minor impact on the light variation of a dEB except in specific circumstances, so any reasonable values can be used for a preliminary analysis.

Figs. 104 and 105 display a set of model light curves generated using the EBOP code for sets of photometric parameters designed to illustrate the effect each parameter has on the light curve for typical dEBs. For convenience Table 10 contains the values of these parameters for each displayed light curve. All light curves have: a sum of the fractional radii, $r_A + r_B$, of 0.4 (towards the limit of capability of EBOP but chosen for display purposes); gravity brightening coefficients, β_A and β_B , of 1.0 (appropriate for radiative atmospheres); a mass ratio, q , of 1.0 (this parameter is unimportant for well-detached EBs); no third light, $L_3 = 0.0$ (the effect of third light is simply to reduce the total magnitude of variation without changing the shape of the light curve); and equal limb darkening coefficients for both stars, $u_A = u_B$.

Panels (a), (b) and (c) of Fig. 104 each show three sets of

³⁶ <http://www.konkoly.hu/IBVS/IBVS.html>

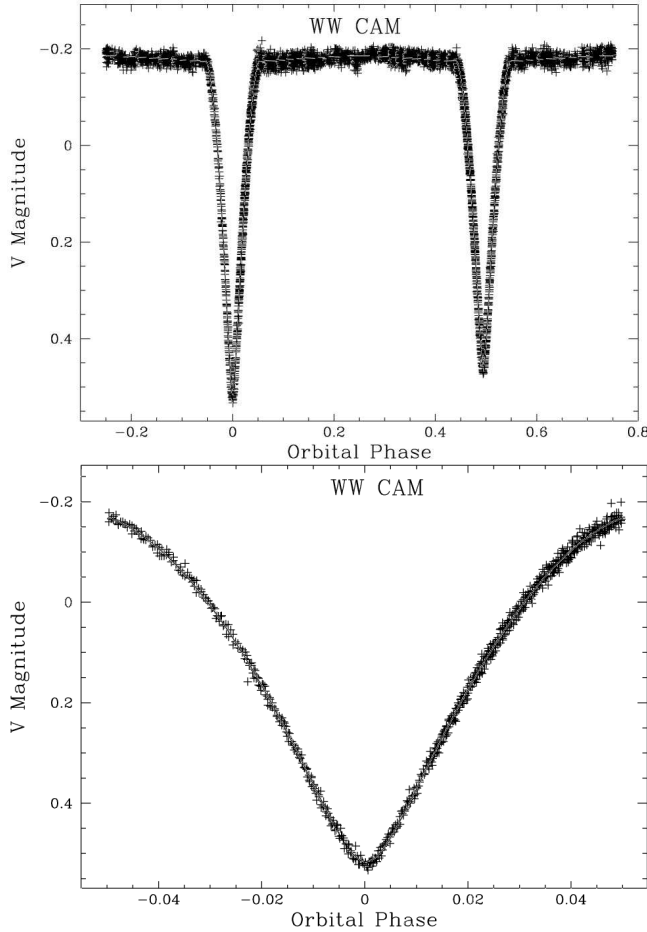


Figure 103. The most complete light curve obtained for a dEB by the year 2002. This light curve of WWCamelopardalis was observed using the Johnson *V* passband and consists of 5759 separate observations. The upper panel shows the whole light curve and the lower panel concentrates on the primary eclipse. Taken from Lacy et al. (2002).

parameters for typical MS dEBs, illustrating how the ratio of the radii (with a realistic adjustment to the surface brightness ratio also) changes. The orbital inclinations for the panels have been chosen to demonstrate total eclipses, deep eclipses and shallow eclipses. Fig. 105 panel (a) shows how a change of orbital eccentricity affects a light curve, with the longitude of periastron chosen to be 90.0° so the secondary minimum is at phase 0.5 irrespective of the value of orbital eccentricity. Fig. 105 panel (b) demonstrates how different values of the longitude of periastron change the phase of secondary minimum compared to the primary minimum (which has been put to phase 0.0 in all cases). Finally, Fig. 105 panel (c) shows the change in a light curve brought about by a large change in limb darkening coefficients for both stars. The effect is very small, demonstrating that an exceptionally good set of observations is needed to make the limb darkening coefficients well determined.

13.2.3 Parameter determinacy and correlations

Once a reasonable fit to the light curves under analysis has been found, the data can be fitted with a model using least-squares minimisation techniques. However, there are a number of well-known difficulties in the fitting of models to light curves of dEBs. Many of these relate to correlated parameters, although solutions exist. This means that choices must be made about which parameters to adjust freely, to fix to reasonable estimates, or to consider a variation of but not include in individual least-squares fits. A list of the problems follows.

The mass ratio is indeterminate in well-detached systems, so

Table 10. Photometric parameters of the EBOP model light curves shown in Figs. 104 and 105. Light curves are identified using the figure number, the panel and the light curve number. The parameters of interest to a particular panel are given in bold. All light curves have been generated using $r_A + r_B = 0.4$ (quite large but within the capability of EBOP), $\beta_A = 1.0$, $\beta_B = 1.0$, $q = 1.0$, $L_3 = 0.0$ and $u_A = u_B = u$.

Fig.	Panel	LC	k	i	J	u	e	ω
104	(a)	1	1.0	90.0	1.0	0.4	0.0	90.0
104	(a)	2	0.8	90.0	0.6	0.4	0.0	90.0
104	(a)	3	0.6	90.0	0.2	0.4	0.0	90.0
104	(b)	1	1.0	84.0	1.0	0.4	0.0	90.0
104	(b)	2	0.8	84.0	0.6	0.4	0.0	90.0
104	(b)	3	0.6	84.0	0.2	0.4	0.0	90.0
104	(c)	1	1.0	75.0	1.0	0.4	0.0	90.0
104	(c)	2	0.8	75.0	0.6	0.4	0.0	90.0
104	(c)	3	0.6	75.0	0.2	0.4	0.0	90.0
105	(a)	1	0.8	85.0	0.6	0.4	0.0	90.0
105	(a)	2	0.8	85.0	0.6	0.4	0.25	90.0
105	(a)	3	0.8	85.0	0.6	0.4	0.5	90.0
105	(b)	1	0.8	85.0	0.6	0.4	0.25	90.0
105	(b)	2	0.8	85.0	0.6	0.4	0.25	0.0
105	(b)	3	0.8	85.0	0.6	0.4	0.25	180.0
105	(c)	1	0.8	85.0	0.6	0.4	0.0	90.0
105	(c)	2	0.8	85.0	0.6	0.1	0.0	90.0
105	(c)	3	0.8	85.0	0.6	0.7	0.0	90.0

should be fixed at a spectroscopically-determined value or a good estimate (the latter possibility is allowable because the value of the mass ratio is unimportant). However, for close binaries which exhibit total eclipses, the mass ratio – and indeed the rotational velocity – may be found more easily from light curves than from spectroscopy (Wilson 1994; Fitzpatrick et al. 2003).

The investigation of second-order effects such as limb darkening and gravity brightening is difficult except for certain types of light curves and very good observational data. Third order effects, such as the effect of convection theory on limb darkening coefficients and gravity brightening exponents, are impossible to distinguish (Claret 2000a).

Third light can be very difficult to quantify in well-detached systems, and can be correlated with orbital inclination. Many researchers find no obvious trace of third light so arbitrarily set it to zero. This practice should be avoided when analysing good light curves. Either third light must be included as a free parameter, or an expected maximum possible value must be decided upon and the final parameter uncertainties modified to include a contribution due to this problem.

The light curves of close binary stars generally give better-determined values of the mass ratio, third light and of gravity brightening exponents. This can make them better distance indicators than well-detached binary stars (e.g., Graczyk 2004; Harries, Hilditch & Howarth 2003; Lee 1997) but less good for studying the evolution of single stars as the influence of the binary companion on the evolution of each star is greater.

For dEBs composed of two very similar stars which don't exhibit total eclipses, the ratio of the radii can be very poorly determined (Popper 1984). In this case the sum of the radii is usually well-known but the individual radii are strongly correlated with each other, and the ratio of the radii is strongly correlated with the light ratio of the system. For some dEBs it may not be possible to break this degeneracy, but for others it can be solved by adopting a light ratio found spectroscopically.

The ratio of the radii may be correlated with $e \sin \omega$ (see e.g., Clausen, Giménez & Scarfe 1986; Andersen & Clausen 1989; Clausen 1991; Barmba & Etzel 1995) as they have a similar

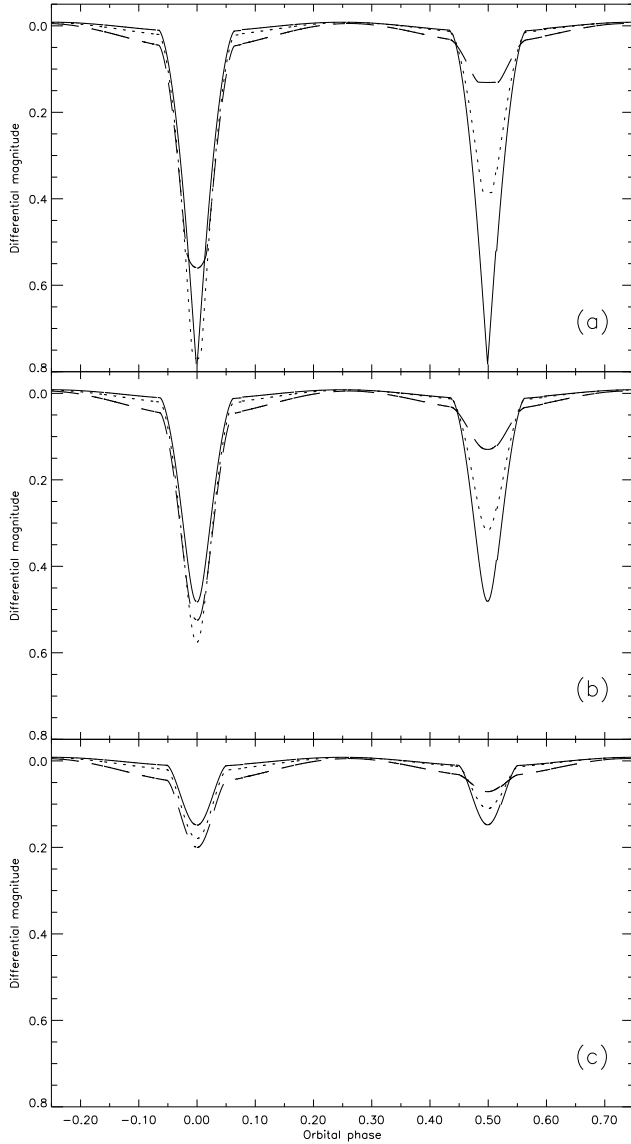


Figure 104. Representative light curves showing how orbital inclination affects the shape of light curves. The theoretical light curves were generated using the EBOP code (Sec. 13.1.2). The parameters of the different models are given in Table 10. In each panel, curve 1 is shown with a solid line, curve 2 with a dotted line, and curve 3 with a dashed line.

effect on the shape of the eclipses. This degeneracy can be broken by using results from a spectroscopic or apsidal motion analysis. Orbital eccentricity and periastron longitude can be also strongly correlated (e.g., Wilson & Woodward 1983). This is why EBOP and WINK solve for $e \cos \omega$ and $e \sin \omega$; these are better determined, particularly in systems with a small eccentricity.

Inclination and third light can be correlated (Popper 1984).

13.2.4 Final parameter values

Once the data have been assembled, the orbital ephemeris found, estimated parameter values determined and the parameters to solve for selected, the light curve fitting algorithm can be unleashed. Usually several different choices of adjustable parameters are made and different solutions obtained, depending on the type of light curve being studied. Once a best solution has been selected and extended to each light curve (assuming they were not solved simultaneously), there exists a set of best-fitting values for each parameter. Whilst some parameters, for example the surface brightness ratio, depend on the passband used to obtain each light curve, other parameters, for example the stellar radii,

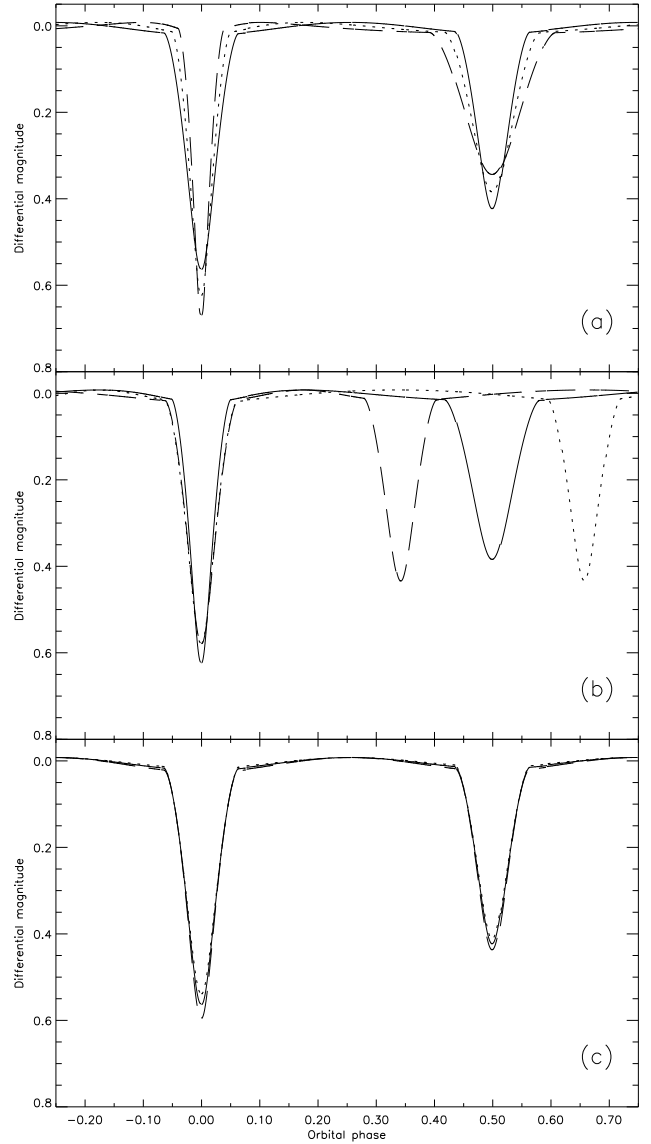


Figure 105. Representative light curves showing how orbital shape affects the shape of light curves. Symbols and references are as in Fig. 104.

are common between light curves. As several different determinations exist (one per light curve), the values can be compared to check that they are consistent. If they are, then the correct quantity to quote as a final result for each is the mean value. If uncertainties have been estimated (see below) then the weighted mean is the appropriate result to adopt.

When the ratio of the radii of the stars is poorly determined, it can be useful to constrain its value with a light ratio derived from spectroscopic observations (e.g., Andersen, Clausen & Nordström 1990a). On the MS, surface brightness decreases as stellar radius decreases, so a spectroscopic light ratio can provide a very accurate constraint on the ratio of the radii. An example of this is in the analysis of the dEB GG Orionis by Torres et al. (2000b). The *B* and *V* light curves for this dEB are shown in Fig. 106; they exhibit a shape which makes the ratio of the radii relatively poorly determined. Fig. 107 shows how a known light ratio (from spectroscopic observations) transforms directly into a constraint on the ratio of the radii for GG Orionis.

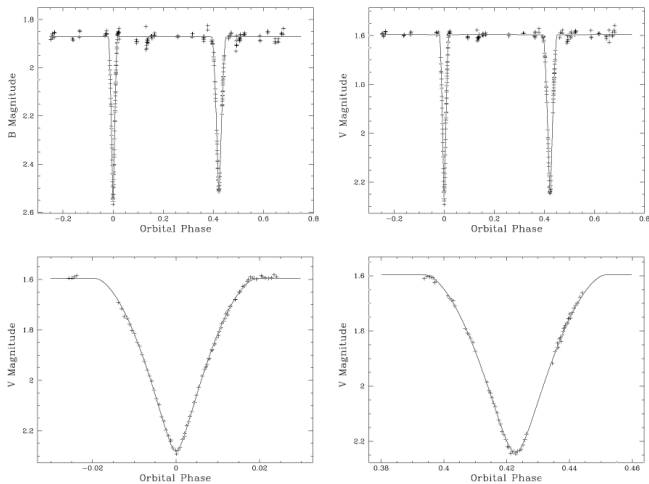


Figure 106. The B and V light curves of the dEB GG Orionis. Taken from Torres et al. (2000b).

13.3 Uncertainties in the parameters

13.3.1 The problem

Uncertainties in the photometric parameters of a light curve fit have not generally been investigated as well as they should be. Whilst a result only has meaning if it is accompanied with a reasonable estimate of its uncertainty, this has been neglected by several researchers. The main cause of this is that all light curve analysis programs, as supplied by their authors, calculate formal uncertainties based on the final fit. Whilst these uncertainties have some use, they are generally very optimistic as they do not fully reflect the correlations between different parameters (Andersen 1991). Some researchers supply the formal errors of the fit as their final error estimates and subsequently cause difficulties, for example Schiller & Milone (1987) (see discussion in Pinsonneault et al. 2003) and Munari et al. (2004) (see Sec. 14). Formal errors can be found, without discussion, in very recent works, for example Munari et al. (2004) and Stassun et al. (2004).

Popper (1984) provided an error analysis of the light curves of dEBs, using Monte Carlo simulations to estimate the magnitudes of errors. He found that no general rules exist to aid in the estimation of realistic uncertainties, but that robust uncertainties were generally no greater than three times the formal (internal) error of the fit. Popper also noted that the secondary eclipse appears to be more sensitive to changes in model parameters than the primary eclipse. He also stated that analyses of the same dEB by different researchers tended to disagree by more than expected given the quoted errors. This occurs for two reasons: correlated errors in observational data (i.e., ‘night errors’) cause systematic errors in the derived parameters, and researchers have been quoting optimistic errors.

13.3.2 The solutions

The best way of estimating uncertainties is to observe many separate light curves, analyse them separately, and consider the values found for each parameter. Unfortunately, a sufficient number of light curves is not in general obtainable to provide an accurate estimation of the uncertainties. If only one or two light curves have been observed, this technique would provide no error estimates whatsoever.

One way of estimating reliable parameter uncertainties from light curves is, for each parameter, to fix it at several values, optimise the other parameters, and analyse the χ^2 of the resulting fits. This has been used by Hensberge, Pavlovski & Verschueren (2000) in their analysis of the high-mass dEB V578 Monocerotis. They found that the uncertainties they derived were roughly five times larger than the formal errors calculated by the WD93 code. They also considered the expected photometric errors and overall

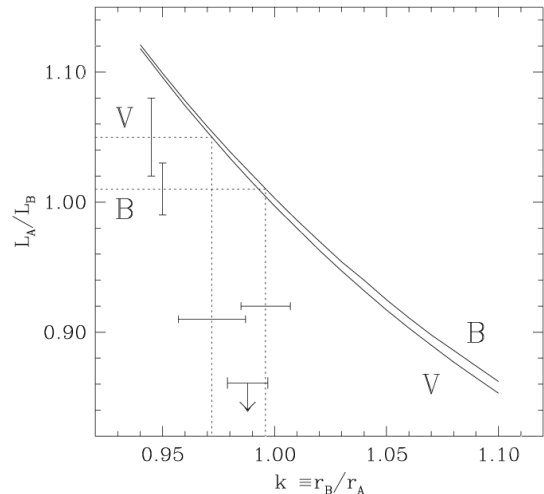


Figure 107. Illustration of the use of a spectroscopic light ratio to find the ratio of the radii of a dEB. A known light ratio (L_A/L_B) is used to find the corresponding ratio of the radii (r_B/r_A) for GG Orionis. Taken from Torres et al. (2000b).

uncertainties and found that the systematic error, i.e., the difference between the two error estimates, was about twice as large as the random error for that study.

A full discussion of error analysis is given by Press et al. (1992, pp.684–700). For the study of the light curves of dEBs, for which the model light curves provide a good representation of the actual light variation, the most reliable technique is Monte Carlo simulations. Once a best fit has been found, the model is evaluated at the phases of observation. Random Gaussian scatter (to simulate observational errors) is added and the resulting light curve is refitted. This process is repeated a large number of times and the spread of values of the derived parameters can then be analysed to determine robust uncertainties. Confidence intervals can then be constructed according to the requirements of the researcher. One problem with this process is that the confidence intervals refer to the point in parameter space where the initial best fit was found, which is in general slightly different to the actual properties of the dEB being studied (Ford 2004). However, this bias is small and generally unimportant for the study of dEBs. A great advantage of Monte Carlo simulations is that study of the sets of parameter values which it provides can give an excellent idea of the relations and correlations between different parameters.

14 ORIGINAL WORK

Detailed analyses have been undertaken for the six detached eclipsing binaries V615 Persei and V618 Persei (Southworth, Maxted & Smalley 2004a), V453 Cygni (Southworth, Maxted & Smalley 2004b), V621 Persei (Southworth et al. 2004c), HD 23642 (Southworth, Maxted & Smalley 2005a) and WW Aurigae (Southworth et al. 2005b); the above works are the primary references for the analyses undertaken. The abstracts are given below.

Eclipsing binaries in open clusters. I. V615 Per and V618 Per in h Persei

We derive absolute dimensions for two early-type main sequence detached eclipsing binaries in the young open cluster h Persei (NGC 869). V615 Persei has a spectral type of B7 V and a period of 13.7 days. V618 Persei is A2 V and has a period of 6.4 days. New ephemerides are calculated for both systems. The masses of the component stars have been derived using high-resolution spectroscopy and are $4.08 \pm 0.06 M_{\odot}$ and $3.18 \pm 0.05 M_{\odot}$ for V615 Per and $2.33 \pm 0.03 M_{\odot}$ and $1.56 \pm 0.02 M_{\odot}$ for V618 Per. The radii have been measured by fitting the available light curves using EBOP and are $2.29 \pm 0.14 R_{\odot}$ and $1.90 \pm 0.09 R_{\odot}$ for V615 Per and $1.64 \pm 0.07 R_{\odot}$ and $1.32 \pm 0.07 R_{\odot}$ for V618 Per. By comparing the observed spectra of V615 Per to synthetic spectra from model atmospheres we find that the effective temperatures of the two stars are 15000 ± 500 K and 11000 ± 500 K. The equatorial rotational velocities of the primary and secondary components of V615 Per are $28 \pm 5 \text{ km s}^{-1}$ and $8 \pm 5 \text{ km s}^{-1}$, respectively. Both components of V618 Per rotate at $10 \pm 5 \text{ km s}^{-1}$. The equatorial rotational velocities for synchronous rotation are about 10 km s^{-1} for all four stars. The timescales for orbital circularisation for both systems, and the timescale for rotational synchronisation of V615 Per, are much greater than the age of h Per. Their negligible eccentricities and equatorial rotational velocities therefore support the hypothesis that they were formed by ‘delayed breakup’ (Tohline 2002). We have compared the radii of these stars to models by the Granada and the Padova groups for stars of the same masses but different compositions. We conclude that the metallicity of the stars is $Z \approx 0.01$. This appears to be the first estimate of the bulk metallicity of h Per. Recent photometric studies have assumed a solar metallicity so their results should be reviewed.

Eclipsing binaries in open clusters. II. V453 Cyg in NGC 6871

We derive absolute dimensions of the early B-type detached eclipsing binary V453 Cygni (B0.4 IV + B0.7 IV, $P = 3.89$ d), a member of the open cluster NGC 6871. From the analysis of new, high-resolution, spectroscopy and the *UBV* light curves of Cohen (1974) we find the masses to be $14.36 \pm 0.20 M_{\odot}$ and $11.11 \pm 0.13 M_{\odot}$, the radii to be $8.55 \pm 0.06 R_{\odot}$ and $5.49 \pm 0.06 R_{\odot}$, and the effective temperatures to be $26\,600 \pm 500$ K and $25\,500 \pm 800$ K for the primary and secondary stars, respectively. The surface gravity values of $\log g = 3.731 \pm 0.012$ and 4.005 ± 0.015 indicate that V453 Cyg is reaching the end of its main sequence lifetime. We have determined the apsidal motion period of the system to be 66.4 ± 1.8 yr using the technique of Lacy (1992) extended to include spectroscopic data as well as times of minimum light, giving a density concentration coefficient of $\log k_2 = -2.254 \pm 0.024$. Contaminating (third) light has been detected for the first time in the light curve of V453 Cyg; previous analyses without this effect systematically underestimate the ratio of the radii of the two stars. The absolute dimensions of the system have been compared to the stellar evolution models of the Granada, Geneva, Padova and Cambridge groups. All model sets fit the data on V453 Cyg for solar helium and metal abundances and an age of 10.0 ± 0.2 Myr. The Granada models also agree fully with the observed $\log k_2$ once general relativistic effects have been accounted for. The Cambridge models with convective core overshooting fit V453 Cyg better than those without. Given this success of the theoretical predictions, we briefly discuss which eclipsing binaries should be studied in order to further challenge the models.

Eclipsing binaries in open clusters. III. V621 Per in χ Persei

V621 Persei is a detached eclipsing binary in the open cluster χ Persei which is composed of an early B-type giant star and a main sequence secondary component. From high-resolution spectroscopic observations and radial velocities from the literature, we determine the orbital period to be 25.5 days and the primary velocity semi-amplitude to be $K = 64.5 \pm 0.4 \text{ km s}^{-1}$. No trace of the secondary star has been found in the spectrum. We solve the discovery light curves of this totally-eclipsing binary and find that the surface gravity of the secondary star is $\log g_B = 4.244 \pm 0.054$. We compare the absolute masses and radii of the two stars in the mass–radius diagram, for different possible values of the primary surface gravity, to the predictions of stellar models. We find that $\log g_A \approx 3.55$, in agreement with values found from fitting Balmer lines with synthetic profiles. The expected masses of the two stars are $12 M_{\odot}$ and $6 M_{\odot}$ and the expected radii are $10 R_{\odot}$ and $3 R_{\odot}$. The primary component is near the blue loop stage in its evolution.

Eclipsing binaries as standard candles: HD 23642 and the distance to the Pleiades

We present a reanalysis of the light curves of HD 23642, a detached eclipsing binary star in the Pleiades open cluster, with emphasis on a detailed error analysis. We compare the masses and radii of the two stars to predictions of stellar evolutionary models and find that the metal and helium abundances of the Pleiades are approximately solar. We present a new method for finding distances to eclipsing binaries, of spectral types A to M, using the empirical calibrations of effective temperature versus surface brightness given by Kervella et al. (2004). We use the calibration for *K*-filter surface brightness to determine a distance of 139.1 ± 3.5 pc to HD 23642 and the Pleiades. This distance is in excellent agreement with distances found from the use of theoretical and empirical bolometric corrections. We show that the determination of distance, both from the use of surface brightness relations and from the use of bolometric corrections, is more accurate and precise at infrared wavelengths than at optical wavelengths. The distance to HD 23642 is consistent with that derived from photometric methods and Hubble Space Telescope parallaxes, but is inconsistent with the distance measured using Hipparcos parallaxes of HD 23642 and of other Pleiades stars.

Absolute dimensions of detached eclipsing binaries. I. The metallic-lined system WW Aurigae

WW Aurigae is a detached eclipsing binary composed of two metallic-lined A-type stars orbiting each other every 2.5 days. We have determined the masses and radii of both components to accuracies of 0.4% and 0.6%, respectively. From a cross-correlation analysis of high-resolution spectra we find masses of $1.964 \pm 0.007 M_{\odot}$ for the primary star and $1.814 \pm 0.007 M_{\odot}$ for the secondary star. From an analysis of photoelectric *uvby* and *UBV* light curves we find the radii of the stars to be $1.927 \pm 0.011 R_{\odot}$ and $1.841 \pm 0.011 R_{\odot}$, where the uncertainties have been calculated using a Monte Carlo algorithm. Fundamental effective temperatures of the two stars have been derived, using the Hipparcos parallax of WW Aur and published ultraviolet, optical and infrared fluxes, and are 7960 ± 420 and 7670 ± 410 K. The masses, radii and effective temperatures of WW Aur are only matched by theoretical evolutionary models for a fractional initial metal abundance, Z , of approximately 0.06 and an age of roughly 90 Myr. This seems to be the highest metal abundance inferred for a well-studied detached eclipsing binary, but we find no evidence that it is related to the metallic-lined nature of the stars. The circular orbit of WW Aur is in conflict with the circularization timescales of both the Tassoul and the Zahn tidal theories and we suggest that this is due to pre-main-sequence evolution or the presence of a circular orbit when the stars were formed.

15 CONCLUSION

15.1 What this work can tell us

We have studied a total of six dEBs in four stellar open clusters. The first conclusion that can be drawn from this research programme is that one telescope observing run can provide a complete spectroscopic dataset for ten or more dEBs (of which only a minority have been studied in this work). Further conclusions divide easily into three categories: analysis techniques for the study of dEBs, what dEBs can tell us about stellar clusters, and what we can find by comparing the properties of dEBs to theoretical stellar models. I will now summarise these three categories.

15.1.1 The observation and analysis of dEBs

Three conclusions concerning the acquisition of data for dEBs:

- Complete spectroscopic observations of many dEBs can be obtained during the same observing run, resulting in a high efficiency in terms of time taken to obtain and reduce data. This is particularly useful for observing a good set of standard stars.
- Photometric data for dEBs requires much more time and effort to obtain, but this can be avoided by using data which has been culled from the literature. These light curves may previously have been analysed using outdated methods (V453 Cyg and WW Aur) or may be previously unpublished (WW Aur).
- dEBs which are located in the same cluster can be studied simultaneously using CCD photometry.

Conclusions concerning the photometric analysis of dEBs:

- The results from the three different light curve modelling codes, EBOP, WINK and WD98, are generally in excellent agreement (HD 23642 and WW Aur) which confirms that they are reliable tools for the study of dEBs.
- Uncertainties in light curve parameters are reported by EBOP, WINK and WD98, but these formal errors are well known to be significantly too optimistic (Popper 1984; Sec. 13.3).
- I have implemented a Monte Carlo algorithm to find robust uncertainties in the light curve parameters of dEBs. Its results agree extremely well with the variation in results for different light curves of the same dEB (V453 Cyg, WW Aur) and it is a powerful way of investigating correlations between different light curve parameters (V453 Cyg, HD 23642, WW Aur). I recommend that the Monte Carlo algorithm becomes the standard technique for finding light curve uncertainties.
- Limb darkening and third light must be considered when fitting light curves of dEBs. The value of the limb darkening coefficients can make a significant difference to the result (V621 Per); this can easily be investigated and quantified using the results of a Monte Carlo analysis. Third light can be assumed to be zero only if this clearly provides the best fit to the light curves (e.g., WW Aur); if not then the uncertainties in the photometric parameters must be increased to reflect this (HD 23642).

Conclusions concerning the spectroscopic analysis of dEBs:

- The two-dimensional cross-correlation algorithm TODCOR (Zucker & Mazeh 1994) is a reliable tool for extracting RVs from observed composite spectra, performing particularly well compared to other methods when the data are of a low signal to noise ratio (V618 Per).
- The use of synthetic template spectra with TODCOR can provide precise results, but systematic errors from the mismatch between the template and the observed spectra must be quantified. One way of estimating these is to run TODCOR using every combination of many template spectra generated for a wide variety of T_{eff} s, surface gravities and rotational and microturbulent velocities (V618 Per; work on NGC 2243 V1 in preparation).
- Template spectra for TODCOR analysis can be obtained by observing the target dEB when the RV difference between the two stars is minimal, or during total eclipse when the spectrum comes entirely from one star (V453 Cyg), avoiding systematic errors due to mismatch between template and target spectra.
- The use of observed template spectra with TODCOR is an

excellent way of deriving accurate spectroscopic orbits as it can avoid the types of systematic errors associated with the use of synthetic templates. If TODCOR is run using every combination of a set of observed templates, systematic errors due to spectral mismatch will average out and the internal errors of the resulting spectroscopic orbit can be found (WW Aur).

- The errors reported by SBOP are an excellent estimate of the actual internal errors of a spectroscopic orbit (WW Aur).

During my analysis of WW Aur I was able to use the techniques covered above to derive accurate masses and radii for both stars using an entirely arithmetical approach. The EBOP code is geometrical by nature and limb darkening coefficients were freely adjusted towards the best fit rather than being fixed at theoretical values. A set of nine observed template spectra were used in the TODCOR analysis, avoiding possible systematic errors due to the use of synthetic spectra or to one observed template spectrum which may or may not match the spectra of the target stars. It is clear to me that these methods are a good way with which to analyse observational data on dEBs.

15.1.2 Studying stellar clusters using dEBs

Knowledge of the masses and radii of a dEB allow us to estimate its age and chemical composition from a comparison with the predictions of theoretical stellar evolutionary models. In the case of the h Persei open cluster, a precise metal abundance was found from the positions of the components of V615 Per and V618 Per in the mass–radius plane even though the radii of these stars are known to accuracies of only about 5%. More accurate dimensions of these four stars would enable estimation of a precise age, metal abundance, helium abundance and possibly convective efficiency parameters. We have also provided further evidence that h Persei and χ Persei are physically related; their systemic velocities are the same (V615 Per, V618 Per and V621 Per). The study of dEBs which are near the MS turn-off of their parent open cluster would allow a detailed investigation into the success of convective overshooting approximations in theoretical models.

dEBs are excellent distance indicators: this was used in the study of HD 23642 to find the distance to the Pleiades open cluster. The resulting distance does not agree with that derived from the parallax observations of the Hipparcos satellite. There are several different ways of finding the distance to a dEB (HD 23642) and the best results are obtained in the IR because of the reduced importance of interstellar reddening, stellar metal abundance, uncertainties in the T_{eff} s of the stars, and a lower ‘cosmic scatter’. Relations between surface brightness and colour index allow an entirely empirical distance to be found to a dEB, but the results can be inaccurate. The use of methods involving bolometric corrections can provide more precise results but this comes with either a dependence on theoretical model atmospheres or inaccurate empirical bolometric corrections. To avoid these problems, we introduced a new method to find the distance to a dEB which uses relations between surface brightness and T_{eff} (HD 23642). Whilst this method is not entirely empirical, it provides results which are as precise as methods using theoretical bolometric corrections but are much less dependent on theoretical calculations.

15.1.3 Theoretical stellar evolutionary models and dEBs

dEBs provide excellent tests of theoretical models because it is possible to derive accurate masses, radii and T_{eff} s of two stars which have the same age, distance and chemical composition. V453 Cyg is a particularly rewarding dEB for a comparison with theoretical models because its masses, radii and T_{eff} s are accurately known, as is the central concentration of the mass of the primary star (from analysis of the apsidal motion of the dEB). The theoretical models of the Granada, Padova, Cambridge and Geneva groups were all able to provide a good fit in the mass–radius and T_{eff} – $\log g$ diagrams to the properties of this high-mass slightly-evolved dEB, whilst the Granada models also successfully predicted the central concentration of the primary star. There was a minor indication that models incorporating convective core

overshooting provide a better fit to V453 Cyg. It is clear that the current generation of theoretical models are very successful at predicting the properties of MS B, A and F stars, and that more evolved, more massive or less massive dEBs must be studied in order to provide useful tests of stellar evolutionary theory.

The formation scenarios of binary stars were investigated for V615 Per and V618 Per. All four component stars have slow rotation and circular orbits despite being only 13 Myr old. This is in complete disagreement with theories of MS tidal evolution but may be explained by strong tidal effects during the PMS phase or by formation of binary stars with these characteristics at birth.

The properties of metallic-lined stars were investigated (WW Aur) and it was found that they have masses and radii characteristic of normal A-type stars, suggesting that the Am phenomenon is a surface characteristic. For WW Aur we were only able to fit the masses and radii using theoretical models with a very high metal and helium abundance ($Z = 0.06$ and $Y = 0.36$). We presented evidence that this was not the case for other metallic-lined dEBs and that indications of such large abundances have been noted elsewhere.

One unique feature of the study of dEBs in stellar clusters is that it is possible to find accurate masses, radii and T_{eff} s for four or more stars with the same age, distance and chemical composition. This was first noted when studying V615 Per and V618 Per and potentially can provide an extremely detailed test of theoretical models in which values may be found for many different theoretical parameters which would otherwise be left fixed at a reasonable estimate. Another way in which the study of dEBs in clusters will be useful is in forcing theoretical models to fit the masses, radii and T_{eff} s of both components of the dEB whilst simultaneously matching the radiative properties of the other member stars in the CMD of the cluster. This requires accurate dimensions for a dEB in a cluster with a well-defined morphology in the cluster CMDs, so we were not able to investigate it further using the dEBs studied in this work.

15.2 Further work

15.2.1 Further study of the dEBs in this work

A definitive study of a dEB is generally expected to provide masses and radii to accuracies of 2% as well as accurate T_{eff} s and a reasonable comparison with theoretical models. The studies of WW Aur and V453 Cyg presented in this work can therefore be regarded as definitive, although the characteristics of V453 Cyg are such that further spectroscopy, photometry and times of minimum light would clearly make the dEB even more interesting.

V615 Per and V618 Per are prime candidates for further study as we have found their masses to within 1.5% but their radii are much more uncertain. As the h Persei open cluster has a well-defined CMD morphology, an improved study of the two dEBs will allow the development of tools for the simultaneous matching of the properties of the dEBs and the cluster to theoretical models. This should be done as soon as possible.

The dimensions of HD 23642 are also less accurate than they could be (contrary to the findings of Munari et al. 2004) and this dEB is also in a nearby and well-known open cluster. It will certainly be the subject of further study in the near future, and I am aware that new light curves have been obtained by another group.

The study of V621 Per presented in this paper is different to the other work in that we were not able to detect the secondary star spectroscopically and so were not able to measure the masses and radii of either star. This dEB may be difficult to study further but the effort would be very worthwhile because it might provide accurate dimensions of a B-type giant star, a system which would be unique amongst well-studied dEBs (see Andersen 1991). The secondary component in the V621 Per system is known to be unevolved as we were able to calculate its surface gravity to be $\log g = 4.244 \pm 0.054$ from the results of the spectroscopic and photometric analysis. dEBs with a small mass ratio (here expected to be around 0.5) are particularly valuable as they are excellent tests of theoretical stellar models.

15.2.2 Other dEBs in open clusters

We have obtained spectroscopic data for a substantial number of dEBs which were not studied in this work, and hope to be able to publish much of this in the near future. A short list of dEBs in open clusters is presented in Table 11; we already have data for some of these systems. I know of two other research groups currently working on dEBs in open clusters.

Many studies have been published on the photometric identification of variable stars by the observation of light curves using telescopes and CCDs. These are often targeted at open clusters to increase the number of stars in the observed field of view, and because variable stars in open clusters are intrinsically more interesting. In particular, the journal *Acta Astronomica* has published many such studies (Table 11). dEBs are usually found towards the MS turn-off as stars increase in radius during the latter stages of the MS evolution (Kaluźny & Rucinski 1993).

15.2.3 dEBs in globular clusters

The usefulness of studying dEBs in globular clusters was demonstrated by Thompson et al. (2001) by their analysis of OGLE GC 17 in the peculiar Galactic globular cluster ω Centauri. The faintness of this dEB meant that the masses and radii were found to accuracies of only 7% and 3%, respectively, but the use of IR surface brightness relations meant that a distance of 5360 ± 300 pc ($(M - m)_0 = 13.65 \pm 0.12$ mag) could be derived. The age of the dEB and of ω Cen was also found to be between 13 and 17 Gyr. Further observations of this dEB have been made (Kaluźny et al. 2002) but have not yet been published, and several more dEBs are known in this cluster.

A significant number of dEBs have been discovered in 47 Tucanae (Albrow et al. 2001; Weldrake et al. 2004), but these are quite faint. Additional candidates have been found in NGC 6641 (Pritzl et al. 2001) and M 22 (Kaluźny & Thompson 2001). A compilation of variable stars in the fields of globular clusters has been given by Clement et al. (2001).

15.2.4 dEBs in other galaxies

A large number of dEBs have been found through time-series photometry of the LMC and SMC by the OGLE, EROS, MACHO and MOA groups (see sec 6.3.4) and several have been studied in order to find the distances to the Large and Small Magellanic Clouds (e.g., Hilditch, Harries & Howarth et al. 2005; Sec. 6.3.4). The MOA group have published details of 167 EBs in the SMC (Bayne et al. 2004). The EROS group have published a list of 79 EBs located towards the bar of the LMC (Grison et al. 1995). The MACHO group have published a list of 611 EBs in the LMC (Alcock et al. 1997).

The OGLE group have obtained by far the largest amount of photometry towards both the LMC and the SMC and have found 2580 EBs in the LMC (Wyrzykowski et al. 2003) and 1459 EBs in the SMC (Udalski et al. 1998). Analysis using a difference image analysis algorithm (Żebruń, Soszyński & Wozniak 2001) has allowed the discovery of a further 455 EBs in the SMC (Wyrzykowski et al. 2004). Most interesting are the 127 EBs which have been found to be in optical coincidence with star clusters in the SMC (Pietrzyński & Udalski 1999).

15.2.5 dEBs in clusters containing δ Cepheids

δ Cephei stars can be used as distance indicators at greater distances than EBs because they are intrinsically brighter objects (with absolute visual magnitudes between about -2 and -6) and can be studied at dimmer apparent magnitudes because spectroscopy is not needed. The distances to Galactic open clusters which contain dEBs and δ Cepheids can be found from the dEB and used to calibrate the δ Cepheid distance scale.

The primary candidate for such an analysis is the dEB QX Cassiopeiae, which is a possible member of the open cluster NGC 7790, which contains three δ Cepheids. However, it is a

Table 11. A list of dEBs in Galactic open clusters and associations for which their study may be very rewarding.

Eclipsing system	α_{2000} (hour, min)	δ_{2000} (degrees)	Cluster or association	Spectral type	Apparent V magnitude	Period (days)	Reference
NGC 188 V12	00 39	+85	NGC 188	F	15.0	2.8	Zhang et al. (2002, 2004)
DS And	01 54	+37	NGC 752	F3 + G0	0.5	1.0	Schiller & Milone (1988)
V818 Tau	04 24	+15	Hyades	G8 + K3	8.3	5.6	Schiller & Milone (1987)
EW Ori	05 33	−01	Collinder 70	G0 + G5	9.8	6.9	Popper et al. (1986)
NGC 2243 V5	06 28	−31	NGC 2243	F	16.3	1.2	Kałużny et al. (1996)
V392 Car	07 58	−61	NGC 2516	A2	9.5	3.2	Debernardi & North (2001)
GV Car	11 04	−58	NGC 3532	A0	8.9	4.3	Kraft & Landolt (1959)
QR Cen	13 57	−59	NGC 5381	A	12.5	2.3	Pietrzyński et al. (1997)
V906 Sco	17 51	−34	Messier 7	B9	6.0	2.8	Alencar, Vaz & Helt (1997)
V1481 Cyg	21 42	+53	NGC 7128	B2	12.3	2.8	Jerzykiewicz et al. (1996)

photometric nonmember (Sandage 1958; Sandage & Tammann 1969) and is distant from the core of the cluster on the sky, so may be a non-member.

Several δ Cepheids are known to be members of Galactic open clusters (see Mermilliod, Mayor & Burki 1987) and these clusters should be photometrically surveyed for dEBs which can then be studied in order to find the distance and chemical composition of the clusters and δ Cepheids.

15.2.6 dEBs which are otherwise interesting

There is a shortage of dEBs which contain well-studied component stars more massive than $10 M_{\odot}$ (Andersen 1991). Such systems are intrinsically rare as massive stars have a low birth rate and very short lifetimes. They can also be very difficult to study, both photometrically as long orbital periods are needed for the stars to be well detached, and spectroscopically due to having very few strong metallic spectral lines and often large rotational velocities. Accurate properties of massive dEBs are needed to provide improved constraints on theoretical stellar models and to ensure that we understand such systems well enough to use them as distance indicators in external galaxies.

There is a shortage of dEBs which contain well-studied component stars less massive than $1 M_{\odot}$ (Andersen 1991). Such systems are difficult to find as low-mass stars are intrinsically faint, and the stars are small so are less likely to eclipse. They can also be very difficult to study, both photometrically as they often exhibit surface inhomogeneities such as starspots, and spectroscopically as their spectra are complex and relatively poorly understood. Mazeh et al. (2001) have stated that the best observational data which can be used to improve stellar evolutionary models for low-mass stars is the study of M-type dEBs in open clusters. This will need nearby clusters for the M dwarf stars to be sufficiently bright for study, but may provide accurate masses and radii of low-mass stars with a known metal abundance and age. Several researchers are obtaining accurate astrophysical parameters of low-mass dEBs (Clausen, Helt & Olsen 2001; Oblak et al. 2004; Hebb, Wyse & Gilmore 2004; Pepper, Gould & DePoy 2004).

dEBs which exhibit apsidal motion are intrinsically more valuable because their orbital parameters may be derived more accurately and the central concentration of the masses of the stars can be investigated (sec 7.2). This allows a more detailed test of theoretical stellar evolutionary models (e.g., V453 Cyg, sec 14).

Some types of stellar peculiarity can be investigated by studying examples which are in dEBs, e.g., metallic-lined stars (WW Aur, Sec. 14) and slowly pulsating B stars (Clausen 1996a).

15.2.7 dEBs found by large-scale photometric monitoring

Wide-field searches for photometrically variable stars is currently an extremely popular subject in astronomy, mainly due to the possibility of detecting extrasolar planetary candidates which transit their parent stars (so are therefore actually members of

EBs). Several of these have targeted nearby open clusters. It is expected that many (possibly thousands of) dEBs will be discovered in the near future, and that the light curves of some of these may be definitive, depending on the observational procedures adopted by the groups involved. A full exposition of the groups pursuing this research is beyond the scope of the current work, but it is relevant to mention some of those groups whose research is either sufficiently advanced to have appeared in published journals or is particularly relevant to the study of dEBs in stellar clusters. A full list of groups who are attempting to detect transiting extrasolar planets through wide-field CCD photometry is maintained by K. D. Horne³⁷.

SuperWasp³⁸ is the brainchild of D. Pollacco³⁹ and currently consists of five CCD cameras and telephoto lenses which are mounted on one telescope mount on La Palma. Each camera-lens combination has a field of view of $(7.8^{\circ})^2$ and can achieve 1% photometric precision for stars with apparent magnitudes between about 7 and 12 (Christian et al. 2004). This project is the successor to the WASP0 project, which consisted of one CCD camera and telephoto lens piggy-backed onto a commercially available Meade telescope (see Kane et al. 2004).

All Sky Automated Survey (ASAS⁴⁰) is a project to survey the whole Southern sky for photometric variability using four telescopes located at Las Campanas Observatory, Chile (Pojmański 1997, 1998, 2000). Several thousand variable stars have already been found (Pojmański 2002, 2003; Pojmański & Maciejewski 2004a, 2004b) and the project is ongoing.

EXPLORE/OC⁴¹ is a project to detect planetary transits around stars located towards nearby open clusters. It has obtained substantial photometry of the open clusters NGC 2660, NGC 6208, IC 2742, NGC 5316 and NGC 6235 (Lee et al. 2004; von Braun et al. 2004) using a 1.0 m telescope and large-format CCD camera. NGC 2660 and NGC 6208 are being analysed.

PISCES⁴² (Planets In Stellar Clusters Extensive Search; it is hoped that less attention will be paid to contrived acronyms in the future) is studying open clusters to find variable stars and transiting planets using a 1.2 m telescope and wide-field camera with a mosaic of four CCDs. Results have been published for NGC 6791 and NGC 2158 (Mochejska et al. 2002, 2004, 2005).

STEPSS⁴³ (Survey for Transiting Extrasolar Planets in Stellar Systems) is studying nearby open clusters using 2.4 m and 1.3 m telescopes equipped with a mosaic of eight CCDs. Results have been published for NGC 1245 (Burke et al. 2003, 2004) and are expected soon for NGC 2099 (M 37) and M 67.

It is hoped that, after a lull during the 1990s, the huge numbers of newly detected dEBs will be used to begin a new golden age of the study of EBs, the properties of which are of such fundamental importance to almost all aspects of astrophysics.

³⁷ <http://star-www.st-and.ac.uk/~kdh1/transits/table.html>

³⁹ <http://star.pst.qub.ac.uk/~dlp/>

⁴⁰ <http://www.astrow.edu.pl/~gp/asas/asas.html>

REFERENCES

- Abt H. A., 1958, *ApJ*, 128, 139.
- Abt H. A., Levato H., 1978, *PASP*, 90, 201.
- Abt H. A., Levy S. G., 1973, *ApJ*, 184, 167.
- Abt H. A., Morrell N. I., 1995, *ApJS*, 99, 135.
- Abt H. A., Levy S. G., Gandet L., 1972, *AJ*, 77, 138.
- Abt H. A., Levato H., Grosso M., 2002, *ApJ*, 573, 359.
- Adelman S. J., Gulliver A. F., Smalley B., Pazder J. S., Younger P. F., Boyd L., Epand D., 2004, in Zverko J., Weiss W. W., Žižňovský J., Adelman S. J., eds., *IAU Symp. 224, The A-Star Puzzle*, Cambridge Univ. Press, 911.
- Albrow M. D., Gilliland R. L., Brown T. M., Edmonds P. D., Guhathakurta P., Sarajedini A., 2001, *ApJ*, 559, 1060.
- Alcock C., Allsman R. A., Alves D., 1997, *AJ*, 114, 326.
- Alencar S. H. P., Vaz L. P. R., 1999, *A&AS*, 135, 555.
- Alencar S. H. P., Vaz L. P. R., Helt B. E., 1997, *A&A*, 326, 709.
- Allen C. W., 1973, *Astrophysical Quantities* (Third edition), The Athlone Press, University of London.
- Allende Prieto C., 2001, *ApJ*, 547, 200.
- Al-Naimiy H. M. K., 1978, *Ap&SS*, 53, 181.
- Alongi M., Bertelli G., Bressan A., Chiosi C., Fagotto F., Greggio L., Nasi E., 1993, *A&AS*, 97, 851.
- Alonso A., Arribas S., Martínez-Roger C., 1996, *A&A*, 313, 873.
- Alves D. R., 2004, *New Astron. Rev.*, 48, 659.
- Alves D. R., Rejkuba M., Minniti D., Cook K. H., 2002, *ApJ*, 573, L51.
- Anders E., Grevesse N., 1989, *Geochimica et Cosmochimica Acta*, 53, 197.
- Andersen J., 1975a, *A&A*, 44, 355.
- Andersen J., 1975b, *A&A*, 44, 445.
- Andersen J., 1975c, *A&A*, 45, 203.
- Andersen J., 1991, *A&AR*, 3, 91.
- Andersen J., 1993, in Weiss W. W., Baglin A., eds., *ASP Conf. 40., Inside the stars*, 347.
- Andersen J., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, 99.
- Andersen J., Clausen J. V., 1989, *A&A*, 213, 183.
- Andersen J., Vaz L. P. R., 1984, *A&A*, 130, 102.
- Andersen J., Vaz L. P. R., 1987, *A&A*, 175, 355.
- Andersen J., Clausen J. V., Giménez A., 1993, *A&A*, 277, 439.
- Andersen J., Clausen J. V., Nordström B., 1980, in M. J. Plavec, D. M. Popper, R. K. Ulrich, eds., *IAU Symp. 88, Close binary stars: Observations and interpretation*, 81.
- Andersen J., Clausen J. V., Nordström B., 1984, *A&A*, 134, 147.
- Andersen J., Clausen J. V., Nordström B., 1990a, *A&A*, 228, 365.
- Andersen J., Clausen J. V., Nordström B., 1990b, *ApJ*, 363, L33.
- Andersen J., Clausen J. V., Nordström B., Reipurth B., 1981, *A&A*, 101, 7.
- Andersen J., Clausen J. V., Jørgensen H. E., Nordström B., 1984, in Maeder A., Renzini A., eds., *IAU Symp. 105, Observational Tests of the Stellar Evolution Theory*, 391.
- Andersen J., Clausen J. V., Nordström B., Popper D. M., 1985, *A&A*, 151, 329.
- Andersen J., Nordström B., García J. M., Giménez A., 1987, *A&A*, 174, 107.
- Andersen J., Clausen J. V., Nordström B., Gustafsson B., Vandenberg D. A., 1988, *A&A*, 196, 128.
- Andersen J., Clausen J. V., Nordström B., Tomkin J., Mayor A., 1991, *A&A*, 246, 99.
- Argelander F., 1903, *Bonner Durchmusterung des nordlichen Himmels*, Eds Marcus and Weber's Verlag, Bonn.
- Arias J. I., Morrell N. I., Barbá R. H., Bosch G. L., Grosso M., Corcoran M., 2002, *MNRAS*, 333, 202.
- Asplund M., Gustafsson B., Kiselman D., Eriksson K., 1997, *A&A*, 318, 521.
- Asplund M., Grevesse N., Sauval A. J., 2004, in Bash F. N., Barnes T. G., eds., *ASP Conf. Ser., Cosmic abundances as records of stellar evolution and nucleosynthesis*, 25.
- Auer L. H., Mihalas D., 1972, *ApJS*, 24, 193.
- Bagnuolo W. G., Gies D. R., 1991, *ApJ*, 376, 266.
- Bagnuolo W. G., Gies D. R., Wiggs M. S., 1992, *ApJ*, 385, 708.
- Bahcall J. N., Basu S., Pinsonneault M., Serenelli A. M., 2005, *ApJ*, 618, 1049.
- Baker R. H., 1910, *Publ. Allegheny Obs.*, 1, 163.
- Baldwin M. E., 1973, *IBVS*, 795.
- Balog Z., Delgado A. J., Moitinho A., Fűrész G., Kaszás G., Vinkó J., Alfaro E. J., 2001, *MNRAS*, 323, 872.
- Balona L. A., 1984, *MNRAS*, 211, 973.
- Balona L. A., 1994, *MNRAS*, 268, 119.
- Balona L. A., Shobbrook R. R., 1984, *MNRAS*, 211, 375.
- Baraffe I., Chabrier G., Allard F., Hauschildt P. H., 1995, *ApJ*, 446, L35.
- Barban C., Goupil M. J., Van't Veer-Menneret C., Garrido R., Kupka F., Heiter U., 2003, *A&A*, 405, 1095.
- Barbosa C. I., Figer D., 2004, in press (*astro-ph/0408491*).
- Barembaum M. J., Etzel P. B., 1995, *AJ*, 109, 2680.
- Barnard A. J., Cooper J., Shamey L. J., 1969, *A&A*, 1, 28.
- Barnes T. G., Evans D. S., 1976, *MNRAS*, 174, 489.
- Barnes T. G., Evans D. S., Parsons S., 1976, *MNRAS*, 174, 503.
- Barnes T. G., Evans D. S., Moffett T. J., 1978, *MNRAS*, 183, 285.
- Barr J. M., 1908, *Journal of the Royal Astron. Soc. Canada*, 2, 70.
- Bayne G., Tobin W., Pritchard J. D., et al., 2002, *MNRAS*, 331, 609.
- Bayne G. P., Tobin W., Pritchard J. D., Pollard K. R., Albrow M. D., 2004, *MNRAS*, 349, 833.
- Becker S. R., Butler K., 1988, *A&AS*, 76, 331.
- Bellazzini M., Ferraro F. R., Pancino E., 2001, *ApJ*, 556, 635.
- Benvenuto O. G., Serenelli A. M., Althaus L. G., Barbá R. H., Morrell N. I., 2002, *MNRAS*, 330, 435.
- Bergbusch P. A., Vandenberg D. A., Infante L., 1991, *AJ*, 101, 2102.
- Bessell M. S., 1979, *PASP*, 91, 589.
- Bessell M. S., 1995, *PASP*, 107, 672.
- Bessell M. S., 2000, *PASP*, 112, 961.
- Bessell M. S., Brett J. M., 1988, *PASP*, 100, 1134.
- Bessell M. S., Castelli F., Plez B., 1998, *A&A*, 333, 231.
- Bessell M. S., Castelli F., Plez B., 1998, *A&A*, 337, 321.
- Bidelman W. P., 1943, *ApJ*, 98, 61.
- Bikmaev I. F., Ryabchikova T. A., Bruntt H., Musaev F. A., Mashonkina L. I., Belyakova E. V., Shimansky V. V., Barklem P. S., Galazutdinov G., 2002, *A&A*, 389, 537.
- Binnendijk L., 1960, *Properties of Double Stars*, University of Pennsylvania Press, Philadelphia.
- Binnendijk L., 1974, *Vistas in Astronomy*, 16, 61.
- Binney J., Merrifield M., 1998, *Galactic Astronomy*, Princeton University Press.
- Bíró I. B., Borkovits T., Hegedüs T., Paragi Z., 1998, *IBVS*, 4555.
- Blackwell D. E., Shallis M. J., 1977, *MNRAS*, 180, 177.
- Blackwell D. E., Shallis M. J., Selby M. J., 1979, *MNRAS*, 188, 847.
- Blackwell D. E., Petford A. D., Shallis M. J., 1980, *A&A*, 82, 249.
- Blackwell D. E., Petford A. D., Arribas S., Haddock D. J., Selby M. J., 1990, *A&A*, 232, 396.
- Boesgaard A. M., Friel E. D., 1990, *ApJ*, 351, 467.
- Böhm-Vitense E., 1958, *Zeitschrift für Astrophysik*, 46, 135.
- Böhm-Vitense E., 1981, *ARA&A*, 19, 295.
- Bonanos A. Z., Stanek K. Z., Udalski A., Wyrzykowski L., Żebruń K., Kubiak M., Szymański M. K., Szewczyk O., Pietrzyński G., Soszyński I., 2004, *ApJ*, 611, L33.
- Bonifazi A., Tosi M., Fusi Pecci F., Romeo G., 1990, *MNRAS*, 245, 15.
- Bonnell I. A., 1999, in undergraduate lecture notes, University of St. Andrews.
- Breger M., 1988, *PASP*, 100, 751.
- Bressan A., Fagotto F., Bertelli G., Chiosi C., 1993, *A&AS*, 100, 647.
- Brogia P., Lenouvel F., 1960, *Mem. Soc. Astron. It.*, 30, 199.
- Brown T. M., Charbonneau D., Gilliland R., Noyes R. W., Burrows A., 2001, *ApJ*, 552, 699.
- Budaj J., 1996, *A&A*, 313, 523.
- Budaj J., 1997, *A&A*, 326, 655.
- Burgasser A. J., Kirkpatrick J. D., Brown M. E., 2002, *ApJ*, 564,

- 421.
- Burkholder V., Massey P., Morrell N., 1997, *ApJ*, 490, 328.
- Burke C. J., DePoy D. L., Gaudi B. S., Marshall J. L., Pogge R. W., 2002, in Deming D., Seager S., eds., *ASP Conf. Ser.* 294, *Scientific Frontiers in Research on Extrasolar Planets*, 379.
- Burke C. J., Gaudi B. S., DePoy D. L., Pogge R. W., Pinsonneault M. H., 2004, *AJ*, 127, 2382.
- Burki G., Mayor M., 1986, in Earnshaw J. B., Cottrell P. L., eds., *IAU Symp. 118, Instrumentation and Research Programmes for Small Telescopes*, 385.
- Butler R. P., Marcy G. W., Williams E., McCarthy C., Dosanjh P., Vogt S. S., 1996, *PASP*, 108, 500.
- Campbell W. W., 1910, *Lick Obs. Bull.*, 6, 17.
- Cannon A. J., Pickering E. C., 1918, *Annals of the Astron. Obs. Harvard College*, 92, 1.
- Cannon A. J., Pickering E. C., 1923, *Annals of the Astron. Obs. Harvard College*, 98, 1.
- Canterna R. W., 1976, *AJ*, 81, 228.
- Canuto V. M., Mazzitelli I., 1991, *ApJ*, 370, 295.
- Canuto V. M., Mazzitelli I., 1992, *ApJ*, 389, 724.
- Capilla G., Fabregat J., 2002, *A&A*, 394, 479.
- Carbon D. F., Gingerich O., 1969, in Gingerich O., ed., *Proc. Third Harvard-Smithsonian Conference on Stellar Atmospheres*, Cambridge, Massachusetts, 377.
- Carpenter J. M., 2001, *AJ*, 121, 2851.
- Carquillat M. J., Jaschek C., Jaschek M., Ginestet N., 1997, *A&AS*, 123, 5.
- Carraro G., Bertelli G., Bressan A., Chiosi C., 1993, *A&AS*, 101, 381.
- Cassisi S., Castellani V., Salaris M., Straniero O., 1994, *A&A*, 282, 760.
- Castellani V., Chieffi A., Straniero O., 1992, *ApJS*, 78, 517.
- Castellani V., Degl'Innocenti S., Prada Moroni P. G., Tordiglione V., 2002, *MNRAS*, 334, 193.
- Castellani V., Degl'Innocenti S., Marconi M., Prada Moroni P. G., Sestito P., 2003, *A&A*, 404, 645.
- Castelli F., Hubrig S., 2004, *A&A*, 425, 263.
- Castelli F., Gratton R. G., Kurucz R. L., 1997, *A&A*, 318, 841.
- Catala C., Foing B. H., Baudrand J., et al., 1993, *A&A*, 275, 245.
- Caton D. B., Burns W. C., Hawkins R. L., 1991, *IBVS*, 3552.
- Cayrel de Strobel G., Soubiran C., Lalite N., 2001, *A&A*, 373, 159.
- Cerruti M. A., 1996, *AcA*, 46, 429.
- Cester B., Fedel B., Giuricin G., Mardirossian F., Mezzetti M., 1978, *A&AS*, 33, 91.
- Chaboyer B., 1995, *ApJ*, 444, L9.
- Chaboyer B., Green E. M., Liebert J., 1999, *AJ*, 117, 1360.
- Chabrier G., Baraffe I., 1995, *ApJ*, 451, L29.
- Chaffee F. H., 1970, *A&A*, 4, 291.
- Charbonnel C., Meynet G., Maeder A., Schaerer D., 1996, *A&AS*, 115, 339.
- Charbonnel C., Däppen W., Schaerer D., Bernasconi P. A., Maeder A., Meynet G., Mowlavi N., 1999, *A&AS*, 135, 405.
- Chen L., Hou J., Wang J. J., 2003, *AJ*, 125, 1397.
- Chieffi A., Straniero O., 1989, *ApJS*, 71, 47.
- Chieffi A., Straniero O., Salaris M., 1995, *ApJ*, 445, L39.
- Chiosi C., 1990, *PASP*, 102, 412.
- Chiosi C., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, 323.
- Chiosi C., Maeder A., 1986, *ARA&A*, 24, 329.
- Chou K. C., 1959, *AJ*, 64, 468.
- Christian D. J., Pollacco D. L., Clarkson W. L., et al., 2004, in F. Favata ed., *Proceedings of the 13th Cool Stars Workshop*, in press (preprint: astro-ph/0411019).
- Claret A., 1995, *A&AS*, 109, 441.
- Claret A., 1997, *A&AS*, 125, 439.
- Claret A., 1998, *A&AS*, 131, 395.
- Claret A., 2000a, *A&A*, 359, 289.
- Claret A., 2000b, *A&A*, 363, 1081.
- Claret A., 2003, *A&A*, 401, 657.
- Claret A., 2004a, *A&A*, 424, 919.
- Claret A., 2004b, *A&A*, 428, 1001.
- Claret A., Cunha N. C. S., 1997, *A&A*, 318, 187.
- Claret A., Giménez A., 1989, *A&AS*, 81, 1.
- Claret A., Giménez A., 1990a, *A&A*, 230, 412.
- Claret A., Giménez A., 1990b, *Ap&SS*, 169, 223.
- Claret A., Giménez A., 1991, *A&A*, 244, 319.
- Claret A., Giménez A., 1992, *A&AS*, 96, 255.
- Claret A., Giménez A., 1993, *A&A*, 277, 487.
- Claret A., Giménez A., 1995, *A&AS*, 114, 549.
- Claret A., Giménez A., 1998, *A&AS*, 133, 123.
- Claret A., Hauschildt P. H., 2003, *A&A*, 412, 241.
- Claret A., Díaz-Cordovés J., Giménez A., 1995, *A&AS*, 114, 247.
- Claret A., Giménez A., Cunha N. C. S., 1995, *A&A*, 299, 724.
- Clausen J. V., 1991, 246, 397.
- Clausen J. V., 1996a, *A&A*, 308, 151.
- Clausen J. V., 1996b, in Milone E. F., Mermilliod J.-C., eds., *ASP Conf. Ser. 90, The origins, evolution, and destinies of binary stars in clusters*, 154.
- Clausen J. V., 1998, in Kjeldsen H., Bedding T. R., eds., *The First MONS Workshop: Science with a Small Space Telescope*, 105.
- Clausen J. V., 2000, in Bergeron J., Renzini A., eds., *From Extrasolar Planets to Cosmology: The VLT Opening Symposium*, 225.
- Clausen J. V., 2004, *New Astron. Rev.*, 48, 679.
- Clausen J. V., Giménez A., 1987, in J. Palous, ed., *10th European Regional Astronomy Meeting of the IAU*, Prague, 185.
- Clausen J. V., Giménez A., 1991, *A&A*, 241, 98.
- Clausen J. V., Giménez A., Scarfe C., 1986, *A&A*, 167, 287.
- Clausen J. V., Giménez A., van Houten C. J., 1995, *A&AS*, 109, 425.
- Clausen J. V., Helt B. E., Olsen E. H., 2001, *A&A*, 374, 980.
- Clausen J. V., Helt B. E., Olsen E. H., García J. M., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, poster proceedings p. 56.
- Clausen J. V., Baraffe I., Claret A., Vandenberg D. A., 1999, in A. Giménez, E. F. Guinan & B. Montesinos, eds., *ASP Conf. Ser. 173, Stellar Structure: Theory and Test of Connective Energy Transport*, 265.
- Clausen J. V., Storm J., Larsen S. S., Giménez A., 2003, *A&A*, 402, 509.
- Clement C. M., Muzzin A., Dufton Q., et al., 2001, *AJ*, 122, 2587.
- Clementini G., Gratton R., Bragaglia A., Carretta E., Di Fabrizio L., Maio M., 2003, *AJ*, 125, 1309.
- Code A. D., Bless R. C., Davis J., Brown R. H., 1976, *ApJ*, 203, 417.
- Cohen H. L., 1969, *AJ*, 74, 1168.
- Cohen H. L., 1971, *PASP*, 83, 677.
- Cohen H. L., 1974, *A&AS*, 15, 181.
- Colavita M., Akeson R., Wizinowich P., et al. 2003, *ApJ*, 592, L83.
- Collins G. W., Truax R. J., 1995, *ApJ*, 439, 860.
- Cordier D., Lebreton Y., Goupil M.-J., Lejeune T., Beaulieu J.-P., Arenou F., 2002, *A&A*, 392, 169.
- Cousins A. W. J., 1980, *SAO Circular*, 1, 234.
- Cowley C. R., 1995, *An Introduction to Cosmochemistry*, Cambridge University Press.
- Cox A. N., Stewart J. N., 1962, *AJ*, 67, 113.
- Cox A. N., Stewart J. N., 1965, *ApJS*, 11, 22.
- Cox A. N., Stewart J. N., 1970a, *ApJS*, 19, 243.
- Cox A. N., Stewart J. N., 1970b, *ApJS*, 19, 261.
- Cox A. N., Tabor J. E., 1976, *ApJS*, 31, 271.
- Cox A. N., 2000, *Allen's Astrophysical Quantities (Fourth Edition)*, AIP Press, New York.
- Crawford D. L., 1958, *ApJ*, 128, 185.
- Crawford D. L., 1975, *AJ*, 80, 955.
- Crawford D. L., 1978, *AJ*, 83, 48.
- Crawford D. L., 1979, *AJ*, 84, 1858.
- Crawford D. L., 1980, *AJ*, 85, 621.
- Crawford D. L., 1994, *Rev. Mex. Astron. Astroph.*, 29, 115.
- Crawford D. L., Barnes J. V., 1970, 75, 987.
- Crawford D. L., Mander J. V., 1966, *AJ*, 71, 114.
- Crawford D. L., Mandwewala N., 1976, *PASP*, 88, 917.
- Crawford D. L., Perry C. L., 1976, *AJ*, 81, 419.

- Crawford D. L., Barnes J. V., Warren W. H., 1974, *AJ*, 79, 623.
- Crawford D. L., Glaspey J. W., Perry C. L., 1970, *AJ*, 75, 822.
- Crawford D. L., Barnes J. V., Gibson J., Golson J. C., Perry C. L., Crawford M. L., 1972, *A&AS*, 5, 109.
- Crinklaw G., Etzel P. B., 1989, *AJ*, 98, 1418.
- Daflon S., Cunha K., Butler K., Smith V. V., 2001, *ApJ*, 563, 325.
- D'Antona F., Mazzitelli I., 1994, *ApJS*, 90, 467.
- Dall'Ora M., Storm J., Bono G., et al., 2004, *ApJ*, 610, 269.
- Daniel S. A., Latham D. W., Mathieu R. D., Twarog B. A., 1994, *PASP*, 106, 281.
- Davis J., Tango W. J., Booth A. J., 2000, *MNRAS*, 318, 387.
- Davis J., Tango W. J., Booth A. J., Ten Brummelaar T. A., Minard R. A., Owens S. M., 1999a, *MNRAS*, 303, 773.
- Davis J., Tango W. J., Booth A. J., Thorvaldson E. D., Giovannis J., 1999b, *MNRAS*, 303, 783.
- de Zeeuw P. T., Hoogerwerf R., de Bruijne J. H. J., Brown A. G. A., Blaauw A., 1999, *AJ*, 117, 354.
- Debernardi Y., North P., 2001, *A&A*, 374, 204.
- Demarque P., Woo J.-H., Kim Y.-C., Yi S. K., 2004, *ApJS*, 155, 667.
- di Benedetto G. P., 1993, *A&A*, 270, 315.
- di Benedetto G. P., 1998, *A&A*, 339, 858.
- Di Folco E., Thévenin F., Kervella P., Domiciano de Souza A., Coudé du Foresto V., Ségransan D., Morel P., 2004, *A&A*, 426, 601.
- Dias W. S., Alessi B. S., Moitinho A., Lépine J. R. D., 2002, *A&A*, 389, 871.
- Díaz-Cordovés J., Giménez A., 1992, *A&A*, 259, 227.
- Díaz-Cordovés J., Claret A., Giménez A., 1995, *A&AS*, 110, 329.
- Dinescu D. I., Girard T. M., van Altena W. F., Yang T.-G., Lee Y.-W., 1996, *AJ*, 111, 1205.
- Draper P. W., Gray N., Berry D. S., 2004, *Software User Note SUN/214.15*, Starlink.
- Dravins D., Lindegren L., Madsen S., 1999, *A&A*, 348, 1040.
- Drechsel H., Haas S., Lorenz R., Gayler S., 1995, *A&A*, 294, 723.
- Ducati J. R., Ribeiro D., Rembold S. B., 2003, *ApJ*, 588, 344.
- Dufton P. L., Brown P. J. F., Fitzsimmons A., Lennon D. J., 1990, *A&A*, 232, 431.
- Dugan T. S., 1930, *Contributions from the Princeton University Observatory*, 10.
- Duquenooy A., Mayor M., 1991, *A&A*, 248, 485.
- Duquenooy A., Mayor M., Halbwachs J.-L., 1991, *A&AS*, 88, 281.
- Dworetzky M. M., Moon T. T., 1986, *MNRAS*, 220, 787.
- Eaton J. A., Poe C. H., 1984, *AcA*, 34, 97.
- Eaton N., Draper P. W., Allen A., 1999, *Software User Note SUN/45.9*, Starlink.
- Ebersberger J., Pohl E., Kizilirmak A., 1978, *IBVS*, 1449.
- Edvardsson B., Andersen J., Gustafsson B., Lambert D. L., Nissen P. E., Tomkin J., 1993, *A&A*, 275, 101.
- Eggen O. J., 1965, *AJ*, 70, 19.
- Eggleton P. P., 1971, *MNRAS*, 151, 351.
- Eggleton P. P., 1972, *MNRAS*, 156, 361.
- Eggleton P. P., Faulkner J., Flannery B. P., 1973, *A&A*, 23, 325.
- Eisberg R., Resnick R., 1985, *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles (Second Edition)*, John Wiley & Sons, USA.
- Elias J. H., Frogel J. A., Matthews K., Neugebauer G., 1982, *AJ*, 87, 1029.
- Etzel P. B., 1975, *Masters Thesis*, San Diego State University.
- Etzel P. B., 1980, *EBOP Users Guide* (San Diego State University).
- Etzel P. B., 1981, *Photometric and Spectroscopic Binary Systems*, NATO ASI, 111.
- Etzel P. B., 1993, in Milone E. F., ed., *Light Curve Modelling of Eclipsing Binary Stars*, Springer-Verlag, 113.
- Etzel P. B., 2004, *SBOP: Spectroscopic Binary Orbit Program*, San Diego State Univ.
- Fagotto F., Bressan A., Bertelli G., Chiosi C., 1994a, *A&AS*, 104, 365.
- Fagotto F., Bressan A., Bertelli G., Chiosi C., 1994b, *A&AS*, 105, 29.
- Fagotto F., Bressan A., Bertelli G., Chiosi C., 1994c, *A&AS*, 105, 39.
- Feast M. W., 2003, *Lecture Notes in Physics*, 635, 45 (preprint: astro-ph/0301100).
- Ferluga S., Floreano L., Bravar U., Bédalo C., 1997, *A&AS*, 121, 201.
- Fernandes J., Lebreton Y., Baglin A., Morel P., 1998, *A&A*, 338, 455.
- Fitch W. S., 1964, *AJ*, 69, 316.
- Fitzpatrick E. L., 1999, *PASP*, 111, 63.
- Fitzpatrick E. L., Massa D., 1986, *ApJ*, 307, 286.
- Fitzpatrick E. L., Massa D., 1988, *ApJ*, 328, 734.
- Fitzpatrick E. L., Massa D., 1990, *ApJS*, 72, 163.
- Fitzpatrick E. L., Massa D., 1999, *ApJ*, 525, 1011.
- Fitzpatrick E. L., Ribas I., Guinan E. F., DeWarf L. E., Maloney F. P., Massa D., 2002, *ApJ*, 564, 260.
- Fitzpatrick E. L., Ribas I., Guinan E. F., Maloney F. P., Claret A., 2003, *ApJ*, 587, 685.
- Flower P. J., 1996, *ApJ*, 469, 355.
- Frémat Y., Lampens P., Hensberge H., 2005, *MNRAS*, 356, 545.
- Ford E. B., 2005, *AJ*, 129, 1706.
- Freedman W. L., Madore B. F., Gibson B. K., et al., 2001, *ApJ*, 553, 47.
- Friel E. D., 1995, *ARA&A*, 33, 381.
- Fukugita M., Ichikawa T., Gunn J. E., Doi M., Shimasaku K., Schneider D. P., 1996, *AJ*, 111, 1748.
- Gal-Yam A., Maoz D., 2004, *MNRAS*, 347, 942.
- Gaposchkin S., 1962, *AJ*, 67, 358.
- Gaposchkin S., 1970, *IBVS*, 496.
- Garmay C. D., Stencel R. E., 1992, *A&AS*, 94, 211.
- Gatewood G., de Jonge J. K., Han I., 2000, *ApJ*, 533, 938.
- Gieren W. P., Barnes T. G., Moffett T. J., 1993, *ApJ*, 418, 135.
- Gies D. R., Lambert D. L., 1992, *ApJ*, 387, 673.
- Giménez A., 1985, *ApJ*, 297, 405.
- Giménez A., 1992, in Kondo Y., Sistero R. F., Polidan R. S., eds., *IAU Symp. 151, Evolutionary Processes in Interacting Binary Stars*, 31.
- Giménez A., Claret A., 1992, in Kondo Y., Sistero R. F., Polidan R. S., eds., *IAU Symp. 151, Evolutionary Processes in Interacting Binary Stars*, 277.
- Giménez A., Clausen J. V., 1994, *A&A*, 291, 795.
- Giménez A., Clausen J. V., 1996, in Milone E. F., Mermilliod J.-C., eds., *ASP Conf. Ser. 90, The origins, evolution, and destinies of binary stars in clusters*, 44.
- Giménez A., Garcia-Pelayo J. M., 1983, *Ap&SS*, 92, 203.
- Giménez A., Quintana J. M., 1992, *A&A*, 260, 227.
- Giménez A., Scaltriri F., 1982, *A&A*, 115, 321.
- Girardi L., Bressan A., Chiosi C., Bertelli G., Nasi E., 1996, *A&AS*, 117, 113.
- Girardi L., Groenewegen M. A. T., Weiss A., Salaris M., 1998, *MNRAS*, 301, 149.
- Girardi L., Bressan A., Bertelli G., Chiosi C., 2000, *A&AS*, 141, 371.
- Girardi L., Bertelli G., Bressan A., Chiosi C., Groenewegen M. A. T., Marigo P., Salasnich B., Weiss A., 2002, *A&A*, 391, 195.
- Girardi L., Grebel E. K., Odenkirchen M., Chiosi C., 2004, *A&A*, 422, 205.
- Giuricin G., Mardirossian F., Mezzetti M., 1984a, *A&A*, 131, 152.
- Giuricin G., Mardirossian F., Mezzetti M., 1984b, *A&A*, 134, 365.
- Giuricin G., Mardirossian F., Mezzetti M., 1984c, *A&A*, 135, 393.
- Giuricin G., Mardirossian F., Mezzetti M., 1984d, *A&A*, 141, 227.
- Giuricin G., Mardirossian F., Mezzetti M., 1985, *A&AS*, 59, 37.
- Glass I. S., 1973, *MNRAS*, 164, 155.
- Goldman I., Mazeh T., 1991, *ApJ*, 376, 260.
- Goldreich P., Nicholson P. D., 1989, *ApJ*, 342, 1079.
- Golay M., 1966, in Loden K., Loden L. O., Sinnerstand U., eds., *IAU Symp. 24, Spectral Classification and Multicolour Photometry*, 262.
- González J. F., Lapasset E., 2002, *AJ*, 123, 3318.
- Graczyk D., 2003, *MNRAS*, 342, 1334.
- Gray D. F., 1992, *The Observation and Analysis of Stellar Photospheres (Second Edition)*, Cambridge University Press.
- Gray D. F., Toner C. G., 1985, *PASP*, 97, 543.
- Gray R. O., Napier M. G., Winkler L. I., 2001, *AJ*, 121, 2148.
- Grevesse N., Noels A., Sauval A. J., 1996, in Holt S. S., Sonneborn

- G., eds., ASP Conf. Ser. 99, Cosmic Abundances, 117.
- Griffin R. F., 1967, *ApJ*, 148, 465.
- Griffin R. F., 2001, *The Observatory*, 121, 315.
- Griffin R. F., 2003, *The Observatory*, 123, 203.
- Griffin R. F., 2004, *The Observatory*, 124, 97.
- Griffin R. F., Emerson B., 1975, *The Observatory*, 95, 23.
- Griffin R. F., Carquillat J.-M., Ginestet N., 2003, *The Observatory*, 123, 69.
- Griffin R. F., Griffin R. E. M., Gunn J. E., Zimmerman B. A., 1985, *AJ*, 90, 609.
- Grisson P., Beaulieu J.-P., Pritchard J. D., 1995, *A&AS*, 109, 447.
- Groenewegen M. A. T., Salaris M., 2001, *A&A*, 366, 752.
- Groenewegen M. A. T., Salaris M., 2003, *A&A*, 410, 887.
- Groenewegen M. A. T., 2004, *MNRAS*, 353, 903.
- Grygar J., 1965, *Bull. Astron. Inst. Czechoslovakia*, 16, 195.
- Güdür N., 1978, *Ap&SS*, 57, 17.
- Guinan E. F., 2004, *New Astron. Rev.*, 48, 647.
- Guinan E. F., Fitzpatrick E. L., DeWarf L. E., et al., 1998, *ApJ*, 509, L21.
- Gustafsson B., Bell R. A., Eriksson K., Nordlund Å., 1975, *A&A*, 42, 407.
- Habets G. M. H. J., Heintze J. R. W., 1981, *A&AS*, 46, 193.
- Hadrava P., 1990, *Contr. Astron. Obs. Skalnaté Pleso*, 20, 23.
- Hadrava P., 1995, *A&AS*, 114, 393.
- Hanbury Brown R., Davis J., Allen L. R., 1967, *MNRAS*, 137, 375.
- Hanbury Brown R., Davis J., Allen L. R., 1974, *MNRAS*, 167, 121.
- Hanbury Brown R., Davis J., Allen L. R., Rome J. M., 1967, *MNRAS*, 137, 393.
- Hanbury Brown R., Davis J., Lake R. J. W., Thompson R. J., 1974, *MNRAS*, 167, 475.
- Harmanec P., 1988, *Bull. Astron. Inst. Czechoslovakia*, 39, 329.
- Harries T. J., Hilditch R. W., Howarth I. D., 2003, *MNRAS*, 339, 157.
- Hauck B., Mermilliod M., 1998, *A&AS*, 129, 431.
- Haywood M., 2002, *MNRAS*, 337, 151.
- Hebb L., Wyse R. F. G., Gilmore G., 2004, *AJ*, 128, 2881.
- Hejlesen P. M., 1980a, *A&A*, 84, 135.
- Hejlesen P. M., 1980, *A&AS*, 39, 347.
- Hejlesen P. M., 1987, *A&AS*, 69, 251.
- Hensberge H., Pavlovski K., Verschueren V., 2000, *A&A*, 358, 553.
- Herrero A., Puls J., Najjarro F., 2002, *A&A*, 396, 949.
- Herrero A., Puls J., Villamariz M. R., 2000, *A&A*, 354, 193.
- Herrero A., Kudritzki R. P., Vilchez J. M., Kunze D., Butler K., Haser S., 1992, *A&A*, 261, 209.
- Heyrovský D., 2003, *ApJ*, 594, 464.
- Hidas M. G., Ashley M. C. B., Webb J. K., et al., 2005, *MNRAS*, 360, 703.
- Hilditch R. W., 1973, *MNRAS*, 164, 101.
- Hilditch R. W., 1997, in *undergraduate lecture notes*, University of St. Andrews.
- Hilditch R. W., 2001, *An Introduction to Close Binary Stars*, Cambridge University Press.
- Hilditch R. W., 2004, in Hilditch R. W., Hensberge H., Pavlovski K., eds., ASP Conf. Ser. 318, *Spectroscopically and Spatially Resolving the Components of Close Binary Stars*, 318.
- Hilditch R. W., Harries T. J., Howarth I. D., 2004, *New Astron. Rev.*, 48, 687.
- Hilditch R. W., Harries T. J., Howarth I. D., 2005, *MNRAS*, 357, 304.
- Hill G., 1979, *Pub. Dominion Astroph. Obs. Victoria*, 15, 322.
- Hill G., Hutchings J. B., 1970, *ApJ*, 162, 265.
- Hillier D. J., Miller D. L., 1998, *ApJ*, 496, 407.
- Høg E., Kuzmin A., Bastian U., Fabricius C., Kuimov K., Lindgren L., Makarov V. V., Röser S., 1998, *A&A*, 335, L65.
- Høg E., Fabricius C., Makarov V. V., et al., 2000, *A&A*, 355, L27.
- Hoag A. A., Johnson H. L., Iriarte B., Mitchell R. I., Hallam K. L., Sharpless S., 1961, *Publ. U. S. Naval Obs.*, 17, 450.
- Holmgren D., Wolf M., 1996, *Obs*, 116, 307.
- Holmgren D. E., Scarfe C. D., Hill G., Fisher W., 1990, *A&A*, 231, 89.
- Horne K., 1986, *PASP*, 98, 609.
- Howarth I. D., 1993, *The Observatory*, 113, 75.
- Howarth I. D., Siebert K. W., Hussain G. A., Prinja R. K., 1997, *MNRAS*, 284, 265.
- Hron J., 1987, *A&A*, 176, 34.
- Huebner W. F., Merts A. L., Magee N. H., Argo M. F., 1977, *Los Alamos Scientific Laboratory Report*, LA-6760-M.
- Huffer C. M., Kopal Z., 1951, *ApJ*, 114, 297.
- Humphreys R. M., 1978, *ApJS*, 38, 309.
- Hurley J. R., Pols O. R., Tout C. A., 2000, *MNRAS*, 315, 543.
- Hurley J. R., Tout C. A., Pols O. R., 2002, *MNRAS*, 329, 897.
- Hut P., 1981, *A&A*, 99, 126.
- Hutsemekers D., 1993, *ApJ*, 417, 97.
- Hynes R. I., Maxted P. F. L., 1998, *A&A*, 331, 167.
- Iglesias C. A., Rogers F. J., 1991, *ApJ*, 371, 408.
- Ilijić S., 2003, *MSc. Thesis*, University of Zagreb.
- Irwin J. B., 1947, *ApJ*, 106, 380.
- Ivanova N., Belczynski K., Fregeau J. M., Rasio F. A., 2005, *MNRAS*, 358, 572.
- Ivezić, Ž., Lupton R. H., Anderson S., et al., 2003, *Mem. Soc. Astron. It.*, 74, 978.
- Jacoby G. H., Hunter D. A., Christian C. A., 1984, *ApJS*, 56, 257.
- Jensen K. S., Clausen J. V., Giménez A., 1988, *A&AS*, 74, 331.
- Jeffries R. D., Thurston M. R., Hambly N. C., 2001, *A&A*, 375, 863.
- Jerzykiewicz M., Pigulski A., Kopacki G., Miałkowska A., Niczyporuk S., 1996, *AcA*, 46, 253.
- Johnson H. L., 1957, *ApJ*, 126, 121.
- Johnson H. L., 1958, *Lowell. Obs. Bull.*, 4, 37.
- Johnson H. L., 1965, *ApJ*, 141, 923.
- Johnson H. L., 1966, *ARA&A*, 4, 193.
- Johnson H. L., Mitchell R. I., 1958, *ApJ*, 128, 31.
- Johnson H. L., Morgan W. W., 1953, *ApJ*, 117, 313.
- Jones T. J., Hyland A. R., 1982, *MNRAS*, 200, 509.
- Joy A. H., 1918, *PASP*, 30, 253.
- Kallrath J., Linnell A. P., 1987, *ApJ*, 313, 364.
- Kallrath J., Milone E. F., Terrell D., Young, A. T., 1998, *ApJ*, 508, 308.
- Kałużny J., Rucinski S. M., 1993, *MNRAS*, 265, 34.
- Kałużny J., Rucinski S. M., 1995, *A&AS*, 114, 1.
- Kałużny J., Thompson I. B., 2001, *A&A*, 373, 899.
- Kałużny J., Thompson I. B., 2003, *AJ*, 125, 2534.
- Kałużny J., Udalski A., 1992, *AcA*, 42, 9.
- Kałużny J., Krzemiński W., Mazur B., 1996, *A&AS*, 118, 303.
- Kałużny J., Stanek K. Z., Krockenberger M., Sasselov D. D., Tonry J. L., Mateo M., 1998, *AJ*, 115, 1016.
- Kałużny J., Mochejska B. J., Stanek K. Z., Krockenberger M., Sasselov D. D., Tonry J. L., Mateo M., 1999, *AJ*, 118, 346.
- Kałużny J., Thompson W., Krzemiński A., Olech W., Pych B., Mochejska B., 2002, in F. van Leeuwen, J. D. Hughes and G. Piotto, eds., ASP Conf. Proc. 265, *Omega Centauri, A Unique Window into Astrophysics*, 155.
- Kane S. R., Collier Cameron A., Horne K. D., James D., Lister T. A., Pollacco D. L., Street R. A., Tsapras Y., 2004, *MNRAS*, 353, 689.
- Katz D., Munari U., Cropper M., 2004, *MNRAS*, 354, 1223.
- Kaufmann W. J., 1994, *Universe (Fourth Edition)*, W. H. Freeman and Company.
- Keller S. C., Wood P. R., 2002, *ApJ*, 578, 144.
- Keller S. C., Grebel E. K., Miller G. J., Yoss K. M., 2001, *AJ*, 122, 248.
- Kervella P., Thévenin F., Morel P., Bordé P., Di Folco E., 2003, *A&A*, 408, 681.
- Kervella P., Thévenin F., Morel P., Berthomieu G., Bordé P., Provost J., 2004a, *A&A*, 413, 251.
- Kervella P., Nardetto N., Bersier D., Mourard D., Coudé du Foresto V., 2004b, *A&A*, 416, 941.
- Kervella P., Bersier D., Mourard D., Nardetto N., Coudé du Foresto V., 2004c, *A&A*, 423, 327.
- Kervella P., Thévenin F., Di Folco E., Ségransan D., 2004, *A&A*, 426, 297.
- Khaliullin Kh. F., 1985, *ApJ*, 299, 668.
- Kharitonov A. V., Tereshchenko V. M., Knjazeva L. N., 1988,

- Alma-Ata, Nauka, 484 (CDS catalogue: III/202).
- Khopolov P. N., Samus N. N., Frolov M. S., et al., 1999, *VizieR On-line Data Catalogue II/214A*.
- Kilian J., Montenbruck O., Nissen P. E., 1991, *A&AS*, 88, 101.
- Kilkenny D., O'Donoghue D., Koen C., Lynas-Gray A. E., van Wyk F., 1998, *MNRAS*, 296, 329.
- Kim C., 1990, *Ap&SS*, 168, 153.
- Kippenhahn R., Weigert A., Hofmeister E., 1967, in *Methods in Computational Physics*, Vol. 7 (Academic Press, New York), 129.
- Kirkpatrick J. D., Reid I. N., Liebert J., et al., 1999, *ApJ*, 519, 802.
- Kiss L. L., Szabó Gy. M., Sziládi K., Fűrész G., Sárnecky K., Csák B., 2001, *A&A*, 376, 561.
- Kitamura M., 1967, *Tables of the characteristic functions of the eclipse and the related delta-functions for solution of light curves of eclipsing binary systems*, University of Tokyo Press, Tokyo.
- Kitamura M., Kondo M., 1978, *Ap&SS*, 56, 341.
- Kitamura M., Kim T.-H., Kiyokawa M., 1976, *Ann. Tokyo Astron. Obs.*, 16, 22.
- Kiyokawa M., Kitamura M., 1975, *Ann. Tokyo Astron. Obs.*, 15, 117.
- Kizilirmak A., Pohl E., 1974, *IBVS*, 937.
- Kleinmann S. G., Lysaght M. G., Pughe W. L., et al., 1994, *Ap&SS*, 217, 11.
- Klinglesmith D. A., Sobieski S., 1970, *AJ*, 75, 175.
- Koch R. H., Hrivnak B. J., 1981, *AJ*, 86, 438.
- Kochukhov O., Khan S., Shulyak D., 2005, *A&A*, 433, 671.
- Koestler A., 1989, *The Sleepwalkers*, Penguin Books, UK.
- Kopal Z., 1939, *ApJ*, 90, 289.
- Kopal Z., 1946, *Harvard Obs. Monograph No. 6*, Harvard College Observatory.
- Kopal Z., 1950, *Harvard Obs. Monograph No. 8*, Harvard College Observatory.
- Kopal Z., 1959, *Close Binary Systems*, Chapman & Hall, London.
- Kraft R. P., Landolt A. U., 1959, *ApJ*, 129, 287.
- Kristenson H., 1966, *Bull. Astron. Inst. Czechoslovakia*, 17, 123.
- Kron G. E., 1952, *ApJ*, 115, 301.
- Kron G. E., Smith J. L., 1951, *ApJ*, 113, 324.
- Kruszewski A., Semeniuk I., 1999, *AcA*, 49, 561.
- Krzesiński J., Pigulski A., 1997, *A&A*, 325, 987 (KP97).
- Krzesiński J., Pigulski A., Kołaczowski Z., 1999, *A&A*, 345, 505 (KPK99).
- Kubát J., Korčáková D., 2004, in Zverko J., Weiss W. W., Žižňovský J., Adelman S. J., eds., *IAU Symp. 224, The A-Star Puzzle*, Cambridge Univ. Press, 13.
- Kudritzki R. P., 1975, *Astron. Gesellschaft*, 36, 81.
- Kudritzki R. P., 1976, *A&A*, 52, 11.
- Kudritzki R. P., Hummer D. G., 1990, *ARA&A*, 28, 303.
- Kunzli M., North P., Kurucz R. L., Nicolet B., 1997, *A&AS*, 122, 51.
- Kupka F., 1996, in Adelman S. J., Kupka F., Weiss W. W., ASP Conf. Ser. 108, *Model Atmospheres and Spectrum Synthesis*, 73.
- Kurucz R. L., 1975, in Philip A. G. D., Hayes D. S., eds., *Multi-colour Photometry and the Theoretical HR Diagram*, Dudley Obs. Rep., 9, 271.
- Kurucz R. L., 1979, *ApJS*, 40, 1.
- Kurucz R. L., 1993a, in Milone E. F., ed., *Light Curve Modelling of Eclipsing Binary Stars*, Springer-Verlag, 93.
- Kurucz R. L., 1993b, *CD-ROM 13*, SAO.
- Kurucz R. L., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, 217.
- Kurucz R. L., 2002c, in Chávez M., Bressan A., Buzzoni A., Mayya D., *New quests in stellar astrophysics: the link between stars and cosmology*, 3.
- Kurucz R. L., 2002a, in Schultz D. R., Predrag S. K., Ownby F., in *AIP Conf. Proc. 636, Atomic and Molecular Data and Their Applications*, 134.
- Kurucz R. L., 2002b, *Baltic Astron.*, 11, 101.
- Kurucz R. L., 2003, in Piskunov N., Weiss W. W., Gray D. F., eds., *IAU Symp. 210, Modelling of Stellar Atmospheres*, 45.
- Kurucz R. L., Bell B., 1995, *CD-ROM 23*, SAO.
- Kwee K. K., van Woerden H., 1956, *Bull. Ast. Inst. Netherlands*, 12, 327.
- Lacy C. H., 1977a, *ApJ*, 213, 458.
- Lacy C. H., 1977b, *ApJ*, 218, 444.
- Lacy C. H., 1977c, *ApJS*, 34, 479.
- Lacy C. H., 1978, *ApJ*, 228, 138.
- Lacy C. H., 1979, *ApJ*, 28, 817.
- Lacy C. H., 1982, *ApJ*, 261, 612.
- Lacy C. H., Frueh M. L., Turner A. E., 1987, *AJ*, 94, 1035.
- Lacy C. H. S., 1992, *AJ*, 104, 2213.
- Lacy C. H. S., Claret A., Sabby J. A., 2004, *AJ*, 128, 1340.
- Lacy C. H. S., Torres G., Claret A., Sabby J. A., 2002, *AJ*, 123, 1013.
- Lacy C. H. S., Torres G., Claret A., Sabby J. A., 2003, *AJ*, 126, 1905.
- Lacy C. H. S., Vaz L. P. R., Claret A., Sabby J. A., 2004, *AJ*, 128, 1324.
- Landolt A. U., 1983, *AJ*, 88, 439.
- Landolt A. U., 1992, *AJ*, 104, 340.
- Lane B. F., Colavita M. M., 2003, *AJ*, 125, 1623.
- Lastennet E., Valls-Gabaud D., 2002, *A&A*, 396, 551.
- Lastennet E., Fernandes J., Lejeune Th., 2002, *A&A*, 388, 309.
- Lastennet E., Valls-Gabaud D., Oblak E., 2000, in Reipurth B., Zinnecker H., eds., *IAU Symp. 200, Birth and Evolution of Binary Stars*, 164.
- Lastennet E., Fernandes J., Valls-Gabaud D., Oblak E., 2003, *A&A*, 409, 611.
- Latham D. W., Mazeh T., Stefanik R. P., Davis R. J., Carney B. W., Krymowski Y., Laird J. B., Torres G., Morse J. A., 1992, *AJ*, 104, 774.
- Latham D. W., Nordström B., Andersen J., Torres G., Stefanik R. P., Thaller M., Bester M. J., 1996, *A&A*, 314, 864.
- Lavrov M. I., 1993, *Tr. Kazansk. Gor. Astron. Obs.*, 53, 34.
- Lebovitz N. R., 1974, *ApJ*, 190, 121.
- Lebovitz N. R., 1984, *ApJ*, 284, 364.
- Lebreton Y., 2000, *ARA&A*, 38, 35.
- Lebreton Y., Fernandes J., Lejeune T., 2001, *A&A*, 374, 540.
- Lee B. L., von Braun K., Mallén-Ornelas G., Yee H. K. C., Seager S., Gladders M. D., 2004, in Holt S. S., Deming D., eds., *AIP Conf. Proc. 713, The Search for Other Worlds*, 177.
- Lee E. B., 1997, *AJ*, 113, 1106.
- Leggett S. K., Allard F., Burgasser A. J., Jones H. R. A., Marley M. S., Tsuji T., 2004, *Proc. 13th Cool Stars Workshop*, in press (preprint *astro-ph/0409389*).
- Lennon D. J., Brown P. J. F., Dufton P. L., 1988, *A&A*, 195, 208.
- Leung K.-C., Schneider D. P., 1978, *AJ*, 83, 618.
- Leung K.-C., Wilson R. E., 1977, *ApJ*, 211, 853.
- Levato H., Morrell N., 1983, *Astroph. Lett.*, 23, 183.
- Levenberg K., 1944, *Q. Appl. Math.*, 2, 164.
- Lindgren L., Perryman M. A. C., 1996, *A&AS*, 116, 579.
- Linnell A. P., 1984, *ApJS*, 54, 17.
- Linnell A. P., 1986, *ApJ*, 300, 304.
- Lindgren L., Dravins D., 2003, *A&A*, 401, 1185.
- Littlefair S. P., Naylor T., Jeffries R. D., Devey C. R., Vine S., 2003, *MNRAS*, 345, 1205.
- Liu T., Janes K. A., Bania T. M., 1989, *AJ*, 98, 626.
- Liu T., Janes K. A., Bania T. M., 1991, *AJ*, 102, 1103.
- Lorenz R., Mayer P., Drechsel H., 1998, *A&A*, 332, 909.
- Lucy L. B., 1967, *Zeitschrift für Astrophysik*, 65, 89.
- Lucy L. B., Sweeney M. A., 1971, *AJ*, 76, 544.
- Ludwig H.-G., Kučinskas A., 2004, in F. Favata, et al., eds., *Proc. 13th Cool Stars Workshop*, in press (preprint: *astro-ph/0409712*).
- Ludwig H.-G., Salaris M., 1999, in Giménez A., Guinan E. F., Montesinos B., eds., *ASP Conf. 173, Stellar Structure: Theory and Test of Convective Energy Transport*, 229.
- Ludwig H.-G., Freytag B., Steffen M., 1999, *A&A*, 346, 111.
- Lyngå G., 1987, *Computer Based Catalogue of Open Cluster Data (Fifth Edition)*, (CDS catalogue: VII/92A).
- Maceroni C., Montalbán J., 2004, *A&A*, 426, 577.

- Macri L. M., 2004a, in Kurtz D. W., Pollard K. R., eds., IAU Coll. 193, Variable Stars in the Local Group, 33.
- Macri L. M., 2004b, *New Astron. Rev.*, 48, 675.
- Macri L. M., Stanek K. Z., Sasselov D. D., Krockenberger M., Kałużny J., 2001, *AJ*, 121, 870.
- Maeder A., 1976, *A&A*, 47, 389.
- Maeder A., 1981, *A&A*, 102, 401.
- Maeder A., 1998, in Bedding T. R., Booth A. J., Davis J., eds., IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory, Kluwer, Dordrecht, 313.
- Maeder A., Meynet G., 1988, *A&AS*, 76, 411.
- Maeder A., Meynet G., 1989, *A&A*, 210, 155.
- Maeder A., Meynet G., 2000, *ARA&A*, 38, 143.
- Maeder A., Meynet G., 2003, *A&A*, 411, 543.
- Maeder A., Meynet G., 2004, *A&A*, 422, 225.
- Magain P., 1984, *A&A*, 134, 189.
- Makarov V. V., 2002, *AJ*, 124, 3299.
- Malagnini M. L., Morossi C., Rossi L., Kurucz R. L., 1986, *A&A*, 162, 140.
- Malkov O. Yu., 2003, *A&A*, 402, 1055.
- Marquardt D. W., 1963, *Journal for the Society of Industrial and Applied Mathematics*, 11, 431.
- Marques J. P., Fernandes J., Monteiro M. J. P. F. G., 2004, *A&A*, 422, 239.
- Marco A., Bernabeu G., 2001, *A&A*, 372, 477.
- Marcocci M., Mazzitelli I., 1976, *Ap&SS*, 40, 141.
- Marschall L. A., Stefanik R. P., Lacy C. H., Torres G., Williams D. B., Agerer F., 1997, *AJ*, 114, 793.
- Marsh T. R., 1989, *PASP*, 101, 1032.
- Martell S., Smith G. H., 2004, *PASP*, 116, 920.
- Martell S., Laughlin G., 2002, *ApJ*, 577, L45.
- Massey P., Johnson J., 1993, *AJ*, 105, 980.
- Massey P., Johnson K. E., DeGioia-Eastwood K., 1995, *ApJ*, 454, 151.
- Massey P., Penny L. R., Vukovich J., 2002, *ApJ*, 565, 982.
- Massey P., Waterhouse E., DeGioia-Eastwood K., 2000, *AJ*, 119, 2214.
- Massey P., Silkey M., Garmany C. D., DeGioia-Eastwood K., 1989, *AJ*, 97, 107.
- Massey P., Bresolin F., Kudritzki R. P., Puls J., Pauldrach A. W. A., 2004, *ApJ*, 608, 1001.
- Mathieu R. D., Mazeh T., 1988, *ApJ*, 326, 256.
- Mathieu R. D., Latham D. W., Griffin R. F., 1990, *AJ*, 100, 1859.
- Mathieu R. D., Meibom S., Dolan C. J., 2004, *ApJ*, 602, L121.
- Maxted P. F. L., Moran C. K. J., Marsh T. R., Gatti A. A., 2000, *MNRAS*, 311, 877.
- Mayor M., Mermilliod J.-C., 1984, in Maeder A., Renzini A., eds., IAU Symp. 105, Observational tests of the Stellar Evolution Theory, 411.
- Mazeh T., 1990, *AJ*, 99, 675.
- Mazeh T., Latham D. W., Goldberg E., 2001, *MNRAS*, 325, 343.
- Mazeh T., Goldberg D., Duquenois A., Mayor M., 1992, *ApJ*, 401, 265.
- Mazeh T., Zucker S., Goldberg D., Latham D. W., Stefanik R. P., Carney B. W., 1995, *ApJ*, 449, 909.
- Mazeh T., Simon M., Prato L., Markus B., Zucker S., 2003, *ApJ*, 599, 1344.
- Mazur B., Kałużny J., Krzemiński W., 1993, *MNRAS*, 265, 405.
- Mazur B., Krzemiński W., Kałużny J., 1995, *MNRAS*, 273, 59.
- Mazzitelli I., 1989, *ApJ*, 340, 249.
- Mégessier C., 1994, *A&A*, 289, 202.
- Meibom S., Andersen J., Nordström B., 2002, *A&A*, 386, 187.
- Mermilliod J.-C., 1976, *A&A*, 53, 289.
- Mermilliod J.-C., 1981, *A&AS*, 44, 467.
- Mermilliod J.-C., Paunzen E., 2003, *A&A*, 410, 511.
- Mermilliod J.-C., Mayor M., Burki G., 1987, *A&AS*, 70, 389.
- Mermilliod J.-C., Rosvick J. M., Duquenois A., Mayor M., 1992, *A&A*, 265, 513.
- Meynet G., Maeder A., 2002, *A&A*, 390, 561.
- Meynet G., Maeder A., Ekström, 2004, in R. Humphreys, K. Stanek, eds., ASP Conf. Ser., The Fate of the Most Massive Stars, 232.
- Meynet G., Mermilliod J.-C., Maeder A., 1993, *A&AS*, 98, 477.
- Meynet G., Maeder A., Schaller G., Schaerer D., Charbonnel C., 1994, *A&AS*, 103, 97.
- Milone E. F., Schiller S. J., 1984, *PASP*, 96, 791.
- Milone E. F., Schiller S. J., 1988, *PASP*, 100, 1223.
- Milone E. F., Schiller S. J., 1991, in Janes K., ed., ASP. Conf. Ser. 13, The formation and evolution of star clusters, 427.
- Milone E. F., Wilson R. E., Hrivnak B. J., 1987, *ApJ*, 319, 325.
- Milone E. F., Stagg C. R., Sugars B. A., McVean J. R., Schiller S. J., Kallrath J., Bradstreet D. H., 1995, *AJ*, 109, 359.
- Milone E. F., Schiller S. J., Munari U., Kallrath J., 2000, *AJ*, 119, 1405.
- Mitchell R. C., Baron E., Branch D., Hauschildt P. H., Nugent P. E., Lundqvist P., Blinnikov S., Pun C. S. J., 2002, *ApJ*, 574, 293.
- Mochejska B. J., Kałużny J., Stanek K. Z., Krockenberger M., Sasselov D. D., 1999, *AJ*, 118, 2211.
- Mochejska B. J., Stanek K. Z., Sasselov D. D., Szentgyorgyi A. H., 2002, *AJ*, 123, 3460.
- Mochejska B. J., Stanek K. Z., Sasselov D. D., Szentgyorgyi A. H., Westover M., Winn J. N., 2004, *AJ*, 128, 312.
- Mochejska B. J., Stanek K. Z., Sasselov D. D., et al., 2005, *AJ*, 129, 285.
- Mokiem M. R., Martín-Hernández N. L., Lenorzer A., de Koter A., Tielens A. G. G. M., 2004, *A&A*, 419, 319.
- Montalbán J., Kupka F., D'Antona F., Schmidt W., 2001, *A&A*, 370, 982.
- Moon T. T., 1984, *MNRAS*, 211, 21.
- Moon T. T., 1985, *Commun. Univ. London Obs. No. 78*.
- Moon T. T., 1985b, *Ap&SS*, 117, 261.
- Moon T. T., Dworetsky M. M., 1984, *The Observatory*, 104, 273.
- Moon T. T., Dworetsky M. M., 1985, *MNRAS*, 217, 305.
- Morgan W. W., Keenan P. C., Kellman E., 1943, *An Atlas of Stellar Spectra*, Univ. Chicago Press.
- Morel P., 1997, *A&AS*, 124, 597.
- Moro D., Munari U., 2000, *A&AS*, 147, 361.
- Mowlavi N., Schaerer D., Meynet G., Bernasconi P. A., Charbonnel C., Maeder A., 1998, *A&AS*, 128, 471.
- Mozurkewich D., Johnston K. J., Simon R. S., Bowers P. F., Gaume R., Hutter D. J., Colavita M. M., Shao M., Pan X. P., 1991, *AJ*, 101, 2207.
- Mozurkewich D., Armstrong J. T., Hindsley R. B., et al., 2003, *AJ*, 126, 2502.
- Munari U., Dallaporta S., Siviero A., Soubiran C., Fiorucci M., Girard P., 2004, *A&A*, 418, L31.
- Muthsam H., 1979, *A&AS*, 35, 253.
- Napiwotzki R., Schönberner D., Wenske V., 1993, *A&A*, 268, 653.
- Narayanan V. K., Gould A., 1999, *ApJ*, 523, 328.
- Naylor T., 1998, *MNRAS*, 296, 339.
- Nelder J. A., Mead R., 1965, *Computer Journal*, 7, 308.
- Nelson B., Davis W. D., 1972, *ApJ*, 174, 617.
- Nilakshi S. R., Pandey A. K., Mohan V., 2002, *A&A*, 383, 153.
- Nissan P. E., 1976, *A&A*, 50, 343.
- Nordgren T. E., Sudol J. J., Mozurkewich D., 2001, *AJ*, 122, 2707.
- Nordgren T. E., Germain M. E., Benson J. A., et al., 1999, *AJ*, 118, 3032.
- Nordström B., Andersen J., Andersen M. I., 1997, *A&A*, 322, 460.
- North P., Nicolet B., 1990, *A&A*, 228, 78.
- North P., Zahn J.-P., 2004, *A&A*, 405, 677.
- North P., Zahn J.-P., 2004, *New Astron. Rev.*, 48, 741.
- North P., Studer M., Künzli M., 1997, *A&A*, 324, 137.
- O'Connell D. J. K., 1951, *Pub. Riverview College Obs.*, 2, 85.
- O'Dell M. A., Hendry M. A., Collier Cameron A., 1994, *MNRAS*, 268, 181.
- O'Donoghue D., Koen C., Kilkeny D., Stobie R. S., Koester D., Bessell M. S., Hambly N., MacGillivray H., 2003, *MNRAS*, 345, 506.
- Oblak E., Lastennet E., Fernandes J., Kurpinska-Winiarska M., Valls-Gabaud D., 2004, in Hilditch R. W., Hensberge H., Pavlovski K., eds., ASP Conf. Ser. 318, Spectroscopically and Spatially Resolving the Components of Close Binary Stars, 175.
- Oke J. B., Gunn J. E., 1983, *ApJ*, 266, 713.

- Olsen E. H., 1984, *A&AS*, 57, 443.
Olsen E. H., 1988, *A&A*, 189, 173.
Olsen E. H., 1994a, *A&AS*, 104, 429.
Olsen E. H., 1994b, *A&AS*, 106, 257.
Olson E. C., 1984, *PASP*, 96, 376.
Olson E. C., Etzel P. B., 1993, *AJ*, 106, 1162.
Oosterhoff P. Th., 1937, *Ann. Sternw. Leiden*, 17, 1.
Pace G., Pasquini L., 2004, *A&A*, 426, 1021.
Paczynski B., 2003, *AcA*, 53, 209.
Paczynski B., Sienkiewicz R., 1984, *ApJ*, 286, 332.
Padalia T. D., Srivastava R. K., 1975, *Ap&SS*, 32, 285.
Palmieri R., Piotto G., Saviane I., Girardi L., Castellani V., 2002, *A&A*, 392, 115.
Pan K.-K., 1997, *A&A*, 321, 202.
Pan K.-K., Tan H., Shan H., 1998, *A&A*, 335, 179.
Pan X., Shao M., Kulkarni S. S. 2004, *Nature*, 427, 326.
Patience J., Ghez A. M., Reid I. N., Weinberger A. J., Matthews K., 1998, *AJ*, 115, 1972.
Pauldrach A. W. A., Hoffmann T. L., Lennon M., 2001, *A&A*, 375, 161.
Pearce J. A., 1941, *Pub. Amer. Astron. Soc.*, 10, 223.
Pearce J. A., 1957, *Pub. Dominion Astroph. Obs. Victoria*, 10, 435.
Penny L. R., Gies D. R., Bagnuolo W. G., 1999, *ApJ*, 518, 450.
Pepper J., Gould A., DePoy D. L., 2004, in Holt S. S., Deming D., eds., *AIP Conf. Proc.* 713, *The Search for Other Worlds*, 185.
Percival S. M., Salaris M., Groenewegen M. A. T., 2005, *A&A*, 429, 887.
Percival S. M., Salaris M., Kilkeny D., 2003, *A&A*, 400, 541.
Perryman M. A. C., Lindegren L., Kovalevsky J., et al., 1997, *A&A*, 323, L49.
Peters A. R., Hoffleit D. E., 1992, *Bull. Inf. CDS*, 40, 71.
Petrie R. M., Andrews D. H., 1966, *AJ*, 71.
Phelps R. L., Janes K. A., 1994, *ApJS*, 90, 31.
Phillips A. C., 1999, *The Physics of Stars* (Second Edition), John Wiley and Sons Ltd., West Sussex, UK.
Pietrzyński G., Udalski A., 1999, *AcA*, 49, 149.
Pietrzyński G., Kubiak M., Udalski A., Szymanski M., 1997, *AcA*, 47, 437.
Pinsonneault M. H., Terndrup D. M., Hanson R. B., Stauffer J. R., 2003, *ApJ*, 598, 588.
Pinsonneault M. H., Terndrup D. M., Hanson R. B., Stauffer J. R., 2004, *ApJ*, 600, 946.
Piotrowski S., Serkowski K., 1956, *AcA*, 6, 205.
Pohl E., Kizilirmak A., 1966, *Astron. Nachr.*, 289, 191.
Pohl E., Kizilirmak A., 1970, *IBVS*, 456.
Pohl E., Kizilirmak A., 1972, *IBVS*, 647.
Pohl E., Evren S., Tumer O., Sezer C., 1982, *IBVS*, 2189.
Pojmański G., 1997, *AcA*, 47, 467.
Pojmański G., 1998, *AcA*, 48, 35.
Pojmański G., 2000, *AcA*, 50, 177.
Pojmański G., 2002, *AcA*, 52, 397.
Pojmański G., 2003, *AcA*, 53, 341.
Pojmański G., Maciejewski G., 2004, *AcA*, 54, 153.
Pojmański G., Maciejewski G., 2005, *AcA*, in press (preprint: astro-ph/0412645).
Pols O. R., Tout C. A., Eggleton P. P., Han Z., 1995, *MNRAS*, 274, 964.
Pols O. R., Tout C. A., Schröder K.-P., Eggleton P. P., Mannes J., 1997, *MNRAS*, 289, 869.
Pols O. R., Schröder K.-P., Hurley J. R., Tout C. A., Eggleton P. P., 1998, *MNRAS*, 298, 525.
Popovici C., 1968, *IBVS*, 322.
Popovici C., 1971, *IBVS*, 508.
Popper D. M., 1967, *ARA&A*, 5, 85.
Popper D. M., 1968, *ApJ*, 154, 191.
Popper D. M., 1971, *ApJ*, 169, 549.
Popper D. M., 1974, *AJ*, 79, 1307.
Popper D. M., 1980, *ARA&A*, 18, 115.
Popper D. M., 1981, *Rev. Mex. Astron. Astroph.*, 6, 99.
Popper D. M., 1982, *ApJ*, 254, 203.
Popper D. M., 1984, *AJ*, 89, 132.
Popper D. M., 1993, *ApJ*, 404, L67.
Popper D. M., 2000, *AJ*, 119, 2391.
Popper D. M., Etzel P. B., 1981, *AJ*, 86, 102.
Popper D. M., Guinan E. F., 1998, *PASP*, 110, 572.
Popper D. M., Hill G., 1991, *AJ*, 101, 600.
Popper D. M., Jørgensen H. E., Morton D. C., Leckrone D. S., 1970, *ApJ*, 161, L57.
Popper D. M., Lacy C. H., Frueh M. L., Turner A. E., 1986, *AJ*, 91, 383.
Pourbaix D., Nidever D., McCarthy C., et al., 2002, *A&A*, 386, 280.
Prato L., Simon M., Mazeh T., McLean I. S., Norman D., Zucker S., 2002, *ApJ*, 569, 863.
Press W. H., Wiita P. J., Smarr L. L., 1975, *ApJ*, 202, L135.
Press W. H., Teukolsky S. A., Vetterling, W. T., Flannery B. P., 1992, *Numerical Recipes in Fortran 77: The Art of Scientific Computing*, Cambridge University Press, p. 402.
Pritzl B. J., Smith H. A., Catelan M., Sweigart A. V., 2001, *AJ*, 122, 2600.
Raboud D., Cramer N., Bernasconi P. A., 1997, *A&A*, 325, 167.
Rafert J. B., 1982, *PASP*, 94, 485.
Rafert J. B., Twigg L. W., 1980, *MNRAS*, 193, 79.
Rastorguev A. S., Glushkova E. V., Dambis A. K., Zabolotskikh M. V., 1999, *Ast. Lett.*, 25, 595.
Rauw G., De Becker M., Nazé Y., Crowther P. A., Gosset E., Sana H., van der Hucht K. A., Vreux J.-M., Williams P. M., 2004, *A&A*, 420, L9.
Relyea L. J., Kurucz R. L., 1978, *ApJS*, 37, 45.
Reimann H.-G., 1989, *Astron. Nachr.*, 310, 273.
Ribas I., 2003, *A&A*, 398, 239.
Ribas I., 2004, *New Astron. Rev.*, 48, 731.
Ribas I., Guinan E. F., Fitzpatrick, E. L., 2000, *ApJ*, 528, 692.
Ribas I., Jordi C., Giménez A., 2000, *MNRAS*, 318, L55.
Ribas I., Jordi C., Torra J., 1999, *MNRAS*, 309, 199.
Ribas I., Jordi C., Torra J., Giménez A., 1997, *A&A*, 327, 207.
Ribas I., Giménez A., Torra J., Jordi C., Oblak E., 1998, *A&A*, 330, 600.
Ribas I., Jordi C., Torra J., Giménez A., 2000, *MNRAS*, 313, 99.
Ribas I., Fitzpatrick E. L., Maloney F. P., Guinan E. F., Udalski A., 2002, *ApJ*, 574, 771.
Ribas I., Jordi C., Vilardell F., Giménez A., Guinan E. F., 2004, *New Astron. Rev.*, 48, 755.
Richer J., Michaud G., Turcotte S., 2000, *ApJ*, 529, 338.
Richichi A., Roccataglia V., 2005, *A&A*, 433, 305.
Robinson L. J., Ashbrook J., 1968, *IBVS*, 247.
Rogers F. J., Iglesias C. A., 1992, *ApJS*, 79, 507.
Rogers F. J., Nayfonov A., 2002, *ApJ*, 576, 1064.
Rogers F. J., Swenson F. J., Iglesias C. A., 1996, *ApJ*, 456, 902.
Rolleston W. R. J., Smartt S. J., Dufton P. L., Ryans R. S. I., 2000, *A&A*, 363, 537.
Romaniello M., Salaris M., Cassisi S., Panagia N., 2000, *ApJ*, 530, 738.
Romaniello M., Primas F., Mottini M., Groenewegen M., Bono G., François P., 2005, *A&A*, 429, L37.
Romeo G., Fusi Pecci F., Bonifazi A., Tosi M., 1989, *MNRAS*, 240, 459.
Rossiter R. A., 1924, *ApJ*, 60, 15.
Rucinski S. M., Kałużny J., Hilditch R. W., 1996, *MNRAS*, 282, 705.
Rufener F., 1976, *A&AS*, 26, 275R.
Russell H. N., 1912a, *ApJ*, 35, 315.
Russell H. N., 1912b, *ApJ*, 36, 54.
Russell H. N., Merrill J. E., 1959, *Contr. Princeton Univ. Obs.*, No. 24.
Russell H. N., Shapley H., 1912a, *ApJ*, 36, 239.
Russell H. N., Shapley H., 1912b, *ApJ*, 36, 385.
Russell H. N., Shapley H., 1914, *ApJ*, 39, 405.
Sahade J., Ber Dávila F., 1963, *Annales d'Astrophysique*, 26, 153.
Salaris M., Groenewegen M. A. T., 2002, *A&A*, 381, 440.
Salaris M., Weiss A., Percival S. M., 2004, *A&A*, 414, 163.
Sana H., Rauw G., Gosset E., 2001, *A&A*, 370, 121.
Sandage A., 1958, *ApJ*, 128, 150.
Sandage A., Tammann G. A., 1969, *ApJ*, 157, 683.

- Santolaya-Rey A. E., Puls J., Herrero A., 1997, *A&A*, 323, 488.
- Sarajedini A., Grocholski A. J., Levine J., Lada E., 2002, *AJ*, 124, 2625.
- Schaerer D., Meynet, G., Maeder A., Schaller G., 1993a, *A&AS*, 98, 523.
- Schaerer D., Charbonnel C., Meynet, G., Maeder A., Schaller G., 1993b, *A&AS*, 102, 339.
- Schaller G., Schaerer D., Meynet G., Maeder A., 1992, *A&AS*, 96, 269.
- Schild R. E., 1965, *ApJ*, 142, 979.
- Schild R. E., 1967, *ApJ*, 148, 449.
- Schiller S. J., Milone E. F., 1987, *AJ*, 93, 1471.
- Schiller S. J., Milone E. F., 1988, *AJ*, 95, 1466.
- Schmidt-Kaler T., 1982, in *Stars and Star Clusters*, Springer-Verlag, Berlin.
- Scholz M., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, 51.
- Schröder K.-P., Eggleton P. P., 1996, *Rev. in Modern Astron.*, 9, 221.
- Schuh S. L., Handler G., Drechsel H., et al., 2003, *A&A*, 410, 649.
- Schuster W. J., Nissen P. E., 1989, *A&A*, 221, 65.
- Schwab F., 1918, *Astron. Nachr.*, 206, 67.
- Schwan H., 1991, *A&A*, 243, 386.
- Seaton M. J., Yan Y., Mihalas D., Pradhan A. K., 1994, *MNRAS*, 266, 805.
- Seaton M. J., 1997, *MNRAS*, 289, 700.
- Ségransan D., Kervella P., Forveille T., Queloz D., 2003, *A&A*, 397, L5.
- Semeniuk I., 2000, *AcA*, 50, 381.
- Shallis M. J., Blackwell D. E., 1980, *A&A*, 81, 336.
- Shamey L. J., 1969, Ph.D. Thesis, University of Colorado.
- Shan H.-G., 2000, *Chinese A&A*, 24, 81.
- Shanks T., Allen P. D., Hoyle F., Tanvir N. R., 2002, in Metcalfe N., Shanks T., eds., *ASP Conf. Proc. 283, A New Era in Cosmology*, 274.
- Shara M. M., Smith L. F., Potter M., Moffat A. F. J., 1991, *AJ*, 102, 716.
- Shara M. M., Moffat A. F. J., Smith L. F., Niemela V. S., Potter M., Lamontagne R., 1999, *AJ*, 118, 390.
- Siess L., Dufour E., Forestini M., 2000, *A&A*, 358, 593.
- Simkin S. M., 1974, *A&A*, 31, 129.
- Simon K. P., Sturm F., 1994, *A&A*, 281, 286.
- Simon K. P., Sturm F., Fiedler A., 1994, *A&A*, 292, 507.
- Slesnick C. L., Hillenbrand L. A., Massey P., 2002, *ApJ*, 576, 880.
- Smalley B., 1993, *A&A*, 274, 391.
- Smalley B., 1993, *MNRAS*, 265, 1035.
- Smalley B., 1996, in Adelman S. J., Kupka F., Weiss W. W., eds., *ASP Conf. Ser. 108, Model Atmospheres and Spectrum Synthesis*, 43.
- Smalley B., 2004, in Zverko J., Weiss W. W., Žižňovský J., Adelman S. J., eds., *IAU Symp. 224, The A-Star Puzzle*, Cambridge Univ. Press, 131.
- Smalley B., Dworetsky M. M., 1993, *A&A*, 271, 515.
- Smalley B., Dworetsky M. M., 1995, *A&A*, 293, 446.
- Smalley B., Kupka F., 1997, *A&A*, 328, 349.
- Smalley B., Kupka F., 1998, *Contr. Astron. Obs. Skalnaté Pleso*, 27, 233.
- Smalley B., Kupka F., 2003, in Piskunov N., Weiss W. W., Gray D. F., eds., *IAU Symp. 210, Modelling of Stellar Atmospheres*, poster C10.
- Smalley B., Smith K. C., Dworetsky M. M., 2001, *UCLSYN User-guide*.
- Smalley B., Gardiner R. B., Kupka F., Bessell M. S., 2002, *A&A*, 395, 601.
- Smartt S. J., Rolleston W. R. J., 1997, *ApJ*, 481, L47.
- Smith K. C., 1992, Ph.D. Thesis, University of London.
- Soderblom D. R., Nelan E., Benedict G. F., McArthur B., Ramirez I., Spiesman W., Jones B. F., 2005, *AJ*, 129, 1616.
- Soloviev A., 1918, *Poulkovo Bull.*, 8, 6 (4), No. 86.
- Southworth J., Maxted P. F. L. M., Smalley B., 2004a, *MNRAS*, 349, 547.
- Southworth J., Maxted P. F. L. M., Smalley B., 2004b, *MNRAS*, 351, 1277.
- Southworth J., Zucker A., Maxted P. F. L. M., Smalley B., 2004c, *MNRAS*, 355, 986.
- Southworth J., Maxted P. F. L. M., Smalley B., 2005a, *A&A*, 429, 645.
- Southworth J., Maxted P. F. L. M., Smalley B., Claret A., Etzel P. B., 2005b, *MNRAS*, 363, 529.
- Srivastava R. K., Sinha B. K., 1985, *Ap&SS*, 111, 225.
- Stanek K. Z., Kałużny J., Krockenberger M., Sasselov D. D., Tonry J. L., Mateo M., 1998, *AJ*, 115, 1894.
- Stanek K. Z., Kałużny J., Krockenberger M., Sasselov D. D., Tonry J. L., Mateo M., 1999, *AJ*, 117, 2810.
- Stassun K. G., Mathieu R. D., Vaz L. P. R., Stroud N., Vrba F. J., 2004, *ApJS*, 151, 357.
- Stauffer J. R., Schultz G., Kirkpatrick J. D., 1998, *ApJ*, 499, L199.
- Stauffer J. R., Jones B. F., Backman D., Hartmann L. W., Barado y Navascués D., Pinsonneault M. H., Terndrup D. M., Muench A. A., 2003, *AJ*, 126, 833.
- Stebbins J., 1910, *ApJ*, 32, 185.
- Stebbins J., 1911, *ApJ*, 34, 112.
- Stebbins J., Whitford A. E., 1943, *ApJ*, 98, 20.
- Stello D., Nissen P. E., 2001, *A&A*, 374, 105.
- Sterne T. E., 1939, *MNRAS*, 99, 662.
- Stothers R. B., 1991, *ApJ*, 383, 820.
- Stothers R. B., Chin C.-W., 1991, *ApJ*, 381, L67.
- Straizys V., Kuriliene G., 1981, *Ap&SS*, 80, 353.
- Strömgren B., 1963, *QJRAS*, 4, 8.
- Strömgren B., 1966, *ARA&A*, 4, 433.
- Struve O., 1944, *ApJ*, 100, 189.
- Sturm F., Simon K. P., 1994, *A&A*, 282, 93.
- Taş G., Evren S., Çakirli Ö., İbanoglu C., 2003, *A&A*, 411, 161.
- Tapia M., Roth M., Costero R., Navarro S., 1984, *Rev. Mex. Astron. Astroph.*, 9, 65.
- Tassoul J.-L., 1987, *ApJ*, 322, 856.
- Tassoul J.-L., 1988, *ApJ*, 324, L71.
- Tassoul J.-L., 1990, *ApJ*, 358, 196.
- Tassoul J.-L., 1995, *ApJ*, 444, 338.
- Tassoul J.-L., 1997, *ApJ*, 481, 363.
- Tassoul J.-L., Tassoul M., 1992, *ApJ*, 395, 259.
- Tassoul M., Tassoul J.-L., 1990, *ApJ*, 359, 155.
- Taylor B. J., 2001, *A&A*, 377, 473.
- Thompson I. B., Kałużny J., Pych W., Burley G., Krzemiński W., Paczyński B., Persson S. E., Preston G. W., 2001, *AJ*, 121, 3089.
- Titus J., Morgan W. W., 1940, *ApJ*, 92, 256.
- Tohline J. E., 2002, *ARA&A*, 40, 349.
- Tonry J., Davis M., 1979, *AJ*, 84, 1511.
- Torres G., 2001, *AJ*, 121, 2227.
- Torres G., 2003, *IBVS*, 5402.
- Torres G., Ribas I., 2002, *ApJ*, 567, 1140.
- Torres G., Stefanik R. P., Latham D. W., 1997a, *ApJ*, 474, 256.
- Torres G., Stefanik R. P., Latham D. W., 1997b, *ApJ*, 479, 268.
- Torres G., Stefanik R. P., Latham D. W., 1997c, *ApJ*, 485, 167.
- Torres G., Lacy C. H. S., Claret A., Zakirov M. M., Arzumanyants G. C., Bayramov N., Hojaev A. S., Stefanik R. P., Latham D. W., Sabby J. A., 1999, *AJ*, 118, 1831.
- Torres G., Andersen J., Nordström B., Latham D. W., 2000a, *AJ*, 119, 1942.
- Torres G., Lacy C. H. S., Claret A., Sabby J. A., 2000b, *AJ*, 120, 3226.
- Tosi M., Di Fabrizio L., Bragaglia A., Carusillo P. A., Marconi G., 2004, *MNRAS*, 354, 225.
- Trundle C., Lennon D. J., Puls J., Dufton P. L., 2004, *A&A*, 417, 217.
- Turcotte S., 2002, *ApJ*, 573, L129.
- Turon C., 1998, in Bedding T. R., Booth A. J., Davis J., eds., *IAU Symp. 189, Fundamental Stellar Properties: The Interaction between Observation and Theory*, Kluwer, Dordrecht, 9.
- Twarog B. A., Anthony-Twarog B. J., De Lee N., 2003, *AJ*, 125, 1383.
- Udalski A., Szymański I., Szymański M., Kubiak M., Pietrzyński G., Woźniak P., Żebruń K., 1998, *AcA*, 48, 563.
- Underhill A. B., 1980, *ApJ*, 239, 220.

- Uribe A., García-Varela J.-A., Sabogal-Martínez B.-E., Higuera G., Mario A., Brieva E., 2002, *PASP*, 114, 233.
- Valtonen, M. J., 1998, *A&A*, 334, 169.
- van Belle G. T., 1999, *PASP*, 111, 1515.
- van den Bergh S., 1994, *PASP*, 106, 1113.
- van Genderen A. M., 1986, *A&A*, 158, 361.
- Van Hamme W., 1993, *AJ*, 106, 2096.
- Van Hamme W., Wilson R. E., 1984, *A&A*, 141, 1.
- van Leeuwen F., 1999, *A&A*, 341, L71.
- van Leeuwen F., 2004, in Kurtz D. W., Bromage G. E., eds., *IAU Coll. 196, Transit of Venus: New Views of the Solar System and Galaxy*, 347.
- van Leeuwen F., Hansen Ruiz C. S., 1997, in *Proc. ESA Symp. Hipparcos – Venice '97*, 689.
- van Maanen A., 1944, *ApJ*, 100, 31.
- Van't Veer F., 1975, *A&A*, 44, 437.
- VandenBerg D. A., 1983, *ApJS*, 51, 29.
- VandenBerg D. A., 1985, *ApJS*, 58, 711.
- VandenBerg D. A., Swenson F. J., Rogers F. J., Iglesias C. A., Alexander D. R., 2000, *ApJ*, 532, 430.
- Vauclair S., 2004, in Kurtz D. W., Pollard K. R., eds., *IAU Coll. 193, Variable Stars in the Local Group*, 413.
- Vaz L. P. R., Andersen J., 1984, *A&A*, 132, 219.
- Vaz L. P. R., Cunha N. C. S., Vieira E. F., Myrrha M. L. M., 1997, *A&A*, 327, 1094.
- Vergely J.-L., Köppen J., Egret D., Bienaymé O., 2002, *A&A*, 390, 917.
- von Braun K., Mateo M., Chiboucas K., Athey A., Hurley-Keller D., 2002, *AJ*, 124, 2067.
- von Braun K., Lee B. L., Mallén-Ornelas G., Yee H. K. C., Seager S., Gladders M. D., 2004, in Holt S. S., Deming D., eds., *AIP Conf. Proc. 713, The Search for Other Worlds*, 181.
- von Zeipel H., 1924, *MNRAS*, 84, 665.
- Vrancken M., Lennon D. J., Dufton P. L., Lambert D. L., 2000, *A&A*, 358, 639.
- Wachmann A. A., 1939, *Beob. Zirk. Astron. Nachr.*, 21, 136.
- Wachmann A. A., 1973, *A&A*, 25, 157.
- Wachmann A. A., 1974, *A&A*, 34, 317.
- Wade R. A., Rucinski S. M., 1985, *A&AS*, 60, 471.
- Waelkens C., Lampens P., Heynderickx D., Cuypers J., Degryse K., Poedts S., Polfiet R., Denoyelle J., van den Abeele K., Rufener F., Smeyers P., 1990, *A&AS*, 83, 11.
- Walborn N. R., 1980, *ApJS*, 44, 535.
- Walborn N. R., Fitzpatrick E. L., 1990, *PASP*, 102, 379.
- Walborn N. R., Fitzpatrick E. L., 2000, *PASP*, 112, 50.
- Walborn N. R., Nichols-Bohlin J., Panek R. J., 1985, *International Ultraviolet Explorer Atlas of O-type spectra from 1200 to 1900Å*.
- Walraven Th., Walraven J. H., 1960, *Bull. Astron. Inst. Netherlands*, 15, 67.
- Watson R. D., West S. R. D., Tobin W., Gilmore A. C., 1992, *MNRAS*, 258, 527.
- Weiss A., Schlattl H., 1998, *A&A*, 332, 215.
- Weldrake D. T. F., Sackett P. D., Bridges T. J., Freeman K. C., 2004, *AJ*, 128, 736.
- Wesselink A. J., 1969, *MNRAS*, 144, 297.
- Wiese W. L., Smith M. W., Glennon B. M., 1966, *Atomic Transition Probabilities I*, US Government Printing Office, Washington DC.
- Wildev R. L., 1964, *ApJS*, 8, 439.
- Wilson O. C., 1941, *ApJ*, 93, 29.
- Wilson R. E., 1970, *PASP*, 82, 815.
- Wilson R. E., 1979, *ApJ*, 234, 1054.
- Wilson R. E., 1983, *Ap&SS*, 92, 229.
- Wilson R. E., 1990, *ApJ*, 356, 613.
- Wilson R. E., 1993, in Leung K.-C., Nha I.-S., eds., *ASP Conf. Ser. 38, New Frontiers in Binary Star Research*, 91.
- Wilson R. E., 1994, *PASP*, 106, 921.
- Wilson R. E., 1998, *Computing Binary Star observables*, unpublished.
- Wilson R. E., 2004, *New Astron. Rev.*, 48, 695.
- Wilson R. E., Biermann P., 1976, *A&A*, 48, 349.
- Wilson R. E., Caldwell C. N., 1978, *ApJ*, 221, 917.
- Wilson R. E., Devinney E. J., 1971, *ApJ*, 166, 605.
- Wilson R. E., Devinney E. J., 1973, *ApJ*, 182, 539.
- Wilson R. E., Hurley J. R., 2003, *MNRAS*, 344, 1175.
- Wilson R. E., Sofia S., 1976, *ApJ*, 203, 182.
- Wilson R. E., Van Hamme W., 2004, *Computing Binary Star observables*, unpublished.
- Wilson R. E., Woodward E. J., 1983, *Ap&SS*, 89, 5.
- Wilson R. E., de Luccia M., Johnston K., Mango S. A., 1972, *ApJ*, 177, 191.
- Wittkowski M., Aufdenberg J. P., Kervella P., 2004, *A&A*, 413, 711.
- Wolfe R. H., Horak H. G., Storer N. W., 1967, in Hack M., ed., *Modern Astrophysics: A Memorial to Otto Struve*, 251.
- Wolff S. C., 1983, *The A-stars: Problems and perspectives*, Monograph series on nonthermal phenomena in stellar atmospheres, CNRS/NASA.
- Wolff S. C., Heasley J. N., 1985, *ApJ*, 292, 589.
- Woo J.-H., Gallart C., Demarque P., Yi S., Zoccali M., 2003, *AJ*, 125, 754.
- Wood D. B., 1971a, *AJ*, 76, 701.
- Wood D. B., 1971b, *PASP*, 83, 286.
- Wood D. B., 1972, *A computer program for modelling non-spherical eclipsing binary star systems*.
- Wood D. B., 1973a, *MNRAS*, 164, 53.
- Wood D. B., 1973b, *PASP*, 85, 253.
- Wyithe J. S. B., Wilson R. E., 2001, *ApJ*, 559, 260.
- Wyithe J. S. B., Wilson R. E., 2002, *ApJ*, 571, 293.
- Wylie, C. C., 1923, *Popular Astronomy*, 31, 93.
- Wyrzykowski L., Pietrzyński G., Szewczyk O., 2002, *AcA*, 52, 105.
- Wyrzykowski L., Udalski A., Kubiak M., Szymański M. K., Żebruń K., Soszyński I., Wozniak P. R., Pietrzyński G., Szewczyk O., 2003, *AcA*, 53, 1.
- Wyrzykowski L., Udalski A., Kubiak M., Szymański M. K., Żebruń K., Soszyński I., Wozniak P. R., Pietrzyński G., Szewczyk O., 2004, *AcA*, 54, 1.
- Wyse A. B., Kron G. E., 1939, *Lick Obs. Bull.*, 29, 17 (No. 496).
- Yi S. K., Kim Y.-C., Demarque P., 2003, *ApJS*, 144, 259.
- Young P. A., Arnett D., 2004, *ApJ*, 618, 908.
- Young P. A., Mamajek E. E., Arnett D., Liebert J., 2001, *ApJ*, 556, 230.
- Zahn J.-P., 1970, *A&A*, 4, 452.
- Zahn J.-P., 1975, *A&A*, 41, 329.
- Zahn J.-P., 1977, *A&A*, 57, 383.
- Zahn J.-P., 1978, *A&A*, 67, 162.
- Zahn J.-P., 1984, in Maeder A., Renzini A., eds., *IAU Symp. 105, Observational Tests of the Stellar Evolution Theory*, 379.
- Zahn J.-P., 1989, *A&A*, 220, 112.
- Zahn J.-P., Bouchet L., 1989, *A&A*, 223, 112.
- Zakirov M. M., 1992, *Kinematika i Fizika Nebesnykh Tel*, 8, 38.
- Zakirov M. M., 2001, *Ast. Lett.*, 27, 379.
- Żebruń K., Soszyński I., Wozniak P. R., 2001, *AcA*, 51, 303.
- Zeilik M., Gregory S. A., 1998, *Introductory Astronomy and Astrophysics (Fourth Edition)*, Saunders College Publishing.
- Zhang X. B., Deng L., Tian B., Zhou X., 2002, *AJ*, 123, 1548.
- Zhang X. B., Deng L., Zhou X., Xin Y., 2004, *MNRAS*, 355, 1369.
- Zombeck M. V., 1990, *Handbook of Astronomy and Astrophysics (Second Edition)*, Cambridge Univ. Press.
- Zucker S., 2003, *MNRAS*, 342, 1291.
- Zucker S., Mazeh T., 1994, *ApJ*, 420, 806.
- Zucker S., Torres G., Mazeh T., 1995, *ApJ*, 452, 863.
- Zucker S., Mazeh T., Santos N. C., Udry S., Mayor M., 2003, *A&A*, 404, 775.
- Zwahlen N., North P., Debernardi Y., Eyer L., Galland F., Groenewegen M. A. T., Hummel C., 2004, *A&A*, 425, L45.