

Eclipsing binary stars in open clusters

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Abstract

The study of detached eclipsing binaries allows accurate absolute masses, radii and luminosities to be measured for two stars of the same chemical composition, distance and age. These data can be used to test theoretical stellar models, investigate the properties of peculiar stars, and calculate its distance using empirical methods. Detached eclipsing binaries in open clusters provide a more powerful test of theoretical models, which must simultaneously match the properties of the eclipsing system and the cluster. The distance and metal abundance of the cluster can be found without the problems of main sequence fitting.

Absolute dimensions have been found for V615 Per and V618 Per, which are eclipsing members of h Persei. The fractional metal abundance of the cluster is $Z \approx 0.01$, in disagreement with literature assumptions of a solar chemical composition.

Accurate absolute dimensions have been measured for V453 Cygni, a member of NGC 6871. The current generation of theoretical stellar models can match these properties, as well as the central concentration of mass of the primary star as derived from a study of the apsidal motion of the system.

Absolute dimensions have been determined for HD 23642, a member of the Pleiades. This has allowed an investigation into the usefulness of different methods to find the distances to eclipsing binaries. A new method has been introduced, based on calibrations between surface brightness and effective temperature, and used to find a distance of 139 ± 4 pc. This value is in good agreement with other Pleiades distance measurements but does not agree with the controversial Hipparcos parallax distance.

The metallic-lined eclipsing binary WW Aur has been studied using extensive new spectroscopy and published light curves. The masses and radii have been found to accuracies of 0.6% using completely empirical methods. The predictions of theoretical models can only match the properties of WW Aur by adopting $Z = 0.060 \pm 0.005$.

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1 Detached eclipsing binary stars

1.1 Stars

A star is a sphere of matter held together by its own gravity and generating energy by means of nuclear fusion in its interior. Stars form from large clouds of gas and dust which attain a sufficient density to gravitationally collapse and form a protostar. The gravitational energy of the cloud is converted to thermal energy, which is transported by convection to the surface and then lost in the form of radiation. This gravitational collapse continues until the centre of the protostar is sufficiently hot and dense for thermonuclear fusion of hydrogen to begin. The minimum mass for this to occur is approximately $0.08 M_{\odot}$. The maximum initial mass of a star is strongly dependent on the chemical composition of the material from which it formed, but is of the order of $100 M_{\odot}$ for a solar chemical composition. Once thermonuclear fusion becomes the main source of energy for the protostar, it ceases to contract and settles down into a long-lived steady state called the main sequence (MS) phase.

The fundamental original properties of a star are its initial mass (M), chemical composition, rotational velocity and age. Given these quantities, stellar evolutionary theories can predict the radius (R), effective temperature, luminosity (L), and structure of any star. The radius of a star is actually not a precisely defined quantity, because stars do not have exact radii but merely a progressive loss of density (Scholz 1998), but is usually taken as the radius of the photosphere at an optical depth of $\frac{2}{3}$ (e.g., Siess, Dufour & Forestini 2000).

The properties of a star are often given in units of the equivalent value for the Sun. The fundamental properties of the Sun are given in Table 1.1.

The matter between stars attenuates the light which passes through it. The amount of light which is attenuated is a function of wavelength, so interstellar material affects the colours of stars as well as their apparent brightnesses. The main attenuation is due to scattering, but some light is also absorbed. As blue light is attenuated more

Table 1.1: The fundamental properties of the Sun. Note that the absolute bolometric magnitude of the Sun is a defined quantity and not a measured value.

References: (1) Zombeck (1990); (2) Bessell, Castelli & Plez (1998)

Quantity	Symbol	Value	Units	Ref
Mass	$M_{\odot}$	1.9891×10^{30}	kg	1
Radius	$R_{\odot}$	6.9599×10^8	m	1
Surface gravity	$\log g_{\odot}$	4.4377	(cm s^{-2})	1
Spectral type		G2 V		1
Luminosity	$L_{\odot}$	$3.855(6) \times 10^{26}$	W	2
Effective temperature	$T_{\text{eff}\odot}$	5781	K	2
Absolute bolometric magnitude	$M_{\text{bol}\odot}$	+4.74	(mag)	2
Absolute visual magnitude	$M_{V\odot}$	+4.81	(mag)	2
Bolometric correction	$\text{BC}_{V\odot}$	-0.07	(mag)	2

than red light, this causes stars to appear to be redder than they actually are, a phenomenon which is termed ‘reddening’.

Fitzpatrick (1998) has made a detailed investigation of the effects of interstellar extinction and how these may be removed from astronomical observations. That investigation was based on an analytical fitting function for extinction curves introduced by Fitzpatrick & Massa (1990), consisting of a linear background, a steep rise in extinction at shorter wavelengths, and a ‘bump’ increase in extinction centred at 2176 Å (Figure 1.1). Whilst the centre of the ‘bump’ is very stable, its width depends on the type of material causing the extinction (Fitzpatrick & Massa 1986). An illustration of the total extinction, A_{λ} , for the Johnson *UBVR**IJKLM* and Strömgren *uvby* passbands is given in Figure 1.2.

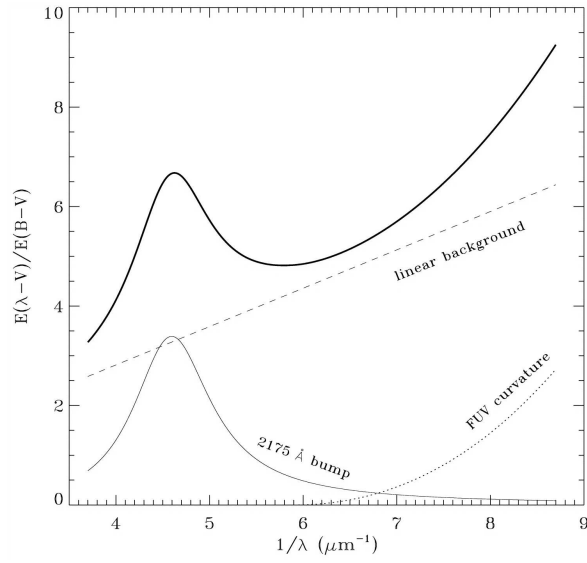


Figure 1.1: Decomposition of the analytical fitting function for extinction curves introduced by Fitzpatrick & Massa (1986, 1988, 1990). Taken from Fitzpatrick (1998).

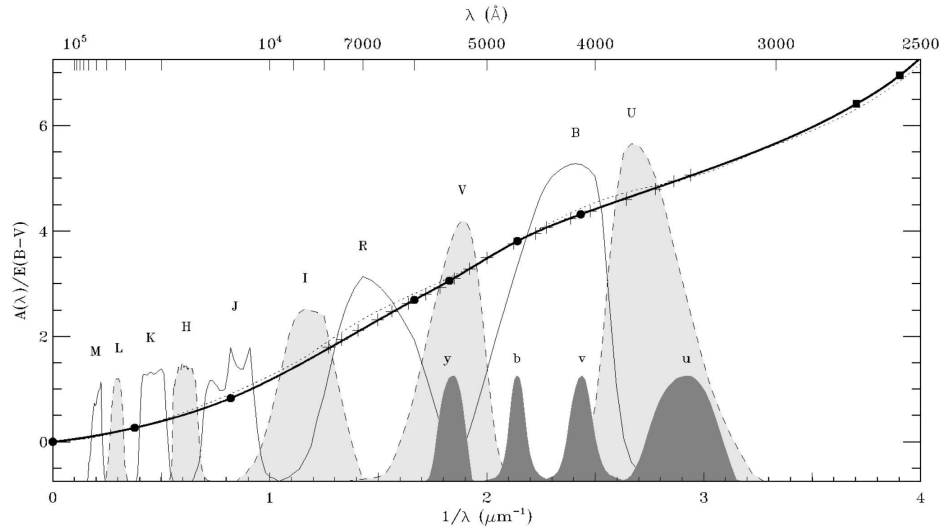


Figure 1.2: Illustration of the wavelength-dependent variation in A_λ and how this affects the Johnson $UBVRIJKLM$, a generic H and the Strömberg $uvby$ passbands. Taken from Fitzpatrick (1998).

1.1.1 Stellar characteristics

1.1.1.1 Stellar interferometry

Interferometric measurements of the radii of nearby stars are of fundamental importance to astrophysics. When combined with good parallax measurements they allow accurate linear radii of stars to be determined. Knowledge of the distance (from parallax) and apparent brightness of a star allows its absolute brightness to be found. If the linear radius of the star is known, its T_{eff} can be calculated directly. This allows calibration of the stellar T_{eff} and bolometric correction scales. The application of interferometry to visual binary stars also allows the masses of such stars to be found, allowing investigation of the mass-luminosity relation.

1.1.1.2 The effective temperature scale

The T_{eff} of a star is defined to be the temperature of a black body emitting the same flux per surface area as the star. The T_{eff} of a star is a precisely defined concept, but as stars are quite different from black bodies, the physical interpretation of T_{eff} is not straightforward. Therefore a scale of T_{eff} s has been established by several researchers.

1.1.1.3 Stellar chemical compositions

Shortly after the Big Bang, the Universe contained mostly hydrogen, with some helium and a trace of lithium. Since this point, the thermonuclear processes inside stars have been converting these light elements into heavier elements, which are ejected back into the interstellar environment when the star dies.

The fractional abundances by mass of hydrogen, helium and ‘metals’ (all other elements) are denoted by X , Y and Z , respectively. The values of these quantities for the Sun are generally taken to be $X_{\odot} = 0.70683$, $Y_{\odot} = 0.27431$ and $Z_{\odot} = 0.01886$ (Anders & Grevesse 1989). $Z_{\odot}$ is found from laboratory studies of pristine meteorites (the ‘C1 chondrite’ class) and from spectroscopic studies of the solar photosphere and

corona, and is dominated by the important volatile elements carbon, oxygen and nitrogen (Grevesse, Noels & Sauval 1996).

Most theoretical studies of stellar evolution adopt metal abundances which are scaled from the solar values, but some studies also adjust the abundances of the ‘ α -elements’. These are the products of α -capture and are ^{24}Mg , ^{28}Si , ^{32}S , ^{36}Ar , ^{40}Ca , ^{44}Ca and ^{48}Ti . They are primarily made by thermonuclear fusion of carbon, oxygen and neon in the later stages of stellar evolution (Cowley 1995).

More recently, solar abundances have been given by Asplund, Grevesse & Sauval (2004) as $X_{\odot} = 0.7392$, $Y_{\odot} = 0.2486$ and $Z_{\odot} = 0.0122$. These values are quite different from those of Anders & Grevesse (1989), and have major implications for stellar astrophysics if they are correct, but are unlikely to be adopted until published in a refereed journal (A. Claret, 2004, private communication). They are in poor agreement with the results of helioseismological investigations (Bahcall et al. 2005).

The abundances of helium and metals are expected to increase over time as stars manufacture them from hydrogen and then eject them into the interstellar medium via winds, binary mass loss and supernovae. Whilst the early Universe contained some helium, negligible amounts of metals were made in the Big Bang. The abundances of helium and metals are therefore expected to be related according to the equation

$$Y = Y_{\text{prim}} + \frac{\Delta Y}{\Delta Z} Z \quad (1.1)$$

where Y_{prim} is the primordial helium abundance and $\frac{\Delta Y}{\Delta Z}$ is the enrichment slope. Ribas et al. (2000) found $Y_{\text{prim}} = 0.225 \pm 0.013$ and $\frac{\Delta Y}{\Delta Z} = 2.2 \pm 0.8$ from fitting theoretical evolutionary models to the properties of several detached eclipsing binaries (dEBs). This is in good agreement with other determinations of both quantities.

1.1.1.4 Bolometric corrections

The bolometric flux produced by a star is the total electromagnetic flux summed over all wavelengths. It follows that luminosity is a bolometric quantity but that the magnitude of a star observed through a photometric passband is not. Transformation between the

bolometric magnitude and a passband-specific magnitude of a star requires bolometric corrections (BCs), which are defined using the formula

$$M_{\lambda} = M_{\text{bol}} - BC_{\lambda} \quad (1.2)$$

where M_{λ} is the absolute magnitude of a star in passband λ and M_{bol} is the star's absolute bolometric magnitude.

The zeropoint of the BC scale is therefore set by the physical properties adopted for the Sun, which means that different sources of BC may adopt different zeropoints. BCs are used in the study of dEBs to aid in determining the distance to a dEB from the luminosities of the stars and the overall apparent passband-specific magnitude of the dEB. For this method there are two types of sources for BCs.

Empirical BCs can be found using two methods. The first method is to obtain spectrophotometric observations of stars over as wide a wavelength range as possible. This method is difficult for hot stars as they emit a significant fraction of their light at ultraviolet wavelengths, and light at wavelengths below 912 Å is not observable as it is strongly absorbed by the interstellar medium. The second method is to resolve the surfaces of stars using interferometry, and find their distances using trigonometrical parallaxes. If their T_{eff} s are known then the absolute bolometric fluxes can be calculated from this and their linear radii.

Empirical BCs have been tabulated by several researchers, including Code et al. (1976), Habets & Heintze (1981), Malagnini et al. (1986) and Flower (1996). The study of dEBs can provide empirically-determined BCs (Habets & Heintze 1981) as the surfaces of the stars are resolved by the analysis of light curves. The disadvantages of empirical BCs is that their values have observational uncertainty and are only relevant to stars of a similar chemical composition to the stars used to find the BCs. As most empirical BCs are determined using interferometry, this limits the chemical composition to approximately solar, as this is the chemical composition of the nearby stars which are resolvable with current interferometric instruments.

Theoretical BCs can be derived using theoretical model atmospheres, meaning they are exact and that they can be derived for any realistic set of atmospheric pa-

rameters, including chemical composition. Although they have no random errors, the use of theoretical calculations in the derivation of BCs means that they are subject to systematic errors. Whilst these systematic errors can be difficult to investigate, the comparison between several different theoretical BC tabulations and empirical BCs can be useful. Theoretical BCs for the V and K passbands have been tabulated by Bessell, Castelli & Plez (1998) for a solar chemical composition. Girardi et al. (2002) have provided BCs for several wide-band photometric systems, including the $UBVR IJHKL$ passbands, for metal abundances, $[\frac{M}{H}]$, of -2.5 to $+0.5$ in steps of 0.5 .

1.1.1.5 Surface brightness relations

The concept of surface brightness was first used in the analysis of EBs almost one century ago (Kruszewski & Semeniuk 1999), when Stebbins (1910) used the known trigonometrical parallax and inferred linear radii of the components of Algol (HD 19356) to estimate the surface brightnesses of both stars relative to the Sun. Stebbins (1911) applied this analysis to the component stars of β Aurigae, which was the first EB with a double-lined spectroscopic orbit (Baker 1910). Kopal (1939) was able to provide a calibration of surface brightness (expressed as an equivalent T_{eff}) in terms of spectral type from the analysis of EBs.

The first analysis to use surface brightness relations to find the distance to an EB, rather than the other way round, was by Gaposchkin (1962), who determined the distance to M 31 from the study of an EB inside this galaxy. Further work was directed towards finding the distance to the Large Magellanic Cloud (LMC) and the Small Magellanic Cloud (SMC) (Gaposchkin 1970). Compared to modern distance values, the results were quite reasonable (although the quoted uncertainties were much too small) but a little large, probably due to the inclusion of more complicated semidetached binaries (Kruszewski & Semeniuk 1999).

Barnes & Evans (1976; Barnes, Evans & Parson 1976; Barnes, Evans & Moffett 1978) used the angular diameters of 52 stars, most of which had been studied using interferometry, to investigate the relations between surface brightness and colour indices

involving the Johnson $BVRI$ broad-band passbands. They discovered that the best relation, in terms of having the smallest scatter, used the $V-R$ colour index. As surface brightness relations in terms of colour index were not originally their idea, it is best to refer to only the surface brightness – $(V-R)$ calibration as being the Barnes-Evans relation (Kruszewski & Semeniuk 1999). Barnes, Evans & Moffett (1978) improved the definition of the relation by adding data for another 40 stars. The relations in $B-V$ and $R-I$ have more scatter due to a dependence on surface gravity and increased “cosmic scatter” (intrinsic variation between similar stars). The relation for $U-B$ is of no use as it is strongly affected by surface gravity, “cosmic scatter”, line blanketing and Balmer line emission. These effects mean that the $U-B$ relation is not monotonic. The $B-V$ relation has a similar problem for stars cooler than mid K -type.

An important aspect of the Barnes-Evans relation is that it is stated to be applicable to all types of stars, including pulsating variables. Thus it can be used to find the distance to, and linear radii of, δ Cepheids, so can be used to calibrate an important distance indicator. However, there is some evidence that the measured angular diameters of late-type stars depend on wavelength, as a result of circumstellar matter (Barnes & Evans 1976) and the spectral characteristics of these stars.

The Barnes-Evans relation was applied by Lacy (1977a) in the determination of the distance moduli to nine dEBs, with accuracies of about 0.2 mag. It was also applied by Lacy (1978) to three dEBs which are members of nearby open clusters or associations. The resulting distances were in reasonable agreement with the distances found by MS fitting methods, although there were suggestions of a systematic discrepancy of 0.1 mag. Lacy (1977c) used the Barnes-Evans relation to find the radii of a large number of nearby single stars. O’Dell, Hendry & Collier Cameron (1994) recalibrated the $F_V - (B-V)$ relation and presented a method to determine the distance to a sample of stars, for example the members of an open cluster, using their recalibration.

The concept of a zeroth magnitude angular diameter was introduced by Mozurkewich et al. (1991) and is the angular diameter of a star with an apparent magnitude of zero.

The surface brightness in passband λ is defined to be

$$S_{m_\lambda} = m_\lambda + 5 \log \phi \quad (1.3)$$

where m_λ is the apparent magnitude in passband λ and ϕ is the stellar angular diameter (milliarcseconds) (Di Benedetto 1998). The zeroth-magnitude angular diameter is

$$\phi^{(m_\lambda=0)} = \phi \times 10^{\frac{m_\lambda}{5}} \quad (1.4)$$

This means that $\phi^{(m_\lambda=0)}$ is actually a measure of surface brightness:

$$\phi^{(m_\lambda=0)} = 10^{\frac{S_{m_\lambda}}{5}} \quad (1.5)$$

Calibrations for $\phi^{(m_\lambda=0)}$ were given for the $B-K$ and $V-K$ indices by van Belle (1999). Calibrations for S_V were constructed by Thompson et al. (2001) for the $V-I$, $V-J$, $V-H$ and $V-K$ indices and used to find the distance to the dEB OGLE GC 17, a member of the globular cluster ω Centauri.

Salaris & Groenewegen (2002) noted that the zeroth-magnitude angular diameter is strongly correlated with the Strömgren c_1 index in B-type stars. They calibrated the relationship using stars in nearby dEBs and found

$$\phi^{V=0} = 1.824(180)c_1 + 1.294(78) \quad (1.6)$$

Salaris & Groenewegen state that this relationship may need a more detailed investigation but that it may be useful in determining the distance to the LMC using dEBs.

Kervella et al. (2004) used interferometric data for nearby stars to provide calibrations for surface brightness based on every photometric index which uses two passbands out of $UBVRIJHKL$ (Figure 1.3). The calibrations are linear, although some are indicated to be a bad representation of nonlinear data. Estimates of “cosmic scatter” are also made; this is below 1% for calibrations based on the $U-L$, $B-K$, $B-L$, $V-K$, $V-L$ and $R-I$ indices. Calibrations for $\phi^{(m_\lambda=0)}$ in terms of T_{eff} are also given for all the passbands mentioned above (Figure 1.4). Further investigation by Groenewegen (2004) has revealed a dependence of $V-K$ on $[\frac{\text{Fe}}{\text{H}}]$; this has been quantified. Groenewegen calibrated S_V against $V-R$ and $V-K$, and S_K against $J-K$; the latter relation has a statistically insignificant dependence on $[\frac{\text{Fe}}{\text{H}}]$.

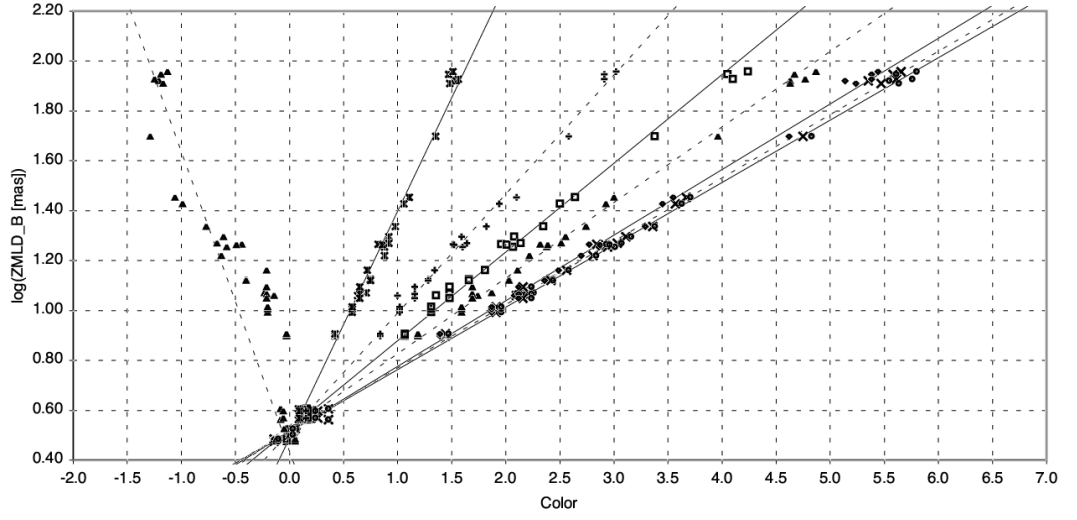


Figure 1.3: Relation between zeroth-magnitude angular diameter and (from left to right on the diagram) $B-U$, $B-V$, $B-R$, $B-I$, $B-J$, $B-H$, $B-K$ and $B-L$. Note the strong nonlinearity in the $B-U$ data. Taken from Kervella et al. (2004).

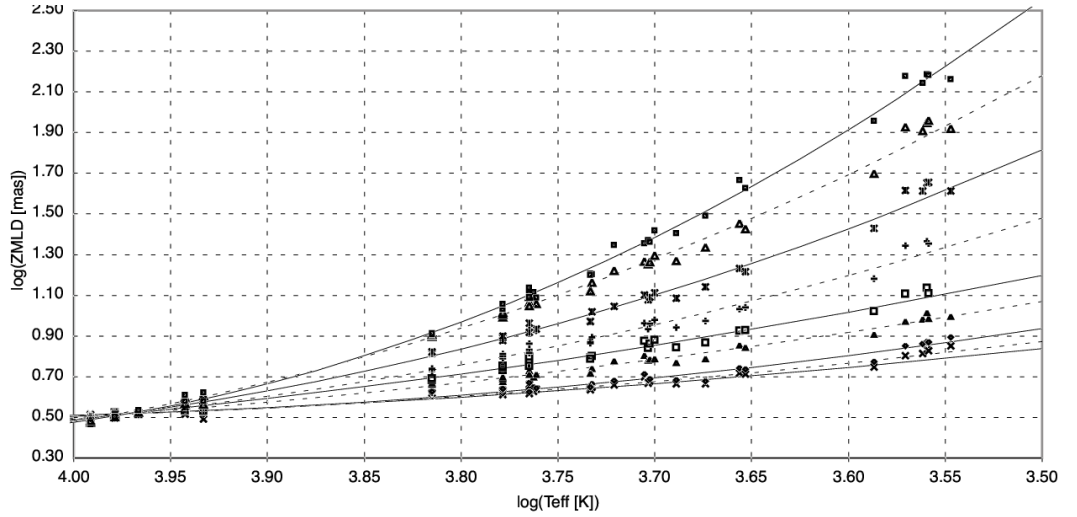


Figure 1.4: Relation between zeroth-magnitude angular diameter and T_{eff} . From top to bottom of the diagram, the lines are for the U , B , V , R , I , J , H , K and L passbands. Taken from Kervella et al. (2004).

1.1.2 Limb darkening

When stars are viewed from a particular direction they do not appear to be uniform discs. Although stars are normally approximately spherically symmetric, towards the edge of their disc they appear to get dimmer. This limb darkening (LD) occurs because when we look obliquely into the surface of a star we are seeing a cooler gas overall than when we look from normal to the surface. As cooler gases are less bright, the limb of a star appears dimmer.

LD is a fundamental effect which must be allowed for when analysing the light curves of EBs. The neglect, or inadequate representation, of LD can create systematic uncertainties in the stellar radii derived from light curve analysis. For the purposes of modelling light curves, the variation in brightness over a stellar disc is represented by various parameterisations called LD laws.

Many tabulations exist of LD coefficients determined theoretically using model atmospheres. Whilst this can introduce a dependence on theoretical models into the analysis of the light curves of EBs, there is no alternative when the observations are not good enough to allow the derivation of LD coefficients from the light curves themselves. The general theoretical method is to derive the emergent flux at different angles from a plane-parallel model atmosphere and fit the resulting curve with the relevant LD law.

1.1.2.1 Limb darkening laws

The simplest LD law is the linear law. This is formulated using $\mu = \cos \theta$ where θ is the angle of incidence of a sight line to the stellar surface. The linear LD law is

$$\frac{I(\mu)}{I(1)} = 1 - u(1 - \mu) \quad (1.7)$$

where $I(\mu)$ is the flux per unit area received at angle θ , $I(1)$ is the flux per unit area from the centre of the stellar disc. The coefficient u depends on the wavelength of observation, the T_{eff} , the surface gravity and the chemical composition of the star.

Two-coefficient laws have been introduced to provide a better representation to

the (theoretically derived) LD characteristics of stars. The quadratic law is

$$\frac{I(\mu)}{I(1)} = 1 - a(1 - \mu) - b(1 - \mu)^2 \quad (1.8)$$

which contains the coefficients a and b . KlingleSmith & Sobieski (1970) introduced the logarithmic LD law

$$\frac{I(\mu)}{I(1)} = 1 - c(1 - \mu) - d\mu \ln \mu \quad (1.9)$$

which contains the coefficients c and d . Díaz-Cordovés & Giménez (1992) introduced the square-root law

$$\frac{I(\mu)}{I(1)} = 1 - e(1 - \mu) - f(1 - \sqrt{\mu}) \quad (1.10)$$

with coefficients e and f . Barban et al. (2003) generalised the cubic law to

$$\frac{I(\mu)}{I(1)} = 1 - p(1 - \mu) - q(1 - \mu)^2 - r(1 - \mu)^3 \quad (1.11)$$

where the fitted coefficients are p , q and r .

Claret (2000b, 2003) investigated a four-coefficient law which is

$$\frac{I(\mu)}{I(1)} = 1 - \sum_{k=1}^4 a_k (1 - \mu^{k/2}) \quad (1.12)$$

where the coefficients are a_k . Claret (2000b) claims that this law is more successful at fitting all types of star than the two-coefficient laws. Claret & Hauschildt (2003) introduced a new biparametric approximation given by

$$\frac{I(\mu)}{I(1)} = 1 - g(1 - \mu) - \frac{h}{(1 - e^\mu)} \quad (1.13)$$

in an attempt to better fit the theoretical LD predicted by recent spherical model atmospheres. The last two laws are notably more successful at short and long wavelengths, where success is measured by the agreement between the predicted LD and the LD law used to fit the predictions. In particular, spherical model atmospheres predict a severe drop in flux significantly before the observed edge of the disc (Claret & Hauschildt 2003), and the last two laws are the most successful at representing this.

Table 1.2: Tabulations of LD coefficients in the literature.

Reference	Linear	Log	Quad	Cubic	Sqrt	Exp	4coeff	Additional remarks
Grygar (1965)	*							
Klinglesmith & Sobieski (1970)	*	*						$T_{\text{eff}} \geq 10000 \text{ K.}$
Al-Naimiy (1978)	*							
Muthsam (1979)			*					
Wade & Rucinski (1985)	*		*					
Claret & Giménez (1990a)	*		*					$T_{\text{eff}} \leq 6730 \text{ K.}$
Claret & Giménez (1990b)	*		*					$T_{\text{eff}} \leq 6730 \text{ K.}$
Díaz-Cordovés & Giménez (1992)	*		*		*			Not tabulated.
van Hamme (1993)	*	*			*			
Díaz-Cordovés, Claret & Giménez (1995)	*		*		*			<i>wby</i> and <i>UBV</i> passbands
Claret, Díaz-Cordovés & Giménez (1995)	*		*		*			<i>RIJHK</i> passbands.
Claret (1998)	*		*		*			
Barban et al. (2003)	*		*	*	*			<i>wby</i> passbands, A and F stars.
Claret (2000b)	*	*	*		*		*	<i>wbyUBVRIJHK</i> passbands
Claret (2003)	*	*	*		*		*	Geneva and Walraven passbands
Claret & Hauschildt (2003)	*	*	*		*	*	*	$5000 \geq T_{\text{eff}} \geq 10000 \text{ K}$
Claret (2004b)	*	*	*		*		*	Sloan <i>u'g'r'i'z</i> passbands

1.1.2.2 Limb darkening and eclipsing binaries

Many tabulations of LD coefficients are collected in Table 1.2. When analysing a light curve, the choice of LD law is restricted to those implemented by the light curve code one is using. It is important to produce results for several different coefficients to determine the uncertainty created by the use of fixed theoretical LD coefficients.

The atmospheres of close binaries are modified by flux incident from the other star in the system, changing the LD characteristics. Theoretical coefficients usually refer to isolated stars but the LD of irradiated atmospheres have been investigated by Claret & Giménez (1990b) and by Alencar & Vaz (1999). These authors also compared theoretical results with linear LD coefficients derived from photometric observations and found reasonable agreement within the (quite large) errors. Other comparisons between theory and observation exist (for example Al-Naimiy 1978) and agreement is generally good. However, the linear LD law does not represent well the flux characteristics of model atmospheres. It is also important to remember that theoretical LD coefficients are known to depend on atmospheric metal abundance (Wade & Rucinski 1985; Claret 1998) and the treatment of convection (Barban et al. 2003). Theoretical and observed linear LD coefficients disagree at ultraviolet wavelengths, which is important to remember when fitting light curves observed through the passbands such as Strömgren u and Johnson U (Wade & Rucinski 1985).

The EBOP light curve analysis code (see Section 2.4.1.1) is restricted to the linear LD law, although attempts have been made by Dr. A. Giménez and Dr. J. Díaz-Cordovés to include nonlinear LD (Etzell 1993). The Wilson-Devinney code (see Section 2.4.1.2) can perform calculations using the linear, logarithmic and the square-root laws (equations 1.7, 1.9 and 1.10). van Hamme (1993) has provided extensive tabulations of the relevant coefficients, and their goodness of fit, to aid the decision as to which law is better in a particular case. In general, the square-root law is better at ultraviolet wavelengths and the logarithmic law is better in the infrared. In the optical, the square-root law is better for hotter stars and the logarithmic law is better for cooler stars, the transition region being between T_{eff} s of 8000 K and 10 000 K.

The incorporation of model atmosphere results into light curve analysis codes allows the direct use of theoretical LD characteristics without parameterisation and approximation into an LD law. This procedure has been implemented by Bayne et al. (2004) using tabulations of Kurucz (1993b) model atmosphere predictions inside a version of the 1993 Wilson-Devinney code.

1.1.3 Gravity darkening

The flux emergent from different parts of a stellar surface is dependent on the local value of surface gravity. This dependence takes the form of the gravity darkening exponent designated β_1 (following the notation of Claret 1998), defined by the relation

$$F \propto T_{\text{eff}}^4 \propto g^{\beta_1} \quad (1.14)$$

where F is the bolometric flux and g is the local surface gravity. An alternative definition, which has often been used, is $T_{\text{eff}} \propto g^{\beta}$ (Hilditch 2001, p.243). Thus the emergent flux from a star which is distorted by surface inhomogeneities or rotation, or the presence of an orbiting companion, is dependent on the position of emergence. Gravity darkening is an important effect in the analysis of the light curves of EBs and also in the study of rotational effects on single stars (Claret 2000a). It also affects the full width at half maxima of the spectral lines of rapidly rotating stars (Shan 2000).

von Zeipel (1924) was the first to investigate this analytically, and found that for a stellar atmosphere in radiative and hydrostatic equilibrium, $\beta_1^{\text{rad}} = 1.0$. Lucy (1967) investigated the properties of convective envelopes, and from numerical methods found an average value of $\beta_1^{\text{conv}} = 0.32$. These values are generally assumed to be correct and were confirmed observationally by Rafert & Twigg (1980), who found mean values of $\beta_1^{\text{rad}} = 0.96$ and $\beta_1^{\text{conv}} = 0.31$ from light curve analyses of a wide sample of dEBs. Hydrodynamical simulations by Ludwig, Freytag & Steffen (1999) found that the value of β_1^{conv} lies between about 0.28 and 0.40. The radiative-convective boundary is around $T_{\text{eff}} = 7250$ K (Claret 2000a).

The canonical assumption of $\beta_1^{\text{rad}} = 1.0$ and $\beta_1^{\text{conv}} = 0.32$ is unsatisfactory because there is a discontinuity in the value at the boundary between convective and radiative envelopes. This is unphysical because in such situations both types of energy transport can exist simultaneously in the envelope of a star (Claret 1998), suggesting that β_1 varies smoothly over all conditions.

Claret (1998, 2000a) presented tabulations of β_1 calculated using the Granada theoretical stellar evolutionary models (see section 1.3.2.1). These works have shown that β_1 is a parameter which depends on surface gravity, T_{eff} , surface metal abundance, the type of convection theory, and evolutionary phase. Claret found that the transition between radiative and convective values is very sharp, but it is continuous. In general β_1^{conv} is between 0.2 and 0.4 for low-mass stars, whereas for stars with masses above about $1.7 M_{\odot}$, $\beta_1^{\text{rad}} \approx 1.0$.

1.2 Stellar evolution

1.2.1 The evolution of single stars

Stellar evolution is generally illustrated using Hertzsprung Russell (HR) diagrams, on which stars are placed according to their T_{eff} and luminosity. Stars form from giant interstellar clouds of gas and dust which collapse if their gravitational energy is larger than their kinetic energy. This requirement is normally met by small parts of a cloud, which individually collapse to form stars. This means that most stars are born in clusters (Phillips 1999, p. 15). Most of the kinetic energy of a cloud is lost by radiation into space. The locus in the HR diagram where stellar objects of different masses become observable is called the Hayashi line. This may even extend beyond the zero-age main sequence (ZAMS) for O-type stars as their evolution is so quick (Maeder 1998).

The protostars continue to contract and lose energy by radiating light. This evolution occurs along the Hayashi track and continues until the core of the protostar

attains a sufficient temperature and density for large-scale thermonuclear reactions to occur. The star has reached the ZAMS, and is in equilibrium between the generation of energy by thermonuclear reactions (the ‘burning’ of hydrogen) and the emission of the energy in the form of radiation from its surface.

1.2.1.1 Main sequence evolution

The ZAMS is the point at which a protostar becomes a star, but is not precisely defined (Torres & Ribas 2002). Alternative definitions include the point at which the radius of a stellar object is a minimum after PMS contraction (Lastennet & Valls-Gabaud 2002) and the point at which 99% of the energy emitted by the stellar object is generated from thermonuclear reactions (Marques, Fernandes & Monteiro 2004).

Whilst on the MS, thermonuclear fusion in the cores of stars converts hydrogen into heavier elements. The energy produced in this way is transported through the envelope of the star by radiative and convective processes. Once it reaches the surface it is emitted, causing the star to be bright.

Stars with masses lower than about $0.4 M_{\odot}$ are completely convective throughout their PMS and MS evolution. Stars with masses below about $1.1 M_{\odot}$ have radiative cores and convective envelopes (Hurley, Pols & Tout 2000). Stars with masses above about $1.3 M_{\odot}$ develop radiative envelopes (Hurley, Tout & Pols 2002) and the convective zone moves towards the centre of the star. More massive stars have convective cores and radiative envelopes. The mass limits quoted above are valid for a solar chemical composition; different chemical abundances cause these limits to change.

As the conversion of hydrogen into helium increases the mean molecular mass of the core of an MS star, the density increases. This causes the amount of thermonuclear fusion to increase, so the core temperature and energy production rise. The increased energy production causes both the luminosity and the radius of the star to go up, the latter as a result of the greater radiation pressure acting on the outer layers of the star. The T_{eff} s of low-mass stars increase as a result of this; high-mass stars get cooler (Hurley, Pols & Tout 2000).

1.2.1.2 Evolution of low-mass stars

At the end of their MS lifetimes, low-mass stars (those with radiative cores) run out of hydrogen in their core. As the core is mainly helium, it is denser and so becomes hotter. The region of hydrogen burning moves outwards to a shell, and the radius of the star increases. The star is now a red giant, a relatively long-lived evolutionary phase. The shell hydrogen burning produces helium, which causes the core to experience an increase in density and temperature. The core becomes degenerate and, once a sufficient temperature has been reached, helium burning abruptly starts in the core in an episode termed the ‘helium flash’ (Kaufmann 1994, p. 385).

After the helium flash, the star becomes a horizontal branch star powered by the thermonuclear fusion of helium in its core. Once helium has been exhausted, the star goes through the asymptotic giant branch and planetary nebula evolutionary phases before ending its life cooling slowly as a white dwarf.

1.2.1.3 Evolution of intermediate-mass stars

For stars which have convective cores on the MS ($M \gtrsim 1.2 M_{\odot}$), the end of their MS evolution is more extreme than for low-mass stars. The exhaustion of hydrogen occurs almost simultaneously over the well-mixed core, leading to a rapid contraction of the core and large increase in radius. As the star climbs the giant branch in the HR diagram, the envelope of the star becomes convective and hydrogen burning moves outwards in a shell, depositing more helium on the core.

Once the conditions in the core have reached a threshold, helium burning begins. For stars of masses above about $2 M_{\odot}$, whose helium cores have not become degenerate, this occurs gently. The star returns along the giant branch to the ‘blue loop’ in the HR diagram and consumes helium in its core and hydrogen in a shell. Once core helium is exhausted, it goes through the asymptotic giant branch phase and either the planetary nebula or supernova phases.

1.2.1.4 Evolution of massive stars

The evolution of massive stars is strongly dependent on the initial chemical composition of the star, mass loss, rotation, magnetic effects and the different mixing process which occur inside a star. Some of these physical phenomena will be discussed later.

Massive stars ($\gtrsim 12 M_{\odot}$) undergo helium burning before reaching the giant branch stage of evolution. The progressively more extreme conditions in the core allow the burning of carbon, oxygen and other elements up to and including iron. Further thermonuclear fusion reactions are endothermic, causing loss of the pressure which was supporting the stellar envelope. The envelope collapses, rebounds, and is ejected in a supernova explosion. The core finishes up as a neutron star or a black hole.

1.3 Modelling of stars

Much of the progress in our understanding of stars has required the construction of theoretical models of their structure and evolution. The intention of a theoretical model is that, for an input mass and chemical composition, it should be able to predict the radius, T_{eff} and internal structure of a star for an arbitrary age. It has recently become clear that the initial rotational velocity is also important (see below) and there remain some physical phenomena which are not incorporated into the current generation of available theoretical models.

The predictive power of the current generation of stellar models is very good for MS and giant stars of spectral types between approximately B and K. The predicted properties of more massive or evolved stars are strongly dependent on several physical phenomena which are simplistically treated, for example convective efficiency and mass loss. Models of less massive stars continue to require work to correct the apparent disagreement between the observed and predicted properties of M dwarfs (Ribas 2003; Maceroni & Montalbán 2004).

Theoretical stellar models generally begin from a reasonable approximation of

a ZAMS or slightly pre-ZAMS stellar structure. The initial chemical composition is decided by assuming a fractional metal abundance, Z , using a chemical enrichment law to find the corresponding helium abundance, Y , and making up the rest with hydrogen, X (see section 1.1.1.3). The metal abundance is normally distributed between the different elements according to the relative elemental abundances of the Sun (‘scaled solar’) although some models have enhanced α -elements.

One-dimensional models are generally used, in which the properties of matter are followed on a radial line from the core of the star to its surface, with the use of roughly 500 discrete ‘mesh points’ (e.g., Bressan et al. 1993) for which the instantaneous temperature, pressure and chemical abundances are calculated. Numerical integration is then used to follow the conditions at these mesh points when physical processes occur. The subsequent evolution of the star is followed until a certain point in its later evolution where it is known that the model has insufficient physics implemented to be able to follow the evolution further. Typically several thousand timesteps are required to follow the evolution of a star (e.g., Bressan et al. 1993).

Theoretical model sets contain several parameterisations of physical effects. The choice of parameter values for these is generally made by forcing the models to match the radius and T_{eff} of the Sun for its mass, chemical composition, and an age of 4.6 Gyr. Helioseismological constraints can also be applied, mainly in specifying the helium abundance of the Sun (Schröder & Eggleton 1996).

The parameterisations incorporated into theoretical models compromise the predictive ability of such models. This predictive power is important to almost all areas of astrophysics (Barbosa & Figer 2004; Young & Arnett 2004).

1.3.1 Details of some of the physical phenomena included in theoretical stellar evolutionary models

1.3.1.1 Equation of state

A central part of a theoretical stellar model is the equation of state, which relates the electron and gas pressure to the temperature and density. Once the pressures have been calculated from the temperature and density, the excitation and ionisation state of each element can be calculated. As the pressures themselves depend on the elemental states, the equation of state must be dealt with using iterative calculation.

1.3.1.2 Opacity

The main effect of most of the species in a stellar interior is to retard the progress of radiative energy from the core of the star to the surface. Photons can be scattered or absorbed and re-emitted by ions and electrons, retarding the photons and causing radiation pressure. The size of this opacity depends on the cross-section of interaction of each different chemical species and is an important ingredient in theoretical models. This has a large influence on the predicted radius of the star and on the conditions in the stellar core, for stars which have large zones where energy transport is radiative.

Determinations of the the strength of stellar opacities have generally increased over time. In the 1980s, matching the properties of massive stars (predominantly in dEBs) often required models with $Z \approx 0.04$ despite having approximately solar chemical compositions found from spectroscopy (Stothers 1991; Andersen et al. 1981). An increase of opacity causes the effect of metals to be increased, so fewer metals are needed to give the same effect. The effect of opacity and metal abundance are difficult to separate when comparing model predictions to observations (Cassisi et al. 1994).

1.3.1.3 Energy transport

Stars consist of plasma at high temperatures and generally at high pressures. The transport of energy through this medium, from its generation in the core to its escape from the stellar surface, is of fundamental importance to the characteristics of stars. Energy transport in stars occurs in two ways: by radiative diffusion and by convective motion. The latter is a particularly complex process to model.

The diffusion of energy can occur by random motion of electrons and of photons. In the typical conditions of a stellar envelope, the energy diffusion by electrons is several orders of magnitudes smaller than the radiative diffusion due to the movement of photons (Phillips 1999, p.91).

Radiative diffusion is the dominant source of energy transport below a certain critical temperature gradient. Convective motions arise when radiative diffusion cannot transport energy quickly enough. Large-scale motions occur once the critical temperature gradient has been reached. These convective currents are very efficient at transporting energy but their characteristics make them very difficult to model.

1.3.1.4 Convective core overshooting

Massive stars tend to have convective cores and radiative envelopes, but there is evidence that the transition between these two modes of energy transport occurs somewhat further out from the core than the point at which the critical temperature gradient is reached. This phenomenon is called convective core overshooting, and may have an important effect on the properties and lifetimes of massive stars. The physical explanation for the effect concerns a pile of material which is undergoing convective motion outwards from the core of the star. Once it reaches the point at which the temperature gradient drops below the critical value, it enters a volume which is formally expected to be free of convective motions. However, the kinetic energy of this material causes it to rise further before it cools sufficiently to begin to sink back towards the core.

The effect of overshooting is to make a larger proportion of the matter in a star

available for thermonuclear fusion in the core. This increases the MS lifetime of the star as it has more hydrogen to burn. The luminosity of the star also increases, its T_{eff} changes more during its MS lifetime (e.g., Alongi et al. 1993; Schröder & Eggleton 1996), and it becomes more centrally condensed (Claret & Giménez 1991). Overshooting has a large effect on the evolution of stars beyond the terminal-age main sequence (TAMS; e.g., Pols et al. 1997). This means that the amount of convective core overshooting can be deduced by comparing observations of stars with the predictions of theoretical stellar evolutionary models (section 1.3.2). These models generally incorporate overshooting by parameterisation, where the overshooting parameter, α_{OV} , is equal to the length of penetration of convective motions into radiative layers in units of the pressure scale height:

$$\alpha_{\text{OV}} = \frac{l_{\text{overshoot}}}{H_p} \quad (1.15)$$

Another effect of overshooting is to modify the surface chemical abundances of evolved stars, as the convective cores of their progenitors are larger so a greater proportion of the star has had its chemical composition modified by thermonuclear fusion.

Andersen, Clausen & Nordström (1990b) also found strong evidence for the presence of overshooting from consideration of the properties of dEBs. Component stars in dEBs with masses of about $1.2 M_{\odot}$, which have small convective cores, are well matched by the predictions of theoretical models but those with masses not much greater than this clearly require models with overshooting to match their properties.

Stothers & Chin (1991) found that the adoption of newer opacity data in their stellar evolutionary code eliminated the need for convective core overshooting when attempting to match predictions to observations. They quoted the maximum amount of overshooting to be $\alpha_{\text{OV}} = 0.20$. Stothers (1991) detailed the results of fourteen tests for the presence of overshooting in medium- and high-mass stars. The results of every test were consistent with $\alpha_{\text{OV}} = 0$, four tests produced the constraint of $\alpha_{\text{OV}} < 0.4$ and one test allowed this constraint to be strengthened to $\alpha_{\text{OV}} < 0.2$. However, Stothers states that matching the amount of apsidal motion exhibited by some well-studied dEBs may continue to require a small amount of overshooting in the evolutionary models.

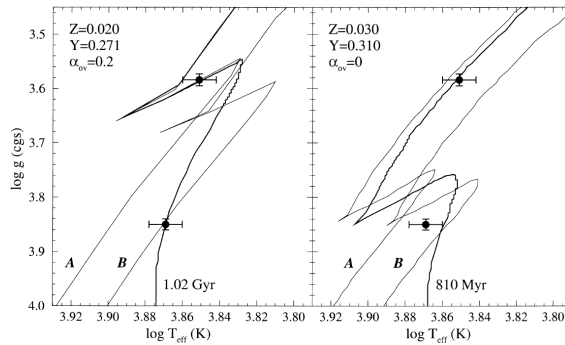


Figure 1.5: $T_{\text{eff}}\text{--}\log g$ plot showing the observed properties of the dEB AI Hya. The panel on the left shows evolutionary tracks and isochrones from the Granada theoretical models (Claret 1995 and subsequent works) for $\alpha_{\text{OV}} = 0.20$. The panel on the right shows the predictions for standard models ($\alpha_{\text{OV}} = 0$). Taken from Ribas et al. (2000).

In their study of the F-type dEB EI Cephei, Torres et al. (2000a) required overshooting to match the properties of the dEB with models. The evolved components of several dEBs can be matched by theoretical models without overshooting, but only in a short-lived state beyond the TAMS (Figure 1.5). If the models include overshooting, these stars can be matched by MS models in an evolutionary phase which lasts much longer (Andersen 1991; Ribas et al. 2000). Evolved dEBs therefore provide strong evidence that overshooting is significant. Figure 1.5 also shows that the value of α_{OV} derived in this way is correlated with metal abundance.

Ribas, Jordi & Giménez (2000) have found evidence that α_{OV} has a dependence on stellar mass (Figure 1.6). This claim is based on the existence of several dEBs with component masses around $2 M_{\odot}$ for which the best match is for theoretical models with $\alpha_{\text{OV}} \approx 0.2$, and two dEBs with larger component masses and a good match for $\alpha_{\text{OV}} \approx 0.6$. It is also thought that overshooting is unimportant for lower-mass stars. On closer examination, though, this work presents only limited evidence of such a mass dependence for α_{OV} . Young et al. (2001) found that overshooting is needed to explain the apsidal motion of massive dEBs and that the best match to the observations may

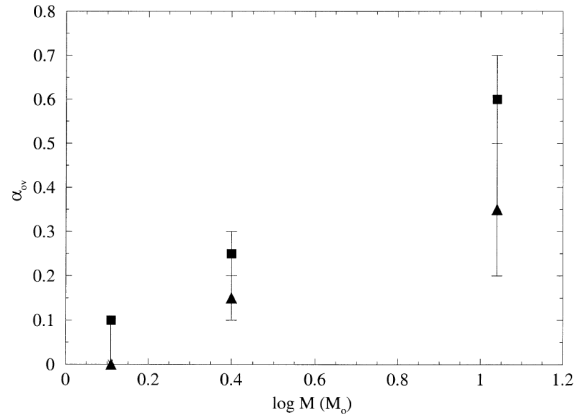


Figure 1.6: Plot of the best-fitting values of α_{OV} for dEBs against stellar mass. Taken from Ribas, Jordi & Giménez (2000).

require an α_{OV} dependent on mass.

Cordier et al. (2002) have presented evidence that α_{OV} depends on chemical composition, with larger metal abundances being accompanied by a smaller amount of overshooting (Figure 1.7). This result is not very robust and could be modified by the inclusion of other effects, such as rotation, in theoretical models (Cordier et al. 2002).

The existence of convective core overshooting seems to be accepted by most of the astronomical community, and it has been included as a free parameter (i.e., fixed at several values) in all major theoretical stellar evolutionary models since the late 1980s. Further work is required to increase our understanding of this effect; for example the ages of globular clusters have an uncertainty of 10% simply due to uncertainty in the treatment of convection in theoretical stellar models (Chaboyer 1995).

1.3.1.5 Convective efficiency

As convection in stars is very difficult to model successfully, the efficiency of convective energy transport in stellar envelopes is normally parameterised using the mixing length

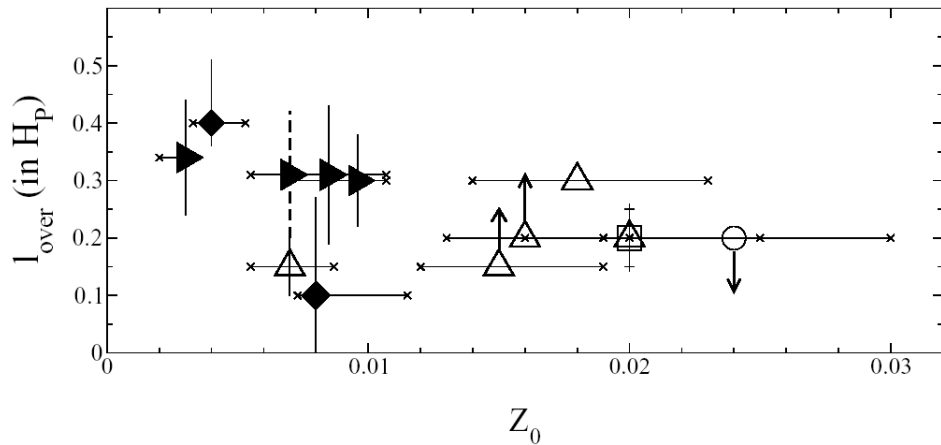


Figure 1.7: Variation of convective core overshooting parameter, α_{OV} , with fractional metal abundance, Z . Taken from Cordier et al. (2002).

theory (MLT) of Böhm-Vitense (1958). The parameter α_{MLT} is defined to be

$$\alpha_{MLT} = \frac{l_{\text{mixing}}}{H_p} \quad (1.16)$$

where l_{mixing} is the mixing length and H_p is the pressure scale height. Convective efficiency is proportional to α_{MLT}^2 (Lastennet et al. 2003).

MLT affects stars whose external layers are convective, which is between $B - V \approx 0.4$ (the boundary with a radiative envelope) and $B - V \approx 1.2$ (where adiabatic convection becomes dominant (Castellani et al. 2002)). In theoretical evolutionary models, α_{MLT} is generally calibrated using the Sun, the only star for which we have an accurate age. However, there is dispute over whether this is applicable to other stars. Fernandes et al. (1998) state that α_{MLT} is independent of mass, age and chemical composition, so that $\alpha_{MLT\odot}$ is valid for all low-mass Population I stars, but D’Antona & Mazzitelli (1994) note that $\alpha_{MLT\odot}$ is not directly relevant to other stars.

Ludwig & Salaris (1999) modelled the dEB AI Phoenicis and found α_{MLT} values which were larger than the solar value, but consistent within the uncertainties. Lastennet et al. (2003) found mixing length values for the component stars of the dEB

UV Piscium of $\alpha_{\text{MLT}}(\text{A}) = 0.95 \pm 0.12 \pm 0.30$ and $\alpha_{\text{MLT}}(\text{B}) = 0.65 \pm 0.07 \pm 0.10$ (where the uncertainties are random and systematic, respectively), which are significantly smaller than the solar value of approximately 1.6. These authors note that α_{MLT} may decrease with mass, and that it may even not be constant throughout the structure of one star. Palmieri et al. (2002) have investigated whether α_{MLT} is dependent on metallicity, but found no evidence for this. However, Chieffi, Straniero & Salaris (1995) have found evidence that α_{MLT} may depend on metallicity.

1.3.1.6 The effect of diffusion on stellar evolution

Diffusion occurs in radiative zones inside stars and is a result of different chemical species having different opacities and masses. Radiation pressure exerts a smaller force on species with lower opacity, and the gravitational force depends on the mass of the species. Because of this, some species are pushed outwards and other species settle inwards, causing chemical composition to vary throughout the radiative zone.

Diffusion causes surface chemical composition anomalies in A-type stars, which have radiative envelopes but less mass loss than more massive stars (lower-mass stars have convective envelopes), creating chemically peculiar objects such as Am, Ap and λ Boötis stars. Thus diffusion causes the spectroscopic chemical composition of stars to differ from the actual envelope chemical composition (e.g., Vauclair 2004)

Diffusion is an essential physical ingredient in theoretical models of the Sun. Whilst the solar envelope is convective towards the surface, the radiative lower layer undergoes diffusion processes. This affects the convective layer by changing the chemical abundances at the boundary between the two layers. The depth of a convective envelope depends on its chemical composition (R. D. Jeffries, 2004, private communication), so the radius of the Sun has a dependence on diffusion processes in the solar interior. Diffusion of hydrogen and helium must be included in solar models, and metal diffusion is also desirable (Weiss & Schlattl 1998).

Table 1.3: Some characteristics of the current generation of theoretical stellar evolutionary models.

Reference	Mass ($M_{\odot}$)	Y	Z	α_{MLT}	α_{OV}
Claret (1995)	1.0 to 40	0.380 0.280 0.180	0.020	1.52	0.20
Claret & Giménez (1995)	1.0 to 40	0.360 0.260 0.190	0.010	1.52	0.20
Claret (1997)	1.0 to 40	0.420 0.320 0.220	0.030	1.52	0.20
Claret & Giménez (1998)	1.0 to 40	0.346 0.252 0.196	0.004	1.52	0.20
Claret (2004a)	0.8 to 125	0.280	0.020	1.68	0.20
Schaller et al. (1992)	0.8 to 120	0.300 0.243	0.020 0.001	1.60	0.20
Schaerer et al. (1993a)	0.8 to 120	0.264	0.008	1.60	0.20
Charbonnel et al. (1993a)	0.8 to 120	0.252	0.004	1.60	0.20
Schaerer et al. (1993b)	0.8 to 120	0.264	0.040	1.60	0.20
Mowlavi et al. (1998)	0.8 to 60	0.480	0.100	1.60	0.20
Bressan et al. (1993)	0.6 to 120	0.280	0.020	1.63	0.50*
Fagotto et al. (1994a)	0.6 to 120	0.240 0.250	0.004 0.008	1.63	0.50*
Fagotto et al. (1994b)	0.6 to 120	0.230 0.352	0.0004 0.050	1.63	0.50*
Girardi et al. (1996)	0.6 to 120	0.230	0.0001	1.63	0.50*
Fagotto et al. (1994c)	0.6 to 9	0.475	0.100	1.63	0.50*
Girardi et al. (2000)	0.15 to 7	0.23 0.23 0.24 0.25 0.273 0.30	0.0004 0.001 0.004 0.008 0.019 0.030	1.68	0.50*
Pols et al. (1998)	0.5 to 50	0.240 0.240 0.242 0.248 0.260 0.280 0.300	0.0001 0.0003 0.001 0.004 0.01 0.02 0.03	2.00	0 and 0.12 [†]

* The overshooting formalism differs in the Padova theoretical models. Their overshooting of $\Lambda_{\text{OV}} = 0.50$ is equivalent to $\alpha_{\text{OV}} = 0.25$ (Bressan et al. 1993).

[†] The overshooting formalism in the Cambridge theoretical models is different to normal. Their overshooting of $\delta_{\text{OV}} = 0.12$ is equivalent to $\alpha_{\text{OV}} = 0.22$ and 0.40 for 1.5 and 7 $M_{\odot}$ stars.

1.3.2 Available theoretical stellar evolutionary models

Some of the most commonly used current theoretical models are detailed below. Some characteristics of the current models are given in Table 1.3.

1.3.2.1 Granada theoretical models

Claret & Giménez (1989) published a set of evolutionary calculations using a code based on that of Kippenhahn (1967). The opacities were taken from the Los Alamos group and the mixing length was $\alpha_{\text{MLT}} = 2.0$. Five chemical compositions were considered and the internal structure constants were given.

Claret & Giménez (1992) updated their previous study by adopting the opacities of OPAL (Iglesias & Rogers 1991). The mixing length was $\alpha_{\text{MLT}} = 1.5$, overshooting was included with $\alpha_{\text{OV}} = 0.2$, and four chemical compositions were given. Internal structure constants were also calculated (section 1.7.2) and mass loss was incorporated.

The current set of theoretical models were published by Claret (1995, 1997) and Claret & Giménez (1995, 1998) and their characteristics are given in Table 1.3. One major advantage of these calculations is that three helium abundances are available for each of the four metal abundances.

Updated theoretical models have been given by Claret (2004a) for an approximately solar chemical composition only. They are optimised for comparison with the properties of dEBs. The effects of stellar rotation have been included.

1.3.2.2 Geneva theoretical models

The Geneva models were developed by Maeder (1976, 1981; Maeder & Meynet 1989). The current generation of theoretical models were introduced by Schaller et al. (1992) and are currently by far the most popular with astrophysicists, with over 1400 citations for the Schaller et al. work alone. They use the opacities of Rogers & Iglesias (1992); characteristics and successive references are given in Table 1.3. Additional consideration has been given to massive star evolution with high mass loss rates (Meynet et

al. 1994), evolved intermediate-mass stars (Charbonnel et al. 1996) and an alternative magnetohydrodynamical equation of state for low-mass stars (Charbonnel et al. 1999).

1.3.2.3 Padova theoretical models

The main rivals to the Geneva models have been developed by the Padova group, culminating in Alongi et al. (1993). The next generation, which remains the current generation for the massive stars, was initiated in Bressan et al. (1993) and uses the OPAL opacities. Further works are given in Table 1.3. The overshooting formalism is different to that in other models in that it is calculated *across* rather than *above* the convective boundary (Girardi et al. 2000). More recent model predictions have been given by Girardi et al. (2000) for masses between 0.15 and $7 M_{\odot}$.

1.3.2.4 Cambridge theoretical models

The original models were produced by Eggleton (1971, 1972; Eggleton, Faulkner & Flannery 1973) and incorporate a simple equation of state which allows evolutionary calculations to be relatively inexpensive in terms of computing time (Pols et al. 1995). The models have been extensively tested using the astrophysical properties of dEBs, and moderate convective core overshooting has been found to best fit the observations (Pols et al. 1997).

The current generation of theoretical models (Pols et al. 1998) uses OPAL opacities. Convective core overshooting is formulated differently to other evolutionary codes; the adoption of δ_{OV} is equivalent to $\alpha_{OV} = 0.22$ and 0.4 for 1.5 and $7.0 M_{\odot}$ stars, respectively. This implicitly includes a mass dependence in α_{OV} . Commendably, the Cambridge models are available both with and without convective core overshooting over their entire mass range. Details of the models are given in Table 1.3.

Analytical formulae which reproduce the results of the models (but are approximations) are given in Hurley, Pols & Tout (2000).

1.3.3 Comments on the currently available theoretical models

Several approximations and parameterisations of complicated physical phenomena allow the construction of theoretical models which are very successful at reproducing the bulk physical properties of many types of stars. However, these approximations and parameterisations are masking a lack of knowledge of the underlying physical processes, and can introduce ‘theoretical uncertainties’ into the results of research which uses predictions from models. Several parameters are set at specific values and published model predictions are not available for alternative values. For example, of the current generation of models only the Cambridge predictions are available both with and without convective core overshooting, and only the Claret (1995) models are published with more than one value of helium abundance for a given metal abundance. This can make it difficult for observational astrophysicists to investigate variations in these parameters. From my own experience I feel that a finer sampling in mass and metal abundance would also be desirable, to reduce the problems associated with interpolating between different predictions. This could easily be managed given the current quality and quantity of the computational resources available to researchers.

1.4 Spectral characteristics of stars

1.4.1 Spectral lines

Early-type stars have relatively few optical-wavelength spectral lines whereas late-type stars have many lines. The blue part of the spectrum is the optical region with the most spectral lines. The phenomenon of ‘line blanketing’ arises when this region contains a sufficient number of lines to significantly affect the amount of flux emitted by the star from over these wavelengths. The flux is redistributed to longer wavelengths and is emitted in the red part of the spectrum, affecting the spectral energy distribution of the star (e.g., Kubát & Korčáková 2004). This effect can cause the T_{eff} of O stars derived from spectral energy distributions to change by up to 3000 K (Mokiem et al.

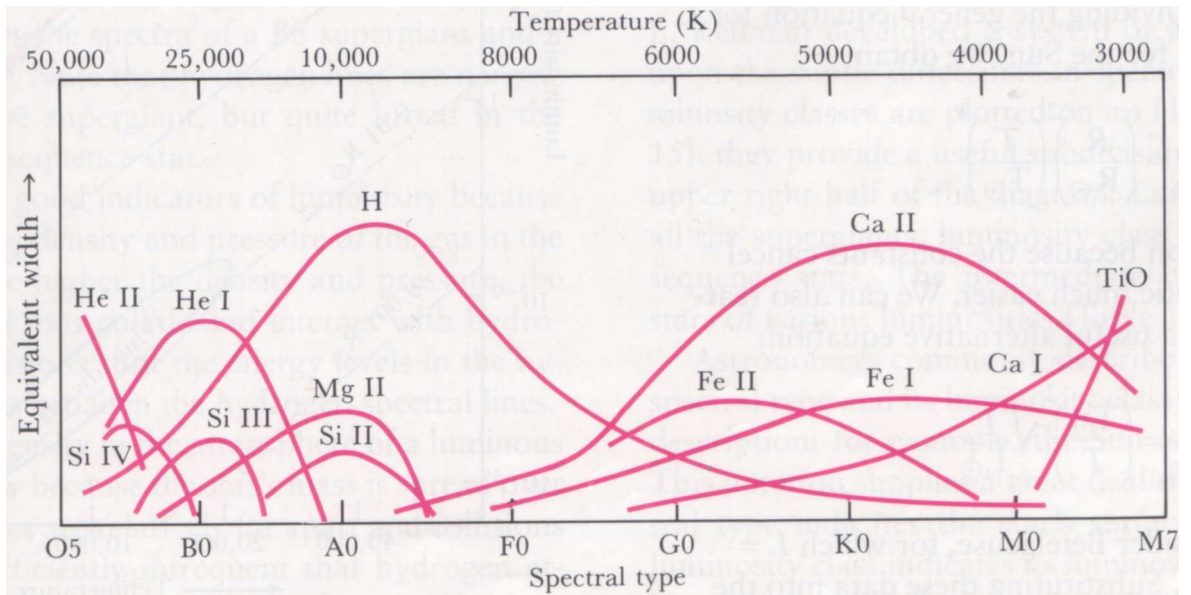


Figure 1.8: The variation of equivalent widths of some important spectral lines with T_{eff} . Taken from Kaufmann (1994, p. 351).

2004). A similar blanketing effect due to stellar winds is significant in very hot stars (Kudritzki & Hummer 1990). It can also have an effect on the temperature structure of a star due to the ‘backwarming’ effect (Smalley 1993).

The spectral classification of stars depends on the relative strengths of different lines in their spectra. A representation of how the strengths of some spectral lines vary over T_{eff} is given in Figure 1.8.

Spectral atlases to aid the classification of stars have been given by Walborn (1980; optical spectral atlas of early-type stars), Walborn Nichols-Bohlin & Panek (1984; ultraviolet atlas for hot stars), Walborn & Fitzpatrick (1990; OB stars), Kilian, Montenbruck & Nissen (1991; early-B stars), Carquillat et al. (1997; infrared atlas for late-type stars), Walborn & Fitzpatrick (2000; peculiar early-type stars) and on the

internet by R. O. Gray¹.

1.4.2 Stellar model atmospheres

Atmospheric models of stars simulate the conditions in a stellar photosphere and predict the variation of the physical conditions throughout the photosphere as a function of optical depth (Gray 1992, p. 146). Important physical conditions include temperature, pressure, density, geometrical depth and various plasma velocity characteristics. These results can then be used to interpret the characteristics of observed stellar spectra in terms of the physical conditions in the outer layers of the star.

Most model atmospheres are calculated with the assumption of local thermodynamic equilibrium (LTE), where the electronic energy level populations of atomic species are dependent entirely on collisional excitation. The Saha equation (which expresses ionisation equilibrium; see Zeilik & Gregory p. 167) and the Boltzmann equation (which expresses excitation equilibrium; see Zeilik & Gregory p. 166) can then be used to determine the excitation and ionization characteristics of the species present.

If radiative excitation and ionisation becomes significant compared to collisional excitation and ionisation then the assumption of LTE breaks down. The excitation and ionization of atomic species depends on both the radiation and the collisional pressure, but unfortunately the amount of radiation pressure depends on the excitation and ionization characteristics of the plasma. Model atmospheres which do not assume LTE are complex, so a large number of iterative calculations are required in order to construct them. The assumption of LTE breaks down above between roughly 10 000 and 20 000 K, depending on surface gravity and on how tolerant the researcher is of the inaccuracy incurred by assuming LTE.

¹<http://nedwww.ipac.caltech.edu/level5/Gray/frames.html>

1.4.2.1 The current status of stellar model atmospheres

The first of the modern generation of theoretical model atmospheres are the ATLAS models which were produced by Kurucz (1979). These are plane-parallel LTE models; they do not contain any contribution to opacity from molecules so significant systematic errors appear at T_{eff} s below about 6000 K (Smalley & Kupka 1997). The currently most popular version of the Kurucz atmospheres is ATLAS9 (Kurucz 1993b) and more details can be found on R. L. Kurucz's homepage². The main competition to the Kurucz models is the MARCS model atmospheres developed by the Uppsala (Sweden) group (Gustafsson et al. 1975; Asplund et al. 1997).

The first non-LTE model atmospheres were produced by Auer & Mihalas (1972) and Kudritzki (1975, 1976) but these were relatively unrealistic as they did not contain metals (Massey et al. 2004). Several more recent non-LTE model atmospheres have been successfully used to interpret the spectra of hot stars. These model atmospheres employ spherical geometry and include the effects of line blanketing and stellar winds, so are far more advanced than the atmospheres of Kurucz. They include FASTWIND (Santolaya-Rey, Puls & Herrero 1997), CMFGEN (Hillier & Miller 1998) and WM-BASIC (Pauldrach, Hoffmann & Lennon 2001).

1.4.2.2 Convection in model atmospheres

Models of stellar atmospheres are similar to evolutionary models of stars (section 1.3) in that convection must be accounted for to provide a more realistic description of the stellar properties. The overshooting of convection zones in the envelope and the efficiency of convective energy transport are both important for stars with $T_{\text{eff}} \lesssim 8500$ K (Smalley 2004). The treatment of convection affects the photometric colours of stars calculated using theoretical model atmospheres (Smalley 1996).

Mixing length theory (MLT, section 1.3.1.5) is commonly used to model convective effects but MLT model atmospheres are generally unable to match the observed

²<http://kurucz.harvard.edu/>

helioseismological oscillation frequencies (Kupka 1996). The Kurucz (1993b) ATLAS9 model atmospheres optionally employ ‘approximate overshooting’, which is more successful in matching some observations (Castelli, Gratton & Kurucz 1997) but not others (Smalley & Kupka 2003). The Canuto & Mazitelli (1991, 1992) turbulent convection theory has been implemented into ATLAS9 by F. Kupka and is generally an improvement on MLT and approximate overshooting (Montalbán et al. 2001; Smalley & Kupka 2003; Smalley 2004).

1.4.2.3 The future of stellar model atmospheres

Model atmospheres are in need of a much better treatment of convection (Kurucz 1998). One-dimensional model atmospheres cannot reproduce convective stellar atmospheres (Kurucz 1998). There is a need for greater knowledge of the energy levels of atoms and ions so more complete spectral line lists can be constructed (Kurucz 2002a). Molecular opacity is also an area where a large amount of work is required – for example, R. L. Kurucz uses line lists for the H₂O and TiO molecules with 38 million and 66 million lines respectively (Kurucz 2002a). CH₄ is even more complex but is important in the study of the characteristics of brown dwarfs and planets. Kurucz (2003) states that “We can produce more science by investing in laboratory spectroscopy rather than by building giant telescopes that collect masses of data that cannot be correctly interpreted.” Kurucz (2002b) states that microturbulence velocity is not constant even in one star, and that half the lines in the spectrum of the Sun remain unidentified.

The effects of magnetic fields have been included in model atmospheres for B and A stars by Kochukhov, Khan & Shulyak (2005), who find that energy transport, diffusion and line formation are significantly modified. They note that the effect of a magnetic field on metal lines can be approximated by a ‘pseudo-microturbulence’.

Three-dimensional hydrodynamical model atmospheres are being developed by several research groups (see Ludwig & Kučinskas 2004). The advantage of these models is that convective energy transport can be modelled directly, so microturbulence and macroturbulence are no longer required (Asplund, Grevesse & Sauval 2004). Mixing

length theory is also bypassed, so the parameter α_{MLT} (section 1.3.1.5) is no longer relevant and the predictive capability of the atmospheres is enhanced. Synthetic spectra calculated using current hydrodynamical model atmospheres provide an ‘almost perfect’ match to the solar spectrum (Ludwig & Kučinskas 2004). The drawback is that a typical three-dimensional hydrodynamical model atmosphere requires about 100 grid points per dimension and some resolution in wavelength, so a lot of computer processor time is required to perform the calculations (of the order of one month for one atmosphere using a desktop PC; Ludwig & Kučinskas 2004).

1.4.3 Calculation of theoretical stellar spectra

Once a theoretical model atmosphere has been constructed for a star, the formation of spectral lines can be modelled using the atmospheric conditions derived using the theoretical model. Apart from a theoretical model atmosphere, the calculation of synthetic spectra requires detailed lists of spectral lines and their characteristics.

Synthetic spectra can be compared to observed spectra to derive the atmospheric parameters of stars. The main problem with this approach is that synthetic spectra are calculated using model atmospheres, so the resulting T_{eff} s, surface gravities and chemical abundances are dependent on theoretical calculations. This problem should usually be quite minor because model atmospheres are generally successful, and many T_{eff} s in the literature are on the T_{eff} scale of the ATLAS9 model atmospheres.

For B and early-A stars, the hydrogen Balmer lines are sensitive both to T_{eff} and to surface gravity, and a $T_{\text{eff}} - \log g$ diagram will have an almost straight line of best fit (Kilian et al. 1991) pointed towards increasing T_{eff} and increasing $\log g$. This degeneracy can be broken by including silicon lines or helium lines in the analysis to provide a measure of T_{eff} through the ionisation balances. The degeneracy can also be avoided if the analysed star is in an EB because its surface gravity may then be determined accurately.

For stars with $T_{\text{eff}} \lesssim 8000$ K the Balmer lines have very little dependence on $\log g$ so can provide accurate T_{eff} values, if there are few enough metal lines for the Balmer

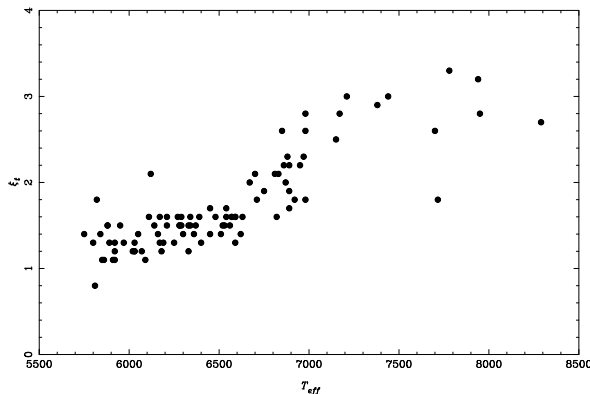


Figure 1.9: Variation of microturbulence with T_{eff} . Taken from Smalley (2004).

line shapes to be well defined, as the Balmer lines are formed at a wide range of depths in stellar atmospheres (Smalley 1996; Smalley & Kupka 2003).

1.4.3.1 Microturbulence velocity

Microturbulence is an effect which is generally required to improve the match between synthetic spectra and observed stellar spectra. It is a line-broadening mechanism caused by small-scale turbulent motions in the photospheres of stars, and in the Sun may result from granulation (Smalley 2004). Microturbulence was originally introduced to make elemental abundances derived from weak and strong spectral lines of the same species agree (Smalley 1993). It can be determined by forcing the abundances from strong and weak lines to agree. Microturbulence increases the widths of spectral lines so has an effect on the opacity in stars (Kurucz 2002b).

A microturbulent velocity of about 2 km s^{-1} is generally found for B and A dwarfs (Smalley 1993; Figure 1.9), but more evolved stars have larger microturbulent velocities (Lennon, Brown & Dufton 1988) which can be up to 12 km s^{-1} for B-type giants (Rolleston et al. 2000). Magain (1984) noted that observational errors generally cause an increase in a derived value of microturbulence.

Non-LTE model atmospheres have been claimed not to need microturbulence (Becker & Butler 1988), but Gies & Lambert (1992) found that microturbulence *is* important in non-LTE analyses (Smartt & Rolleston 1997). Trundle et al. (2004) also find that microturbulence is required when using non-LTE codes.

Hydrodynamical model atmospheres directly simulate convective effects so render the concepts of microturbulence and macroturbulence obsolete (Asplund, Grevesse & Sauval 2004), and are very successful in matching the observed line profiles of stars (Ludwig & Kučinskas 2004).

1.4.3.2 The UCLSYN spectral synthesis code

The UCLSYN (University College London SYNthesis) code uses theoretical model atmospheres and atomic data to calculate synthetic spectra. It also has a binary-star mode (BINSYN) for composite spectra and can calculate telluric-line spectra (TELSYN). UCLSYN was produced by Smith (1992) and is maintained by B. Smalley (Smalley, Smith & Dworcesky 2001). The LTE ATLAS9 model atmospheres of Kurucz (1993b) are used along with the atomic line lists of Kurucz & Bell (1995). The profiles of some of the helium lines are calculated using the work of Barnard, Cooper & Shamey (1969) and Shamey (1969), with $\log gf$ values from Wiese, Smith & Glennon (1966).

1.4.4 Spectral peculiarity

The atmospheres of A-type stars are relatively quiet because they do not have significant winds, like O and B stars, or convection, which occurs in stars later than F0 (Kubát & Korčáková 2004). There is also a large range of formation depths for spectral lines in A stars (Kubát & Korčáková 2004). Most A stars which do not rotate quickly therefore develop peculiar spectra due to elemental diffusion, gravitational settling or the presence of magnetic fields, but are believed to have essentially the same atmospheric structure as normal stars (Bikmaev et al. 2002). Element settling is now included in many theoretical stellar evolutionary codes in order to explain spectrally

peculiar stars (section 1.3.1.6; Vauclair 2004).

1.4.4.1 Metallic-lined stars

Metallic-lined stars (often referred to as Am stars) are dwarfs of spectral types between A4 and F0 (Popper 1971) which show weak calcium and scandium spectral lines but enhanced lines of other metals. The F0 cut-off is linked to the onset of surface convection (Smalley & Dworetsky 1993). The first Am stars were discovered in the Pleiades by Titus & Morgan (1940) as a group of A stars for which spectral types found from the calcium lines and from the metallic lines were earlier and later, respectively, than those found from the Balmer lines. ρ Puppis stars are defined to be subgiant and giant Am stars (Frémat, Lampens & Hensberge 2005).

Am stars have rotational velocities below about 100 km s^{-1} (Budaj 1996); they are often members of short-period binary systems because these stars have the rotation slowed by tidal interactions (Smalley 1993; Abt & Morrell 1995; Budaj 1996, 1997). Am stars appear slightly redder than expected for their Balmer-line spectral types because their enhanced metal lines cause increased line blanketing.

Am stars have often been found to be slightly evolved (e.g., Kitamura & Kondo 1978) but Dworetsky & Moon (1986) found that Am stars in clusters had surface gravities which were negligibly different to those of normal A-stars. The Am phenomenon ends at $\log \approx 3.05$ due to the onset of convection (Richer, Michaud & Turcotte 2000).

The metallic-lined phenomenon in stars is caused by diffusion and gravitational settling which cause metallic ions and atoms to migrate towards the stellar surface. The phenomenon can therefore be likened to a ‘skin disease’ (J. Andersen, 2004, private communication) in which the surface chemical composition does not reflect the interior chemical composition of the star. Several well-studied dEBs show metallic-lined spectral characteristics, so their properties can be used to shed light on the Am phenomenon. Fig. 1.10 compares the characteristics of metallic-lined dEB components to those which exhibit normal spectra. There is no obvious region in parameter space where all stars are Am, which is consistent with the phenomenon being a surface effect

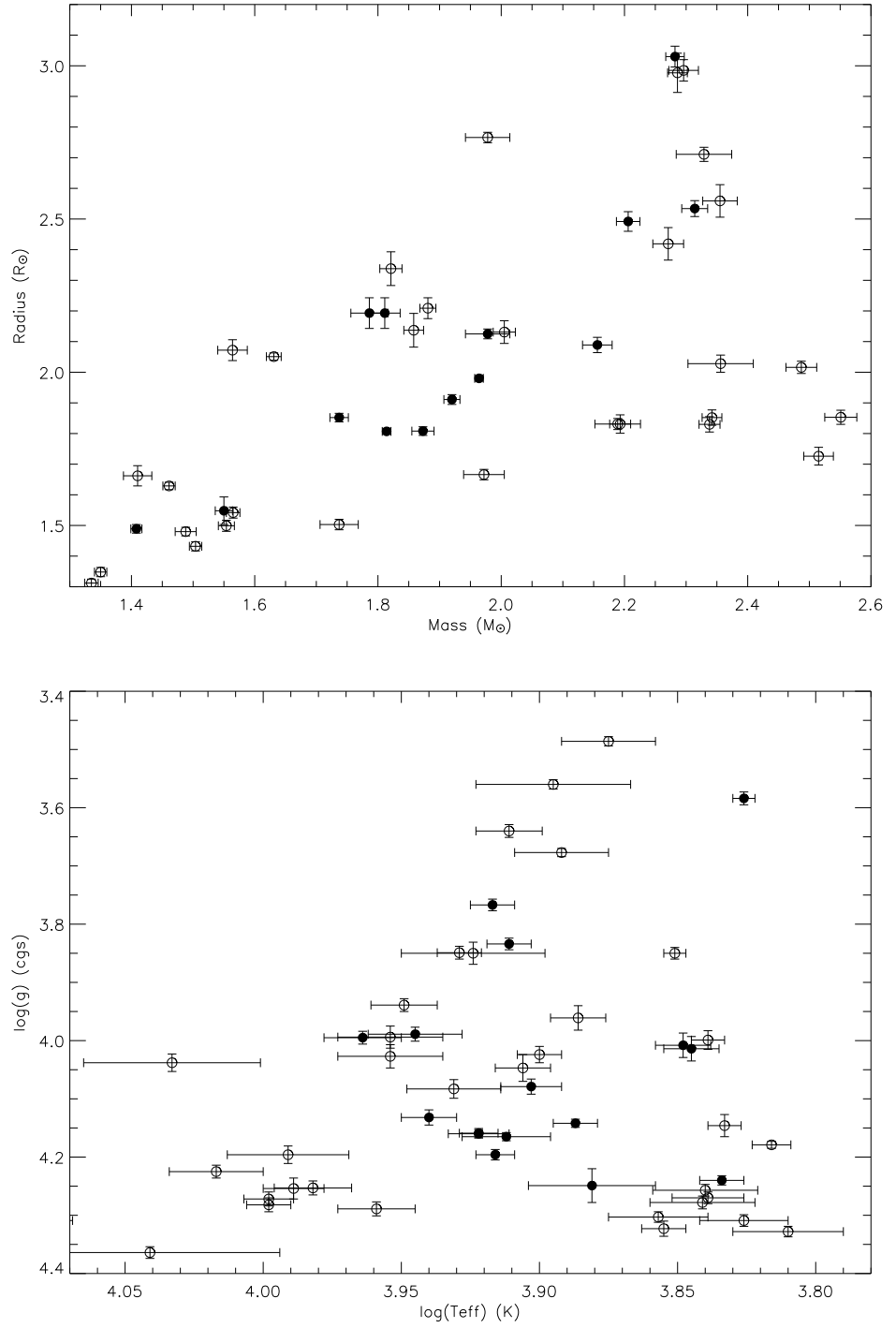


Figure 1.10: Mass–radius and temperature–gravity plots of the components of well-studied dEBs with normal spectra (open circles) and with metallic-lined spectra (filled circles). Data have been taken from Andersen (1991) and updated with the results of more recent studies.

which depends partially on physical properties which have not been considered here.

1.5 Multiple stars

The processes by which stars form naturally also create systems which contain two or more stars. Data on the multiplicity of stars can be used to constrain the theories of the formation of star clusters, single stars, and of other celestial objects such as planets. The evolution of stars in multiple stellar systems can be very different to the evolution of single stars. Close binary systems are the sole progenitors of many exotic objects, so their study and characterisation can be very rewarding. The study of binary systems can also be regarded as a window through which we can study single stars.

The reasons for studying the characteristics of multiple stars include:–

- constraining star formation theory by statistical study of the distribution of orbital elements (e.g., Mazeh et al. 1992)
- characterisation of large stellar populations and the light they produce (which contains a significant contribution from objects which are only formed by interaction between stars in a binary system)
- finding the age of large stellar systems from comparison of the eccentricities of binary systems with tidal evolution theories (section 1.7.1.3)
- investigating the physics of the evolution of close binary star systems
- calibrating the mass–luminosity relation (Duquennoy & Mayor 1991)
- finding high-mass stellar remnants (Duquennoy & Mayor 1991)
- constraining how our Galaxy formed (Duquennoy & Mayor 1991)
- multiple stars play an important role in the evolution of gravitationally bound stellar systems (Mermilliod et al. 1992)

The evolution of the components of binary systems is different to the evolution of single stars which are otherwise similar. This phenomenon seems to arise during formation, where a binary system may have quite different energy characteristics to a single star (Tohline 2002). This is manifested in the fact that even stars in young binary systems rotate more slowly than single stars of the same type (e.g., Levato & Morrell 1983). During evolution as a detached binary, the presence of a companion star affects evolution through tidal effects (which modify the rotational characteristics of the star), irradiation (the reflection effect) and mass transfer in close binary systems. The conditions under which this becomes significant are not accurately known and will not be the same for different research projects.

1.5.1 Binary star systems

Binary star systems present many possibilities for discovering the physical laws which govern the existence of stars. Direct measurements of the characteristics of stars can be made by studying several different types of binary system.

Visual binaries are long-period binary systems which are situated sufficiently close to the Earth that the individual component stars can be observed separately. With the current generation of stellar interferometers, many more binary systems fall into this category, although some researchers call these “interferometric binaries”. Knowledge of the positions of the stars on the sky, as a function of orbital phase, coupled with radial velocity (RV) observations, allow the masses of the stars to be measured directly, along with their luminosity ratio. These stars are therefore good for determining the mass–luminosity relation of stars, but, more importantly, they provide an essentially geometric determination of the distance to the system which is very reliable (Paczynski 2003). Perhaps the best-known studies of such stars allowed Torres, Stefanik & Latham (1997a, 1997b, 1997c) to determine the distance of the Hyades open cluster to be 47.6 ± 1.1 pc from analysis of the visual binaries 51 Tauri, 70 Tauri, θ^1 Tauri and θ^2 Tauri. The data for these visual binaries were also compared to stellar evolutionary models to derive an age and metal abundance from their absolute masses and luminosities.

Spectroscopic binary systems are those for which their binarity is apparent from variation of their RV. The secondary component may also produce spectral lines strong enough to be visible in the spectrum of the system, in which case the spectroscopic binary is “double-lined”. Spectroscopic observations of these systems allows calculation of the orbital period and eccentricity, the mass ratio, and the minimum masses of the components, $M \sin^3 i$, where i is the inclination of the orbit relative to the line of sight of the observer (see section 2.2). These can be studied statistically to constrain tidal evolution theories (section 1.7) but the other uses are minor. It is useful to know which stars are binary when studying the photometric properties of open clusters (section 1.8).

1.5.2 Eclipsing binary systems

Eclipsing binaries (EBs) consist of two stars whose orbit periodically causes one star to eclipse the other, as seen from Earth. As the other star also eclipses the first star once per orbit (except for a few EBs which have very eccentric orbits and orbital inclinations significantly below 90° , e.g., NY Cephei, Holmgren et al. 1990), there are two different eclipses for every orbital period. EBs are classified into a wide variety of types, depending on their evolutionary status and light curve shape.

As approximately 0.2% of binary stars are EBs, it is expected that about 5×10^6 exist in the Milky Way Galaxy, of which about four thousand have been discovered (Guinan 2004). The Hipparcos space satellite found 917 nearby EBs, of which 347 were previously undiscovered (Perryman et al. 1997).

W Ursae Majoris systems are very close binaries composed of two stars which are in contact with each other at the inner Lagrangian point. The absolute masses and radii of W UMa systems can be derived from light curves and RV curves, but the photometric mass ratio and orbital inclination are strongly correlated unless the eclipses are total (Kaluźny & Thompson 2003). They are quite common EBs.

Algol systems are created from a close binary consisting of two MS stars. The more massive component evolves past the TAMS, increases in radius and overflows its Roche Lobe. The secondary star accretes much of the mass lost by the primary star,

and becomes more massive. Algol thus consist of an evolved low-mass star (usually a subgiant), which fills its Roche Lobe, orbiting an early-type MS star. They are relatively common and have mass ratios of the order of 0.3 (Hilditch 2001, p. 288).

Detached eclipsing binaries (dEBs) are composed of two stars which have not interacted by mass transfer and are effectively gravitationally bound single stars. They differ from single stars in their formation (Tohline 2002), and due to tidal interactions, mutual irradiation and interception of each other's stellar winds. dEBs for which these effects are negligible are very important because they allow the direct measurement of absolute masses, radii, T_{eff} s and luminosities of stars which have evolved as single stars. A full characterisation of a dEB requires a significant number of RVs to determine a spectroscopic orbit and a large number of photometric observations to derive the radii of the stars (Figure 1.11). These systems will now be discussed further.

Many exotic objects are exclusively binary systems, for example PG 1336-018, an EB with a period of 0.10 days, containing a pulsating sdB star (Kilkenny et al. 1998). Subdwarf B stars are composed of $0.5 M_{\odot}$ helium cores covered by a thin envelope of hydrogen, and are thought to be created from red giants which lose their envelope due to binary interactions (Maxted et al. 2000) or strong winds.

1.6 Detached eclipsing binary star systems

Double-lined dEBs are of fundamental importance to astronomy and astrophysics as they represent one of the main links between theoretical stellar astrophysics and what happens in the real world (Andersen 1991). Excluding the Sun and a few nearby visual binaries, dEBs are the only systems from which accurate and absolute stellar masses can be found. Accurate absolute stellar radii can also be determined using entirely empirical methods, and they are also amenable to determination of photospheric metal abundance using the same techniques as for single stars. The derivation of accurate T_{eff} s can be more tricky (section 1.4.3), but this knowledge allows the calculation of the luminosities of the two stars and, ultimately, the distance (section 1.6.3).

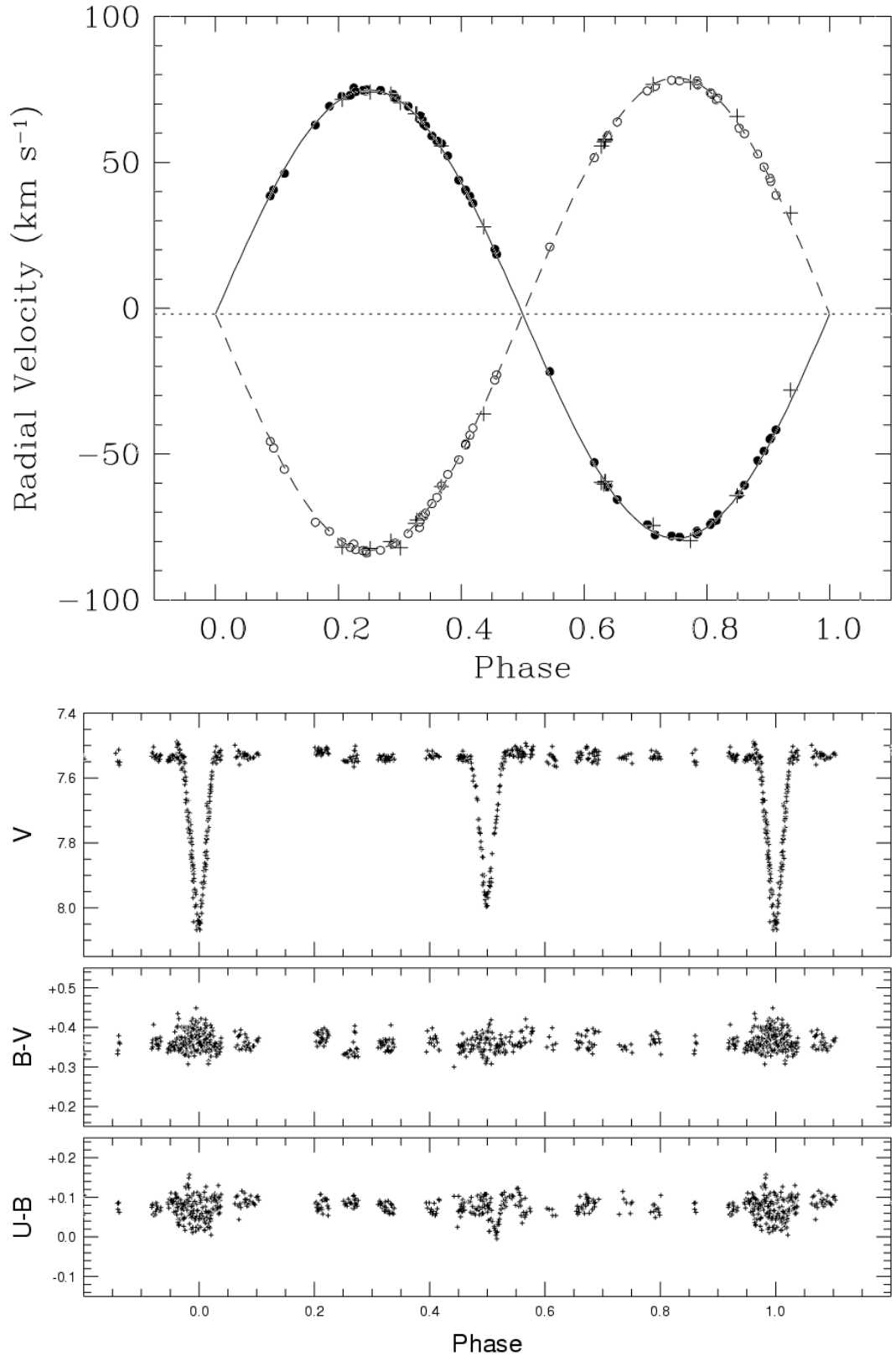


Figure 1.11: Example RV curve (top) and light curves (below) of the dEB V364 Lacertae. RVs for the primary and the secondary stars are given by filled symbols and open symbols, respectively. Taken from Torres et al. (1999).

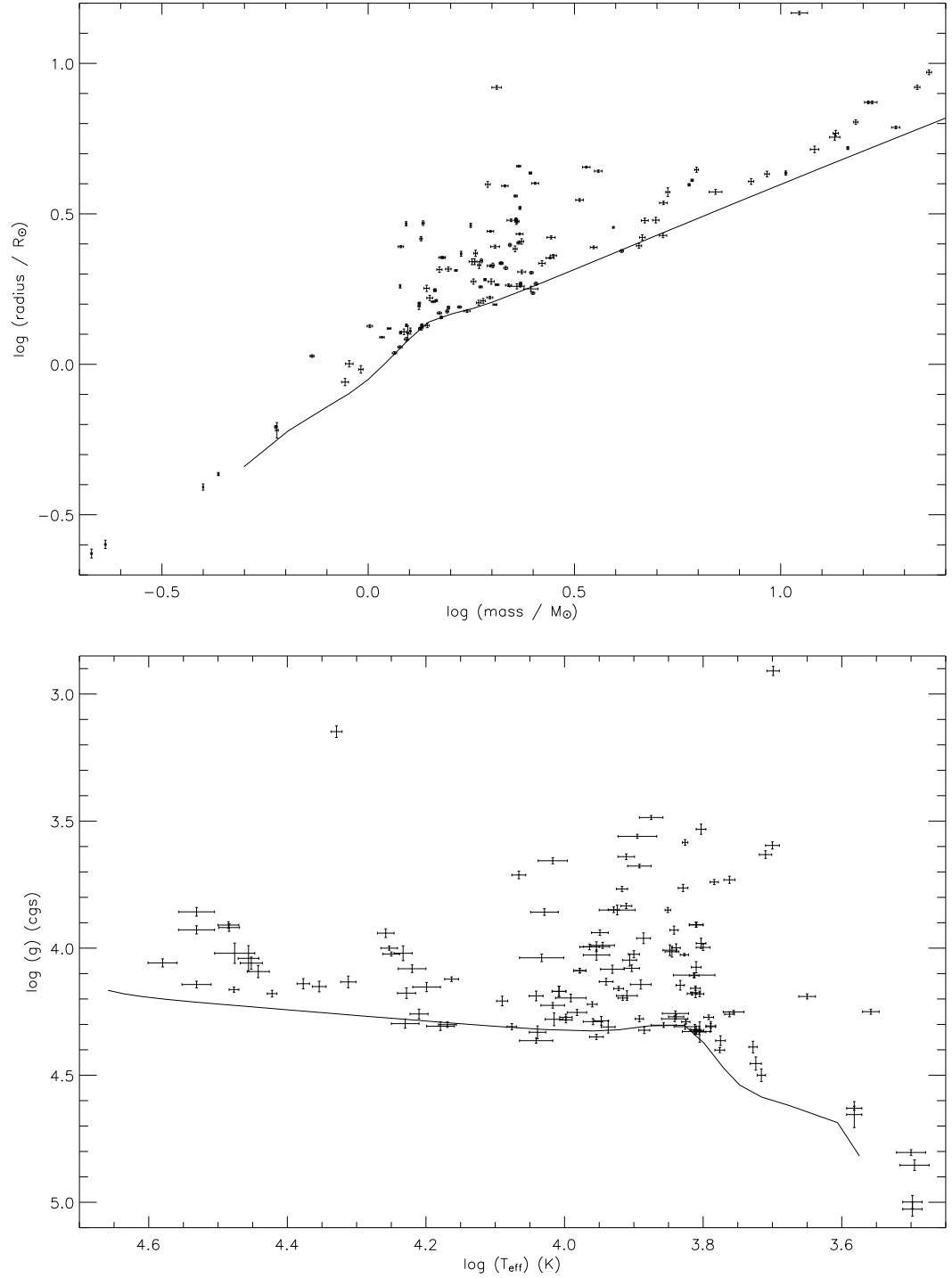


Figure 1.12: Logarithmic mass–radius and temperature–gravity diagrams containing the components of well-studied dEBs. Uncertainties are shown as errorbars and the theoretical ZAMS for a solar composition, taken from the Cambridge stellar evolutionary models (Pols et al. 1998), is given by a solid line.

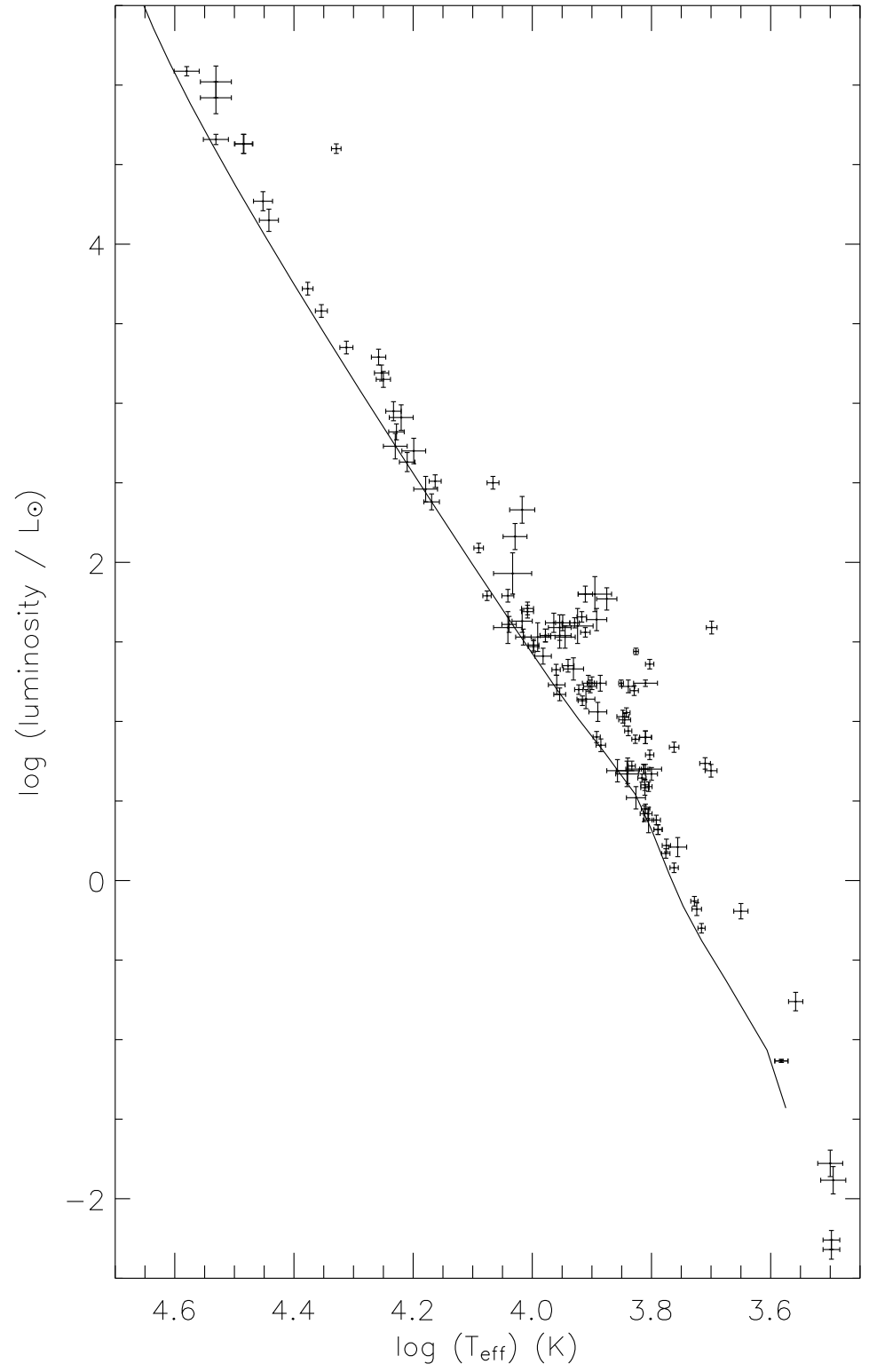


Figure 1.13: HR diagram containing the components of well-studied dEBs. Symbols are as in Figure 1.12.

Excellent reviews of the then-available data on dEBs, techniques for their observation and analysis and general results obtained from their study, have been published by Popper (1967, 1980) and by Andersen (1991). Harmanec (1988) has collected an exhaustive database of the absolute dimensions of dEBs. Whilst the reviews of Popper (1967, 1980) concentrated on the determination of stellar masses and radii, the celebrated work of Andersen (1991) considered only those dEBs for which the masses and radii were known with uncertainties below 2% and effective temperatures to within 5%, the final total being 45 dEBs (containing 90 stars). The reason for the rejection of data on dEBs with more uncertain parameters is that such systems generally have only a limited use compared to the most well-studied dEBs (Andersen, Clausen & Nordström 1980, 1984; Andersen 1993, 1998). Knowledge of the dimensions of a dEB to within 5% is no longer in general useful (e.g., Andersen 1991, Giménez 1992).

Whilst there are a good number of well-studied MS dEBs of spectral types between B and G, very few exist outside these boundaries. Whilst several O star dEBs have been studied (e.g., V1007Scorpii, Sana, Rauw & Gosset 2001), these systems exhibit complications which makes determination of accurate parameters very difficult. dEBs known to contain K or M dwarfs are very rare as the small sizes of these stars means that few exhibit deep eclipses (Popper 1993). Starspots can also be problematic when analysing the light curves of such systems (see e.g., Torres & Ribas 2002; Ribas 2003). There is also a shortage of dEB components which are close to the ZAMS (Andersen 1991, Giménez 1992), particularly for high-mass stars, and beyond the TAMS (with the important exceptions of the giant system TZ Fornacis, Andersen et al. 1991, and SZ Centauri, Andersen 1975c). This is because dEBs which contain an evolved star tend to exhibit single-lined spectra as the unevolved star is a lot dimmer than the evolved star. Therefore dEBs which contain an evolved star but are double-lined must have a mass ratio close to unity (Andersen 1975c), so are very rare. Recent work has begun to focus on dEBs containing substellar objects (e.g., 2MASS J0516288+260738, which appears to be an eclipsing M dwarf – brown dwarf system, Schuh et al. 2003).

The available data on dEBs with masses and radii accurate to 2%, and effective temperatures to within 5%, has been collected from Andersen (1991). Results from

more recent publications have been added, with an emphasis on the inclusion of interesting dEBs rather than those which conform precisely to the above limits on accuracy. These have been plotted Figures 1.12 and 1.13.

There are two main uses of the fundamental astrophysical parameters of dEBs: as calibrators and checks of theoretical models, and as standard candles.

Knowledge of the masses and radii of the components of dEBs has allowed Ribas et al. (1997) to construct photometric calibrations which predict the masses and radii of single stars using Strömgren-Crawford photometric indices (section 2.3.1.3). This study updates the calibration contained in the UVBYBETA code of Moon & Dworetzky (Moon 1985), which was based on finding the absolute magnitude and surface brightness of a star in order to predict its radius. Calibrations of surface brightness could be aided by study of the dEBs suggested by Kruszewski & Semeniuk (1999).

1.6.1 Comparison with theoretical stellar models and atmospheres

The basic stellar properties are mass, radius, luminosity and chemical composition (Andersen 1991). In principle, the mass and chemical composition determine all other stellar properties throughout the lifetime of a star, but the predictions of stellar evolutionary theory are not completely reliable and so must be tested by comparison with observed properties of stars (Andersen 1991). This is because many physical processes are simplistically treated (e.g., convection, mass loss and magnetic activity) and some atomic data (e.g., reaction rates and opacities) are poorly determined.

Theoretical stellar evolutionary models are usually calibrated to predict the radius and T_{eff} of the Sun given its known mass, age and approximately known chemical composition. They are therefore very successful at predicting the properties of solar-type stars. Extension to higher masses, however, depends a lot on the observed properties of well-studied dEBs. Theoretical models for stars much less massive than the Sun can be extremely complex, and the current generation of models do not show a good agreement with each other and with the few well-studied dEBs in this mass

range (e.g., Maceroni & Montalbán 2004). A particular advantage of dEBs is that accurate masses, radii and T_{eff} s can be found for two stars which have a common age, initial chemical composition and distance (according to most star formation theories). This provides a more detailed test of theoretical predictions, as models must match the astrophysical properties of both stars for the same age and chemical composition.

The predictions of stellar evolutionary models are often calibrated, or checked, with the use of accurate astrophysical parameters of dEBs (e.g., Claret 1995; Pols et al. 1995). In particular, the amount of convective core overshooting to use has sometimes been decided using studies of dEBs (e.g., Pols et al. 1997; Hurley, Pols & Tout 2000; Ribas, Jordi & Giménez 2000). Other physical effects incorporated into theoretical models for which the study of dEBs may provide constraints include opacity, mass loss, and characteristic mixing lengths (Shallis & Blackwell 1980).

The use of spectral disentangling (section 2.2.3.4) has made it more straightforward to critically test the success of model atmospheres in predicting stellar spectra. The study of dEBs provides a fundamental and accurate determination of the surface gravity of both stars. The other main atmospheric parameter, T_{eff} , can be inferred in several ways. Given these properties, model atmospheres should enable the calculation of theoretical spectra which are in good agreement with the individual spectra of the two stars, found by disentangling the observed composite spectra (B. Smalley, 2004, private communication; Ribas 2004).

1.6.1.1 The methods of comparison

Once the properties of a dEB have been accurately calculated, they can be compared to the predictions of stellar models. As the two stars are expected to have the same age and chemical composition, stellar models should be able to simultaneously fit their properties for one age and composition. Further constraints can be provided by knowledge of the central concentrations of the stars (from apsidal motion studies) and by the derivation of the chemical compositions of the stars from high-resolution spectroscopic observations (Andersen 1993, 1998; see for example Ribas, Jordi & Torra 1999). The

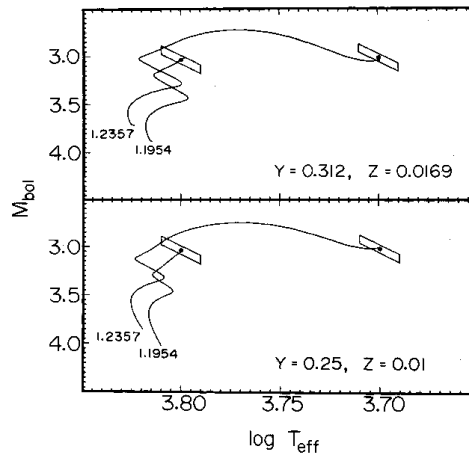


Figure 1.14: HR diagram showing the components of AI Phoenicis (Andersen et al. 1988) compared to predictions of the Vandenberg (1983) evolutionary models. The models were computed for the masses of AI Phe (indicated on the diagram) and two chemical compositions (Y and Z as shown). Taken from Andersen et al. (1988).

determination of the chemical compositions of well-studied dEBs is suggested to be important in the near future to aid the careful study of the success of different sets of stellar model predictions (Andersen 1993, 1998).

The comparison between models and stellar properties is commonly undertaken using HR diagrams (e.g., Figure 1.14), as this method resembles that often used in the photometric study of stellar open clusters (see section 1.8). However, the most directly known fundamental parameters of a dEB are the masses and radii, and the surface gravities which are calculated from them. T_{eff} s must be found using less straightforward methods such as spectral analysis or application of photometric calibrations. The best comparisons are therefore between mass, radius and surface gravity, with comparisons using T_{eff} , luminosity or absolute magnitude being of secondary importance. As stellar radii and surface gravities are quite sensitive to evolution and convection, they are particularly useful properties against which to compare theoretical predictions (Lacy et al. 2003). More detailed comparisons are, however, possible using T_{eff} s or luminosities.

1.6.1.2 Further work

Further work should be concentrated on low-mass stars (Shallis & Blackwell 1980; Clausen, Helt & Olsen 2001), high-mass stars (Herrero, Puls & Najarro 2002), metal-poor stars (such as those found in the LMC and SMC; see section 1.6.3.4) and other types of stars which are poorly represented in the compilation of Andersen (1991). In particular, there exists a discrepancy between the masses of high-mass single stars found from spectroscopic and photometric observations, and the masses inferred from comparison with theoretical evolutionary models (Herrero et al. 1992; Herrero, Puls & Villamariz 2002). Burkholder, Massey & Morrell (1997) studied seven high-mass spectroscopic binaries, of which five are eclipsing, and found that careful analysis did not support this mass discrepancy. However, their study extended only to masses of about $15 M_{\odot}$, because EBs more massive than this usually exhibit major observational complications. Hilditch (2004) has found that the mass discrepancy disappears when several effects, including difficulties related to spectroscopic analysis, RV determination and T_{eff} determination, are allowed for. Major improvements in theoretical model atmospheres of high-mass stars has also helped the situation, but Herrero, Puls & Najarro (2002) find that there are still extremely large random differences between masses found using the two methods. This does suggest that the previous systematic effect has been explained and removed.

Very few late-type dEBs have been found because such stars are small, so are less likely to eclipse, and dim, so it is less likely that their eclipsing nature is discovered (Clausen et al. 1998). An additional problem is that the light curves of late-type dEBs exhibit complexities due to the presence of large starspots, making accurate photometric parameters more difficult to obtain. The Copenhagen Group has a research project to discover and analyse late-type EBs (Clausen et al. 1998; Clausen 1998; Clausen, Helt & Olsen 2001). Initial results suggest that the mass–radius relation suggested by low-mass dEBs is somewhat shallower than that predicted by theoretical evolutionary models (Clausen et al. 1999). This result is confirmed by Lastennet & Valls-Gabaud (2002), who found that this problem exists for well-studied low-mass dEBs. In many

cases the masses and radii of the two components can be fitted by adopting a large metal abundance, suggesting that observations of atmospheric metal abundances for these systems may allow further conclusions to be drawn.

1.6.1.3 The difference between stars in binary systems and single stars

The properties of close binary stars and single stars cannot be assumed to be identical. Due to the effects of mutual irradiation, gravitationally generated tides and mass transfer, single stars are different to the stars in multiple systems. Therefore the comparison between the properties of dEBs and theoretical models of single stars must be restricted to the cases where it is reasonable to assume that the difference between the components of the dEB and single stars of the same mass, age and chemical composition are negligible. This should be the case for well-separated dEBs, but even for close binaries the modification of the properties of the stars can be minor.

Malkov (2003) found that the single-star mass-luminosity relation cannot be determined from dEBs. This analysis is open to criticism for three reasons. Firstly, it was assumed that the components of wide binaries are representative of single stars, although the formation scenarios must have been a little different (Tohline 2002). Secondly, single and binary stars of similar spectral types were directly compared despite spectral type classifications being overly coarse for such a comparison. Thirdly, the components of well-studied dEBs were assumed to be representative of all dEBs, so no corrections for biases were made.

In a study of the discrepancies between theoretically predicted and observed apsidal motion rates, Claret & Giménez (1993) noted that this discrepancy was significant only for a small subset of stars, for which the components occupy more than about 60% of the volume of their Roche lobes at periastron (when the Roche lobes are at their minimum volume) (Figure 1.15).

Lacy, Frueh & Turner (1987) have discovered that the secondary components of some dEBs (with late-A spectral types) have an anomalously low surface brightness compared to the primary components. This suggests that a systematic effect may exist

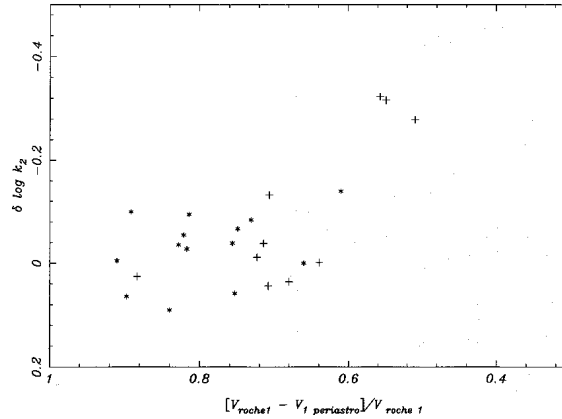


Figure 1.15: Difference between the theoretically predicted and observed central condensations of stars in dEBs against the fraction of the volume of the Roche lobe *not* filled by the primary star at periastron. Taken from Claret & Giménez (1993).

which could be caused by binarity, but the study was based on only six dEBs, none of which had definitive light curves. Further investigation is required to confirm or disprove this anomaly.

1.6.2 The metal and helium abundances of nearby stars

The astrophysical parameters of dEBs allow the age and chemical composition to be derived from comparison with theoretical evolutionary models, so the chemical evolution of the Galaxy can be mapped from the study of dEBs of different ages.

The low-mass dEB CM Draconis is important to the study of galactic chemical evolution because of its age. It has a space motion characteristic of a Population II system, so is expected to be old. As both components have very low masses (both about $0.2 M_{\odot}$; Lacy 1977b) they are completely convective, so their helium abundance can be accurately determined from their absolute dimensions (Paczynski & Sienkiewicz 1984). This has allowed the primordial helium abundance of the Galaxy to be found. An updated study, including YY Geminorum (in which the component masses are close

to $0.6 M_{\odot}$) was given by Chabrier & Baraffe (1995).

Popper et al. (1970) used a similar method to find the helium abundance of seven more massive, nearby, young dEBs, finding a ratio of 0.12 between the number of helium atoms and of hydrogen atoms.

Ribas et al. (2000) used astrophysical properties of the Andersen (1991) list of well-studied dEBs to determine the chemical enrichment law – the relation between the metal abundance and helium abundance of the interstellar medium – and primordial helium abundance in the solar neighbourhood. They found that the chemical enrichment law, $Y(Z)$ where Y and Z are the abundances of helium and metals respectively, is $\Delta Y/\Delta Z = 2.2 \pm 0.8$. The corresponding primordial helium abundance is $Y_p = 0.225 \pm 0.013$. The advantages of their approach over the more usual method of determining both Y and Z from high-resolution spectroscopy is that it reflects the overall chemical composition of the stars, rather than the atmospheric composition, and that Y is difficult or impossible to determine for many stars (including the Sun) from spectroscopic observations alone.

1.6.3 Detached eclipsing binaries as standard candles

An important function of dEBs is that accurate distances can be calculated for them from their physical properties. There are several ways to determine distances to dEBs, and the most reliable of these are calibrated directly from trigonometrical parallax measurements and/or interferometric observations of nearby stars. Using current telescopes, dEBs can give reliable and empirical distances for stellar systems from nearby star clusters to adjacent galaxies such as the Magellanic Clouds. This distance limit is currently being pushed out to more remote galaxies, for example M 33 and M 31 (see below). Methods of determining the distance to dEBs are discussed below.

All methods of distance determination require measurement of reliable reddening-free apparent magnitudes. The effect of interstellar reddening on the final distance can be large, but can be minimised by using infrared photometry (see section 2.3.1). The apparent magnitudes used must be both precise and accurate. The best sources for

these data are well-calibrated large-area surveys, for which the data is both precise and very homogeneous. One good source is the Tycho experiment on board the Hipparcos space satellite (Perryman et al. 1997), which observed the entire sky in the broad-band B_T and V_T passbands down to a limiting magnitude of $V \approx 11.5$. B_T and V_T data can be transformed to the standard Johnson system using the calibration of Bessell (2000). An excellent source of near-infrared JHK photometry is the Two Micron All Sky Survey (2MASS; Kleinmann et al. 1994) which has web-based database access³.

1.6.3.1 Distance determination using bolometric corrections

The most common way of finding the distance to a dEB involves the use of BCs (section 1.1.1.4), e.g., Munari et al. (2004) and Hensberge, Pavlovski & Verschueren (2000). Knowledge of the stellar T_{eff} s and the radii, R , of the stars means that the luminosities, L , can be calculated using the formula which defines T_{eff} :

$$L = 4\pi\sigma_{\text{SB}}R^2T_{\text{eff}}^4 \quad (1.17)$$

where $\sigma_{\text{SB}} = 5.67040(4) \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan-Boltzmann constant. The absolute bolometric magnitudes of the two stars can then be calculated using

$$M_{\text{bol}} = M_{\text{bol}\odot} - 2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right) \quad (1.18)$$

where $M_{\text{bol}\odot}$ and $L_{\odot}$ are the absolute bolometric magnitude and luminosity of the Sun, respectively. Whilst there are no defined values for $M_{\text{bol}\odot}$ and $L_{\odot}$, they are usually adopted to be $M_{\text{bol}\odot} = 4.75$ and $L_{\odot} = 3.826 \times 10^{26} \text{ W}$ (Zombeck 1990). This means that bolometric magnitudes only have significance if they are accompanied by the values of $M_{\text{bol}\odot}$ and $L_{\odot}$ used to calculate them.

The absolute bolometric magnitudes of the two stars then must be transformed to the absolute magnitudes of the stars in a passband for which the apparent magnitude

³2MASS observational data are available from <http://www.ipac.caltech.edu/2mass/>

of the system is available. The absolute magnitudes of the two stars, M_λ^A and M_λ^B , are then combined to determine the passband-specific absolute magnitude of the system:

$$M_\lambda^{\text{TOT}} = -2.5 \log_{10}(10^{-0.4M_\lambda^A} + 10^{-0.4M_\lambda^B}) \quad (1.19)$$

The distance, d (pc), is then calculated from the absolute magnitude of the dEB and the apparent magnitude, m_λ , using the equation

$$\log_{10} d = \frac{m_\lambda - A_\lambda - M_\lambda^{\text{TOT}} + 5}{5} \quad (1.20)$$

where A_λ is the total interstellar extinction in passband λ .

The difficulty with this method lies mainly in complications in obtaining BCs, which depend on T_{eff} and surface gravity, but also on the photospheric metal abundance of a star. Further discussion on BCs can be found in section 1.1.1.4. BCs are quite uncertain for hot stars because these stars emit a large fraction of their radiation in the ultraviolet, where it is strongly absorbed by the interstellar medium, so this method is not a good one for O and B stars (Harries, Hilditch & Howarth 2003).

The zeropoint of the BC scale is set by the assumption of certain values for $M_{\text{bol}\odot}$, $L_\odot$ and the solar BC. This means that BCs are only relevant if they are applied to absolute bolometric magnitudes which have been calculated using the same values for $M_{\text{bol}\odot}$ and $L_\odot$. In this case, the zeropoints coincide and a meaningful answer is found (Bessell, Castelli & Plez 1998). The use of different zeropoints means that the final answer is useless (e.g., Munari et al. 2004; see section 6).

The determination of distances using BCs requires that the T_{eff} s of the stars must be derived consistently with the fundamental definition of T_{eff} ; so the T_{eff} scale has known small or negligible systematic errors. Determination of T_{eff} is discussed in section 1.4.3. This constraint is important as it can be very difficult to quantify systematic errors in T_{eff} scales.

A good example of this distance determination technique, using empirical BCs, is for V578 Monocerotis (Hensberge, Pavlovski & Verschueren 2000). This method has also been discussed by Clausen (2004); he finds that the main uncertainty comes from the calibrations for empirical BCs.

1.6.3.2 Distances from surface brightness calibrations

In this method, the angular diameter of each component of an EB is estimated using the apparent magnitude of the star and calibrations between surface brightness and a photometric property. The linear radius of each star is known from a light and RV curve analysis, and comparison between this and the angular diameter gives its distance (Lacy 1977a). One distance estimate is obtained for each star – the two estimates should agree – and these can be combined using weighted means, although careful consideration of the uncertainties and the correlations in the results is necessary.

Surface brightness relations have been discussed in section 1.1.1.5 and generally comprise a calibration between some measure of the visual surface brightness of a star (in magnitudes) and an observed photometric index. Lacy (1977a) adopted the Barnes-Evans relation (Barnes & Evans 1976) between F_V and the $V - R$ index and applied the above method to nine dEBs for which accurate parallaxes were available, finding that distance moduli derived using the Barnes-Evans relation had accuracies of about 0.2 mag and were in agreement with distances found using the parallax measurements. Lacy (1978) applied the method to CW Cephei, V453 Cygni and AG Persei, all members of nearby open clusters or associations, and found that the distances derived were in agreement with, although slightly larger than, distances found from main-sequence fitting analyses of the stellar associations of which the dEBs were members. Lacy (1979) then applied the tested method to 48 dEBs, finding that their absolute magnitudes were in good agreement with theoretical predictions.

Semeniuk (2000) has compared the method of Lacy (1977a) to other distance methods and found that it is robust as long as it is used on well-behaved dEBs, as single-star surface brightness relations are not applicable to interacting binaries.

1.6.3.3 Distance determination by modelling of the stellar spectral energy distributions

This distance determination method was introduced by Fitzpatrick & Massa (1999) and has been used to find the distance to four EBs in the LMC: HV 2274 (Guinan et al. 1998; Ribas et al. 2000), HV 982 (Fitzpatrick et al. 2002), EROS 1044 (Ribas et al. 2002) and HV 5936 (Fitzpatrick et al. 2003).

The principle of this method is to determine the physical parameters of an early-type EB by fitting Kurucz ATLAS9 theoretical model atmospheres to ultraviolet and optical spectrophotometry. The observed spectral energy distribution of an EB at the Earth is a function of wavelength, λ ;

$$f_{\lambda,\oplus} = \frac{R_A^2 F_{A,\lambda} + R_B^2 F_{B,\lambda}}{d^2} \times 10^{-0.4A_\lambda} \quad (1.21)$$

where $F_{i,\lambda}$ ($i = A, B$) are the emergent fluxes at the surfaces of the two stars, R_i are their radii and A_λ is the total extinction along the line of sight of the EB. Thus

$$f_{\lambda,\oplus} = \left(\frac{R_A}{d}\right)^2 \left[F_{A,\lambda} + \left(\frac{R_B}{R_A}\right)^2 F_{B,\lambda} \right] \times 10^{-0.4E_{B-V} [k(\lambda-V) + R_V]} \quad (1.22)$$

where E_{B-V} is the reddening, $k(\lambda-V) \equiv \frac{E(\lambda-V)}{E_{B-V}}$ is the extinction curve and $R_V = \frac{A_V}{E_{B-V}}$ is the ratio of selective to total absorption in the V passband.

Synthetic spectra from the model atmospheres are fitted to the observed $f_{\lambda,\oplus}$, using nonlinear least squares algorithms, to derive values for $\left(\frac{R_A}{d}\right)^2$, $F_{i,\lambda}$, E_{B-V} and $k(\lambda-V)$. The distance estimate is found directly from $\left(\frac{R_A}{d}\right)^2$ and the radius of the primary star. The ATLAS9 model atmospheres, which represent the surface fluxes, $F_{i,\lambda}$, depend on T_{eff} , surface gravity, metallicity and microturbulence velocity. Fitzpatrick & Massa (1999) found that the ATLAS9 predictions provide a match to observations at a level consistent with current uncertainties in spectrophotometric observations. In addition, it can be assumed that the metallicity and microturbulence velocity of both components is the same. The ratio of the T_{eff} s of the stars is also known from the light curve analysis. Therefore there are only five parameters needed to specify ATLAS9 model spectral energy distributions of the two stars.

1.6.3.4 Recent results for the distance to eclipsing binaries

The main research area currently involving the observation and analysis of EBs is to use their properties as standard candles to determine the distances to Local Group galaxies. The first detailed photometric study of a dEB outside the Milky Way Galaxy was that of Jensen, Clausen & Giménez (1988), who provided the first CCD light curves of dEBs in the Magellanic Clouds.

The Copenhagen (Denmark) group has continued to study dEBs in the Magellanic Clouds (see Clausen 2000 and Clausen et al. 2003) in order to test the predictions of theoretical stellar evolutionary models in the low-metallicity environment of the Magellanic Clouds. The Villanova (USA) group (Guinan et al. 1998; Ribas et al. 2000, 2002; Fitzpatrick et al. 2002, 2003) are continuing their efforts (detailed above). The Mount John (New Zealand) group are also running an observing program to obtain good CCD light curves of Magellanic Cloud EBs (e.g., Bayne et al. 2004). An impressive observing program has been undertaken by Harries, Hilditch & Howarth (2003; see also Hilditch, Harries & Howarth 2004), who used the 2dF multi-object spectrograph at the Anglo-Australian Telescope to obtain RV curves of approximately one hundred high-mass short-period EBs in the SMC.

Recent large-scale photometric surveys have targeted the Magellanic Clouds, obtaining a large number of light curves of distant stars in order to detect and analyse the brightening effects caused by gravitational microlensing phenomena. The Optical Gravitational Microlensing Experiment (OGLE⁴) group have obtained a huge amount of data, through three phases of increasingly sophisticated instrumentation, which is of sufficient quality to derive preliminary results for several thousand EBs. Additional data have also been obtained by the Microlensing Observations in Astrophysics (MOA⁵), Expérience pour la Recherche d'Objets Sombres (EROS⁶) and MASSive Compact Halo Objects (MACHO⁷) groups. As a byproduct of these searches, over five

⁴The OGLE homepage is available on the internet at <http://bulge.princeton.edu/~ogle/>

⁵The MOA homepage is available on the internet at <http://www.physics.auckland.ac.nz/moa/>

⁶The EROS homepage is available on the internet at <http://eros.in2p3.fr/>

⁷The MACHO homepage is available on the internet at <http://www.macho.mcmaster.ca/>

thousand EBs have been detected in the Magellanic Clouds.

Wyithe & Wilson (2001, 2002) have investigated the EBs which have been found in the SMC and suggested that close binaries, including semidetached systems, are very good distance indicators. They are better than dEBs because, given the same quality and quantity of photometric observations, the properties of the system tend to be more accurately determined (Wilson 2004). Graczyk (2003) agrees that close EBs are more useful as the proximity effects in their light curves give useful constraints on the properties of the systems, in particular third light and mass ratio. It is also clear that close binaries spend a greater proportion of their time in eclipse, so a given set of photometric observations will contain more datapoints inside eclipses, and that RV curves are more easy to obtain as the velocity semiamplitudes are greater.

The determination of distance from the study of EBs is being applied to more distant galaxies as observing time on large telescopes becomes more easily available. The large Local Group galaxies M 31 and M 33 (which are gravitationally bound; Guinan 2004) have been targeted by the DIRECT project⁸ (Kałużny et al. 1998 and more recent works) and about 130 EBs have been detected, along with about 600 Cepheids (Macri 2004a). The DIRECT group have begun RV observations of four dEBs in M 31 and M 33, using the 10 m Keck telescopes (Macri 2004b). I. Ribas is also independently leading a research program to study further some EBs discovered by DIRECT, using the 2.5 m Isaac Newton Telescope to obtain light curves and the 8 m Gemini telescopes for spectroscopic observations (Ribas et al. 2004).

1.6.4 Detached eclipsing binaries in stellar systems

The metal abundance, helium abundance, age or distance are often known for nearby stellar open clusters and associations (see section 1.8). If a dEB is a member of the cluster, then it is possible to derive accurate masses, radii and T_{eff} s for two stars of known age, distance or chemical composition. This data can then be used to provide

⁸The DIRECT project homepage is available at <http://cfa-www.harvard.edu/~kstanek/DIRECT/>

a detailed and discriminating test of theoretical stellar evolutionary models. Alternatively, the properties of the dEB can be used to find the age, chemical abundance or distance of the cluster of stars as a whole (e.g., Clausen & Giménez 1991).

The properties of stellar open clusters are generally derived by comparison with the predictions of stellar evolutionary models. The same set of models should be adopted for comparison with the properties of dEBs as are used for the derivation of the properties of their parent cluster. Ideally, models of the same age and chemical composition should be able to simultaneously accurately predict the photometric properties of the cluster and the physical properties of the dEB.

The study of EBs has long been known to be facilitated by their membership of a stellar cluster. Lists of EBs in open clusters have been presented by Kraft & Landolt (1959), Sahade & Dávila (1963) and Clausen & Giménez (1987; Clausen 1996b; Giménez & Clausen 1996).

1.6.4.1 Results on detached eclipsing binaries in clusters

A research project on EBs in open clusters has been undertaken by Milone & Schiller (1991) and collaborators at the Rothney Astrophysical Observatory (Canada), who have studied the dEBs V818 Tauri (HD 27130) in the Hyades (Schiller & Milone 1987) and DS Andromedae in NGC 752 (Schiller & Milone 1988), the contact binary Heine-mann 235 in NGC 752 (Milone et al. 1995) and the curious case of SS Lacertae (Milone et al. 2000), a dEB member of NGC 7209 which no longer shows eclipses due to the perturbations of a third body in the system (Torres 2001). It was stated by Milone & Schiller (1991) that analyses of QX Cassiopeiae (NGC 7790) and CN Lacertae (NGC 7209) were close to completion, but these are yet to be published.

The study of well-detached binaries in open clusters was stated to be able to provide strong constraints on stellar evolutionary theory by Lastennet, Valls-Gabaud & Oblak (2000). These authors considered the Hyades visual binaries 51 Tauri and θ^2 Tauri (section 1.5.1), and the dEBs V818 Tauri (a Hyades member) and CW Cephei (a member of the Cepheus OB3 association). They found that predictions of the

Padova stellar evolutionary models (section 1.3.2.3) were unable to fit the components of V818 Tau in the mass-radius diagram, a conclusion also reached by Pinsonneault et al. (2003). From consideration of the photometric study of this dEB (Schiller & Milone 1987) I would suggest that the problem is probably caused by the analysis of low-quality observations with inadequate consideration of the uncertainties of the resulting photometric parameters.

Lebreton, Fernandes & Lejeune (2001) derived the helium content and the age of the Hyades open cluster from a comparison between the predictions of the CESAM stellar evolutionary models (see section 1.3) and a mass-luminosity relation derived from three double-lined spectroscopic visual binaries (51 Tauri, Finsen 342 and θ^2 Tauri; Torres, Stefanik & Latham 1997a, 1997b, 1997c), a single-lined spectroscopic visual binary (θ^1 Tauri; Torres, Stefanik & Latham 1997c) and the dEB V818 Tauri (referred to as vB 22). They were hampered by correlations between the helium and metal abundances and the mixing length parameter, α_{MLT} , but were able to conclude that the helium abundance was somewhat lower than expected for a given metal abundance, suggesting that the chemical enrichment law in the Hyades is slightly anomalous.

Hurley, Pols & Tout (2000) have found that an overshooting parameter value of $\alpha_{\text{OV}} \approx 0.12$ is supported by the consideration of dEBs in open clusters.

Probably the best-known analysis of a dEB in a stellar cluster is OGLE GC 17 in the globular cluster ω Centauri (Thompson et al. 2001). From a relatively limited amount of observational data – due to the dEB being dimmer than 17th magnitude in the *I* passband – these authors were able to derive masses accurate to 7% and radii accurate to 3%, partially because the dEB exhibits total eclipses. Thompson et al. calibrated several infrared surface brightness relations and used these to find a distance to OGLE GC 17 of 5360 ± 300 pc. Comparison with theoretical stellar evolutionary models gave the age of the dEB to be between about 13 and 17 Gyr. Note that very accurate masses are not vital for the determination of distance because the masses of the stars are not needed for distance calculation. The need for spectroscopy is to find the separation of the two stars, which is better determined than the masses for the same observational data. Accurate masses are needed for a comparison between the

properties of the dEB and the predictions of theoretical stellar evolutionary models. Thompson et al. state that improved observations will be able to give a significantly more accurate distance to ω Cen from study of the dEB OGLE GC 17, and these authors have obtained further observations (Kaluźny et al. 2002).

1.7 Tidal effects

The mutual gravitational attraction between binary stars causes several dynamical phenomena to occur:–

- The orbits of binary stars continuously decrease in eccentricity, so close binary orbits can become circularized.
- The angular rotational velocities of the component stars move towards that of the orbit. As stars are always born with rotational velocities greater than this value (due to the conservation of angular momentum as the stellar radii decrease during evolution towards the ZAMS) their rotational velocities decrease towards synchronization.
- Eccentric binary orbits change orientation continuously (the longitude of periastron increases). This effect is called apsidal motion and can be very useful as it depends on the internal structure of the stars, so the degree of central condensation of stars can be determined observationally.
- The axes of rotation and orbital motion tend to align perpendicular to the plane of the orbit.

1.7.1 Orbital circularization and rotational synchronization

Several theories exist of the magnitude, and indeed existence, of the dynamical effects which cause orbital circularization and rotational synchronization. These theories,

however, do not in general agree with each other or with all observations, and additional effects exist which have not yet been quantitatively investigated.

The equilibrium shapes of the surfaces of single stars are accurately described by equipotential surfaces, where the potential due to gravitational attraction is modified by the effects of rotation. Binary stars have an additional potential due to the gravitational attraction of the other component, causing the surfaces of such stars to bulge outwards in two places: towards and away from the other star. If the orbit is circular and the star's rotation is synchronous with the binary orbit, this bulge is static and has no effect on the dynamics of the stars. If the orbit is eccentric and/or the star has an asynchronous rotational velocity, this bulge does not point straight to the companion star. As stars consist of viscous material, the bulge is pushed by rotation away from the other star and so exerts a force on its own star, due to the gravitational attraction between the bulge and the companion star. This force acts to bring the rotation of the stars towards the synchronous velocity, and to decrease orbital eccentricity.

1.7.1.1 The theory of Zahn

Zahn (1970, 1975, 1977, 1978) considered several physical mechanisms which produce tidal friction in close binary stars. The equilibrium tide is the hydrostatic adjustment of the structure of the star to the perturbing force from the companion. The dynamical tide is the response to the equilibrium tidal force; it depends on the properties of the star and may be resonant over the volume of the star.

The most important tidal evolution mechanism in convective-envelope stars is turbulent viscosity retarding the equilibrium tide. The most important mechanism in radiative-envelope stars is radiative damping on the dynamical tide (Zahn 1984).

The timescales of orbital circularization and rotational synchronization for stars with convective envelopes are derived, for a single star (in years) to be

$$\tau_{\text{circ}}^{\text{conv}} = \frac{1}{84q(1+q)k_2} \left(\frac{MR^2}{L} \right)^{\frac{1}{3}} \left(\frac{a}{R} \right)^8 \quad (1.23)$$

$$\tau_{\text{synch}}^{\text{conv}} = \frac{1}{6q^2k_2} \left(\frac{MR^2}{L} \right)^{\frac{1}{3}} \frac{I}{MR^2} \left(\frac{a}{R} \right)^6 \quad (1.24)$$

where q is the mass ratio, M is the mass, R is the radius, L is the luminosity, I is the moment of inertia, a is the semimajor axis, M , R , L and I are in solar units, and k_2 is the apsidal motion constant of the star (Zahn 1977, 1978). Note the very strong dependence on the fractional stellar radius, $\frac{R}{a}$. Due to uncertainties in the treatment of several physical effects, the formulae are inexact. Approximations are provided which are “probably well within the error margin” (Zahn 1977, 1978):

$$\tau_{\text{circ}}^{\text{conv}} \approx 10^6 \frac{1}{q} \left(\frac{1+q}{2} \right)^{\frac{5}{3}} P^{\frac{16}{3}} \quad (1.25)$$

$$\tau_{\text{synch}}^{\text{conv}} \approx 10^4 \left(\frac{1+q}{2q} \right)^2 P^4 \quad (1.26)$$

where the orbital period, P , is in days.

For stars containing a convective core and a radiative envelope, the theory is more complex and gives the equations

$$\tau_{\text{circ}}^{\text{rad}} = \frac{1}{5} \frac{1}{2^{5/3}} \left(\frac{R^3}{GM} \right)^{\frac{1}{2}} \frac{I}{MR^2} \frac{1}{q^2(1+q)^{5/6}} \frac{1}{E_2} \left(\frac{a}{R} \right)^{\frac{17}{2}} \quad (1.27)$$

$$\tau_{\text{synch}}^{\text{rad}} = \frac{2}{21} \left(\frac{R^3}{GM} \right)^{\frac{1}{2}} \frac{1}{q(1+q)^{11/6}} \frac{1}{E_2} \left(\frac{a}{R} \right)^{\frac{21}{2}} \quad (1.28)$$

where G is the gravitational constant and the constant E_2 depends on the tidal torque and must be determined from stellar structure theory. No suitable approximations for E_2 exist, mainly because it is very sensitive to the mass and evolutionary state of the star. In fact E_2 is proportional to the seventh power of the ratio of the radii of the convective core and the whole star. Tabulations of E_2 are provided by Zahn (1975) and more extensively and accurately by Claret & Cunha (1997).

Zahn (1989) revisited the theory of the equilibrium tide and updated the resulting timescale equations. He suggested that convective effects could cause the orbital circularization timescale to depend on the orbital period according to $\tau_{\text{circ}} \propto P^{\frac{10}{3}}$ for stars with convective envelopes. Goldman & Mazeh (1991) have developed this further and found that it may be a better match to observations.

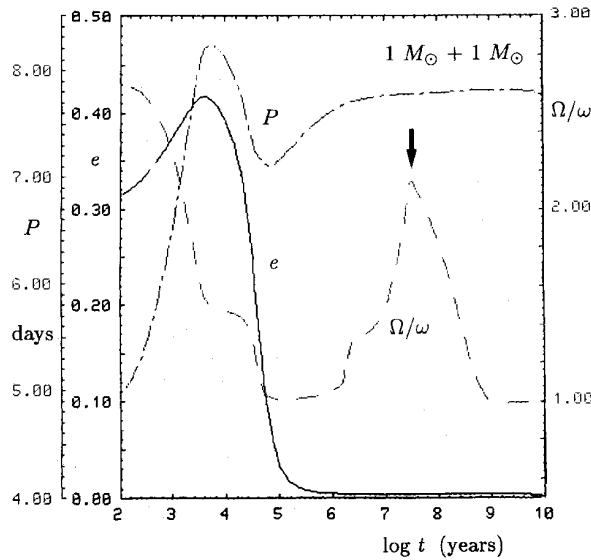


Figure 1.16: Evolution of the period, P , eccentricity, e and the ratio of the rotational to orbital velocity, $\frac{\Omega}{\omega}$, for a close binary containing two $1 M_{\odot}$ stars. The arrow indicates the time at which the ZAMS is reached. Taken from Zahn & Bouchet (1989).

Zahn & Bouchet (1989) investigated the problem of dynamical evolution of binary stars during the PMS evolutionary phase. This is an important effect because of the strong dependence of the magnitude of tidal forces on the separation of the component stars. During PMS evolution the stars have much greater radii, and it appears that the majority of the dynamical evolution of close binary stars occurs during the PMS phase rather than the MS phase. Fig. 1.16 shows the evolution in time of the orbital period, P , eccentricity, e and the ratio of the orbital and rotational velocities, $\frac{\Omega}{\omega}$, for a close binary composed of two $1 M_{\odot}$ stars. The initial parameters were arbitrarily selected and correspond to a well separated system. It is notable that e decreases from an initial value of 0.3 to 0.005 by the time the stars have evolved to the ZAMS. The local maximum of $\frac{\Omega}{\omega}$ at that point is due to the ZAMS being (by definition) the point at which stellar radii attain their minimum value.

1.7.1.2 The theory of Tassoul & Tassoul

Tassoul (1987) developed a theory based on a purely hydrodynamical mechanism which causes orbital circularization and rotational synchronization. The derived spin-down timescale can be expressed in two equivalent ways:

$$\tau_{\text{spin down}} = \frac{1.44 \times 10^{-N/4}}{q(1+q)^{3/8}} \left(\frac{L_{\odot}}{L} \right)^{\frac{1}{4}} \left(\frac{M_{\odot}}{M} \right)^{\frac{1}{8}} \left(\frac{R}{R_{\odot}} \right)^{\frac{9}{8}} \left(\frac{a}{R} \right)^{\frac{33}{8}} \quad (1.29)$$

$$\tau_{\text{spin down}} = 535 \times 10^{-N/4} \frac{1+q}{q} \left(\frac{L_{\odot}}{L} \right)^{\frac{1}{4}} \left(\frac{M}{M_{\odot}} \right)^{\frac{5}{4}} \left(\frac{R_{\odot}}{R} \right)^3 \left(\frac{P}{\text{days}} \right)^{\frac{11}{4}} \quad (1.30)$$

where N depends on the turbulent viscosity. If eddy viscosity in radiative envelopes is ignored then $N = 0$. For turbulent convective envelopes, N is probably between 8 and 12 (Tassoul 1988). Tassoul states that τ_{sync} can be conservatively assumed to be about one order of magnitude larger than $\tau_{\text{spin down}}$. This mechanism is a relatively long-range force [proportional to $(\frac{a}{R})^{33/8}$] compared to the theory of Zahn.

Tassoul (1988) considered the timescale for orbital circularization. This can be obtained by multiplying τ_{sync} by the ratio of the orbital and rotational angular momenta of the stars, to give

$$\tau_{\text{circ}} = \frac{14.4 \times 10^{-N/4}}{(1+q)^{11/8} r_g^2} \left(\frac{L_{\odot}}{L} \right)^{\frac{1}{4}} \left(\frac{M_{\odot}}{M} \right)^{\frac{1}{8}} \left(\frac{R}{R_{\odot}} \right)^{\frac{7}{8}} \left(\frac{a}{R} \right)^{\frac{49}{8}} \quad (1.31)$$

$$\tau_{\text{circ}} = 9.4 \times 10^{4-N/4} \frac{(1+q)^{2/3}}{r_g^2} \left(\frac{L_{\odot}}{L} \right)^{\frac{1}{4}} \left(\frac{M}{M_{\odot}} \right)^{\frac{23}{12}} \left(\frac{R_{\odot}}{R} \right)^5 \left(\frac{P}{\text{days}} \right)^{\frac{49}{12}} \quad (1.32)$$

where r_g is the radius of gyration of the star (for a homogeneous sphere $r_g^2 = \frac{2}{5}$, and for centrally condensed stars $r_g^2 \approx 0.01$ to 0.1).

Tassoul (1990, 1995, 1997) and Tassoul & Tassoul (1990) consider the tidal evolution theory of Tassoul and conclude that its main features are generally confirmed by observations, particularly of high-mass circular-orbit binary stars, with orbital periods of tens of days, which disagree with the theory of Zahn.

1.7.1.3 Comparison with observations

Firstly, the above timescales are applicable to individual stars only. The overall timescale for a binary star must be calculated using

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{prim}}} + \frac{1}{\tau_{\text{sec}}} \quad (1.33)$$

(Claret, Giménez & Cunha 1995) where τ is the characteristic timescale and τ_{prim} and τ_{sec} are the timescales for the individual stars.

Several attempts have been made to compare tidal theories with observations, concentrating mainly on the age-dependent cutoff period, P_{cut} , below which all binary stars in a co-evolutionary sample exhibit circular orbits. This cutoff period has been determined for populations of binaries in the nearby intermediate-age open clusters Hyades and Praesepe (Mayor & Mermilliod 1984; Burki & Mayor 1986) and M67 (Mathieu, Latham & Griffin 1990), the old open cluster NGC 188 (Mathieu, Meibom & Dolan 2004) and for Galactic Population I stars (Latham et al. 1992). The PMS tidal evolution described by Zahn & Bouchet (1989), twinned with the MS evolution theorised by Zahn (1977), would cause all these groups of binaries to display very similar values of P_{cut} , between around seven and nine days, as almost all tidal changes occur before the ZAMS. The observations display a greater range of values of P_{cut} , particularly for NGC 188 and the Population I stars, for which the cutoff periods are 15 and 19 days respectively. It is therefore clear that tidal effects are important on the MS as well as before the ZAMS.

Giuricin, Mardirossian & Mezzetti (1984a, 1984c, 1984d, 1985) compiled lists of eclipsing and non-eclipsing binary stars from the literature and compared their rotational properties to predictions from the theory of Zahn. They found good agreement for late-type stars (with convective envelopes). They also found that there existed early-type binaries in a state of rotational synchronization with periods greater than that allowed by the theory of Zahn. Giuricin, Mardirossian & Mezzetti (1984b) investigated the orbital circularization characteristics of the same binaries and concluded that the observations were compatible with the theory of Zahn. Koch & Hrivnak (1981)

found that Zahn's theory could explain the dynamics of radiative-envelope binaries with small eccentricities and orbital periods below about 20 days.

Claret, Giménez & Cunha (1995) investigated the theory of Tassoul by integration of the relevant differential equations, and concluded that it was in satisfactory agreement with the observations of rotational synchronization and orbital circularization. However, they indicate that the validity of the Tassoul theory has not yet been fully confirmed. Claret & Cunha (1997) treated the Zahn theory in the same way and found that it predicted the majority of the observational results, but was unable to explain some early-type systems which have circular orbits despite τ_{circ} being greater than the MS lifetime of the primary components.

Mathieu & Mazeh (1988) proposed that observations of P_{cut} could be used to find ages of stellar groups. However, tidal theory uncertainties and the difficulty of determining an accurate value of P_{cut} do not allow accurate ages to be derived. Zahn & Bouchet (1989) suggested that PMS tidal interaction makes such a method impossible, but that rotational synchronization could be used instead. However, as stated above, the results of Zahn & Bouchet (1989) are not fully supported by observations.

There exist further problems which are not in general incorporated into the various tidal evolution theories:–

- Magnetic fields may be important contributors to the overall tidal torque.
- Orbital evolution at the PMS stage appears to be more important than evolution after the ZAMS.
- The axes of revolution of the stars may not be parallel to the orbital axis.
- Differential rotation in stars may cause them to appear rotationally synchronized when their interior is not. Synchronization has been suggested to proceed from the surface of a star towards the core (Goldreich & Nicholson 1989).
- Tidal frequencies which are resonant in the stars, and pulsations, have not been included in the above theories.

- Binary stars are created with a range of orbital characteristics but current tidal evolution theories do not fully take this into account, although PMS dynamical evolution will reduce the resulting effect.
- The timescales discussed above are valid for unchanging stars (so no stellar evolution) which are in almost circular orbits and rotating close to synchronously.
- As the conditions of circular orbit and synchronous rotation are approached asymptotically, the tidal timescales are estimates of the amount of time taken for stars to become much closer to these conditions. They are not the time taken for the orbit to become perfectly circular and the rotation to become perfectly synchronous for any initial conditions.
- The circularization timescale depends on rotation in a way which is not explicitly incorporated into the models (Claret & Cunha 1997).
- The time taken to reach circular orbits and synchronous rotation for a particular system must be calculated by integrating its orbital characteristics from their initial values to the present age of the system (Claret & Cunha 1997). As we do not know the initial conditions, this can only be approached in a statistical manner (Tassoul & Tassoul 1992).
- Tidal timescales generally have an abrupt discontinuity at the boundary between radiative and convective envelopes, so at this point the timescales are very uncertain (Claret, Giménez & Cunha 1995).
- The binary components of hierarchical triple systems can have their orbital characteristics significantly modified by the third star. This can cause small eccentricities to exist when tidal theories predict that the orbit should be circular (Mazeh 1990).

In conclusion, several sophisticated tidal theories exist which predict degrees of orbital circularization and rotational synchronization which are in acceptable agreement with the majority of observed binary systems. Of the two commonly investigated

– and somewhat controversial – theories, the basic premise of the theory of Tassoul is not yet fully accepted despite this theory being probably the most successful overall, and the theory of Zahn considers forces which are too weak to explain some observations. Until researchers are able to solve several of the problems listed in the last paragraph, tidal theories are unlikely to become much more successful. A vital part of any implementation of the theory is the time integration of specific systems rather than dependence on one equation valid for all binary stars (Claret & Cunha 1997).

1.7.2 Apsidal motion

The tidal forces which cause orbital circularization and rotational synchronization also affect the orientation of binary orbits, resulting in a constant increase in the value of the longitude of periastron, ω , over time. The apsidal motion period is the time taken for one complete revolution of the line of apsides, and in observed systems varies from a few years, for the very close binaries, to many centuries for well-separated systems. Beyond apsidal periods of about one thousand years the effect becomes too small to be noticed in the comparatively short time interval in which humans have had access to good observing equipment.

Apsidal motion is caused by the fact that stars are not point masses but its magnitude depends strongly on how centrally condensed the stars are. Knowledge of the apsidal period, and the absolute dimensions, of an EB allows us to calculate the internal structure constant $\log k_2$, which can then be compared with theoretical models to see if their internal structure predictions match observations (Hilditch 1973).

The apsidal period can be derived spectroscopically by analysing the increase in the values of ω derived from spectroscopic orbits observed many years apart. For systems with only small eccentricities, e , however, observational errors make this very difficult. In an EB the times of minimum light are dependent on e and ω . The most basic observable is the time difference between a primary and successive secondary light minimum, which depends mainly on the quantity $e \cos \omega$ (e.g., Güdür 1978).

The parameters on which photometric observations of apsidal motion in an EB

depend are the apsidal period, U , the rate of change of ω , $\dot{\omega}$, the value of ω at the reference time of minimum light, ω_0 , the eccentricity, e and the orbital inclination, i . The ephemeris curve takes the form of a sinusoidal variation of the difference between the actual times of eclipse and the times of eclipse predicted using a linear ephemeris. The ephemeris curve does depend on i , but this dependence is weak for $i \gtrsim 70^\circ$ (the effect is shown in Fig. 1.17). As EBs generally have $i \gtrsim 80^\circ$, the exact value of i is unimportant, and this weak dependence makes it impossible to determine i from observations of apsidal motion. However, determinations of e and ω from the study of apsidal motion can be more accurate than direct determinations from the analysis of light curves or RV curves (Clausen, Giménez & van Houten 1995).

Methods of deriving the apsidal motion parameters from observed times of minimum light depend on adjusting the parameters until they best match the observations. The traditional methods (e.g., Sterne 1939) provide easily-calculated approximations to the parameters, which are then optimised by the process of differential corrections or a similar technique. This method was taken to approximations involving the fifth power in eccentricity by Giménez & Garcia-Pelayo (1983). More recently, Lacy (1992) has avoided the use of approximations altogether and provided an exact solution to the problem of deriving apsidal motion parameters from observations. Equations are formulated to predict exact times of eclipse given a set of parameters, and these parameters are adjusted towards the best fit using the Levenberg-Marquart nonlinear least-squares fitting algorithm MRQMIN (Press et al. 1992). Fig. 1.18 shows an example ephemeris curve fitted to observations of the times of minimum light of the dEB V523 Sagittarii, given as an example by Lacy (1992).

1.7.2.1 Relativistic apsidal motion

A general relativistic treatment of the gravitational forces in an EB shows that there is a contribution to the overall apsidal motion of

$$\dot{\omega}_{\text{GR}} = \frac{6\pi G}{c^2} \frac{1}{P} \frac{M_1 + M_2}{a(1-e)^2} \quad (1.34)$$

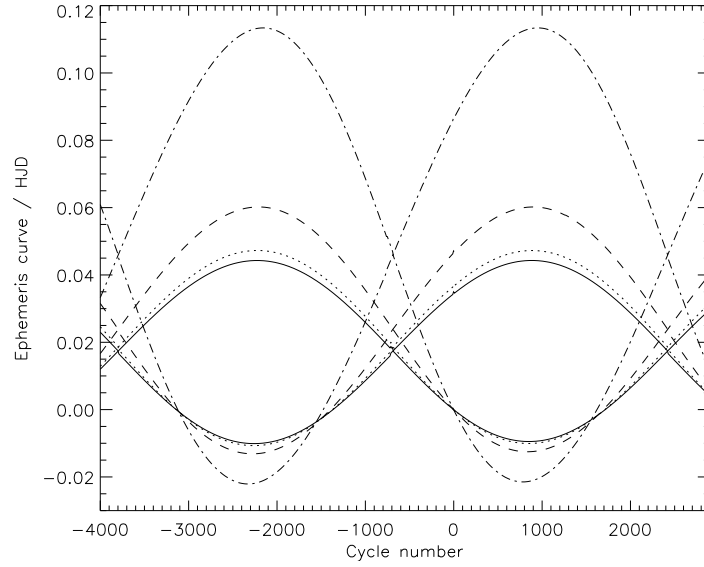


Figure 1.17: The effect of different values of the orbital inclination, i , on the ephemeris curve. The solid lines show the predicted times of primary and secondary eclipse for $i = 90^\circ$. Dotted lines are for 70° , dashed lines for 50° and dot-dash lines for 30° . This figure is based on the parameters of V453 Cygni and was generated using the APSMOT code (see section 4.3).

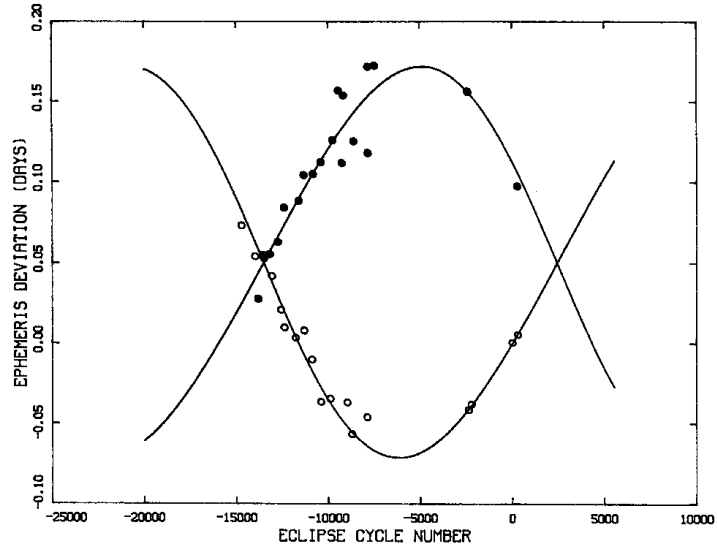


Figure 1.18: The best-fitting ephemeris curve for the dEB V523 Sagittarii. Observed times of minimum light are given by open circles (primary eclipses) and filled circles (secondary eclipses). Taken from Lacy (1992).

where G is the gravitational constant, c is the speed of light, P is the orbital period, a is the orbital semimajor axis and M_1 and M_2 are the masses of the component stars (Giménez 1985). If M_1 and M_2 are expressed in solar masses and P is expressed in days, this equation reduces to (Giménez 1985)

$$\dot{\omega}_{\text{GR}} = 5.45 \times 10^{-4} \frac{1}{1 - e^2} \left(\frac{M_1 + M_2}{P} \right)^{\frac{2}{3}} \quad (1.35)$$

where $\dot{\omega}_{\text{GR}}$ is in units of degrees per orbital cycle. For dEBs with well-known apsidal periods, the general relativistic apsidal motion rate is in general about one order of magnitude smaller than the Newtonian rate.

Giménez (1985) has given a list of EBs which may provide good tests of general relativity. The method requires the total apsidal motion rate to be found and the Newtonian contribution to be removed using theoretical model predictions. This is only reasonable if the general relativistic contribution is similar in size to the Newtonian contribution, which occurs for only well-separated stars, or very eccentric orbits, so is difficult to observe. Giménez & Scaltrini (1982) applied this method to V889 Aquilae and found a relativistic apsidal motion rate in agreement with the theoretical predictions. Khaliullin (1985) undertook the same procedure, using V541 Cygni, also finding agreement with the theory of general relativity.

1.7.2.2 Comparison with theoretical models

Once an apsidal period has been derived, the internal structure constant $\log k_2$ can be calculated for comparison with the predictions of theoretical evolutionary models. However, the two stars in a binary system do not in general have the same $\log k_2$, but the individual contributions to the overall apsidal motion rate are not known.

As discussed in Claret & Giménez (1993), the observed density concentration coefficient can be calculated from the apsidal period using the equation

$$\overline{k_2^{\text{obs}}} = \frac{1}{c_{21} + c_{22}} \frac{P}{U} \quad (1.36)$$

where the constants c_{2i} are weights which depend on the characteristics of each star ($i=1$ refers to the primary star and $i=2$ refers to the secondary). c_{2i} are given by

$$c_{2i} = \left[\left(\frac{\omega_i}{\omega_K} \right)^2 \left(1 + \frac{M_{3-i}}{M_i} \right) f(e) + 15 \frac{M_{3-i}}{M_i} g(e) \right] \left(\frac{R_i}{a} \right)^5 \quad (1.37)$$

$$f(e) = (1 - e^2)^{-2} \quad (1.38)$$

$$g(e) = \frac{8 + 12e^2 + e^4}{8} f(e)^{\frac{5}{2}} \quad (1.39)$$

where ω_i are the rotational velocities of the stars, ω_K are the synchronous (Keplerian) rotational velocities, R_i are the stellar radii and a is the orbital semimajor axis.

The weighted mean theoretical density concentration coefficient must be calculated from the individual theoretical density concentration coefficients using

$$\overline{k_2^{\text{theo}}} = \frac{c_{21}k_{21} + c_{22}k_{22}}{c_{21} + c_{22}} \quad (1.40)$$

to find the weighted average coefficient which is directly comparable to observations.

Once the relativistic apsidal motion contribution, $\dot{\omega}_{\text{GR}}$, has been subtracted from $\overline{k_2^{\text{obs}}}$, this value can then be compared directly with $\overline{k_2^{\text{theo}}}$.

1.7.2.3 Comparison between observed density concentrations and theoretical models

Several dEBs which display apsidal motion have been studied to determine accurate absolute dimensions and apsidal periods. The majority of these were studied by the Copenhagen Group (for example Andersen et al. 1985) and compared to the predictions of the Hejlesen stellar models (Hejlesen 1980, 1987). In general the theoretical values of $\log k_2$ were greater than observed, so the model stars were less centrally condensed than they should be (Young et al. 2001). More recent stellar models (Claret 1995, 1997; Claret & Giménez 1995, 1998), incorporating convective core overshooting, newer opacity data (Stothers & Chin 1991; Rogers & Iglesias 1992) and the effects of stellar rotation, are in much better agreement (Giménez & Claret 1992).

Benvenuto et al. (2002) determined the apsidal motion of the high-mass binary system HD 93205 and, using the predictions of theoretical models, used this information to determine the mass of the primary star to be $60 \pm 19 M_{\odot}$. This method allows the determination of absolute masses of binary stars which are not eclipsing, so is useful for stellar types which are rare in EBs (for example O stars), but is dependent on the predictions of theoretical models.

1.8 Open clusters

When a giant molecular cloud collapses to trigger an episode of star formation, many small parts of it separately contract and subsequently form stars. This creates a cluster of stars which were created at the same time and from material of a uniform chemical composition. Many clusters in the Perseus spiral arm of our Galaxy have similar ages, suggesting that there was a triggering event which caused the collapse of many giant molecular clouds (Phelps & Janes 1994).

Stellar clusters are relatively easy to separate into three different morphological groups. Globular clusters generally contain between 10^5 and 10^7 metal-poor stars, and are very old. Open clusters contain between fifty and several thousand stars which are weakly gravitationally bound and have ages between zero and 10 Gyr. OB associations are collections of stars which formed at a similar time and in a similar place, but are too distant from each other to be gravitationally bound.

As the stars in an open cluster are all the same age, distance and chemical composition, the study of these objects can provide important insights into how stars, clusters and galaxies form and evolve. The usual method of studying these objects is to obtain absolute photometry of the cluster in several passbands, e.g., *UBV*. This allows each observed star to be plotted on colour-magnitude diagrams (CMDs) and colour-colour diagrams. The members of the cluster can then be compared to the radiative properties of nearby stars in order to determine the age and distance of the cluster and the amount of interstellar reddening which affects the light we receive.

The study of open clusters has several uses:—

- To critically test the predictions of theoretical evolutionary models.
- To investigate the radial chemical abundance gradient of galaxies (e.g., Chen, Hou & Wang 2003).
- To investigate the shape and dynamics of galaxies (Romeo et al. 1989).
- To set the distance scale in our Galaxy, which can be used to calibrate other distance indicators such as δ Cepheids (e.g., Sandage & Tammann 1969).
- As most stars are born in clusters, the study of clusters is important to the star formation history of galaxies.
- To investigate the present-day and initial stellar mass functions (Meibom, Andersen & Nordström 2002).
- To provide a lower limit to the age of galaxies and of the Universe (Weiss & Schlattl 1995; Salaris, Weiss & Percival 2004).

There are somewhere over one thousand open clusters in our Galaxy (Balog et al. 2001) and some probably remain undiscovered due to a small size or large interstellar absorption. Large-scale studies and databases of open clusters and associations have been compiled by Mermilliod (1981), Lyngå (1987), Garmany & Stencel (1992), Phelps & Janes (1994), Dias et al. (2002), Chen, Hou & Wang (2003) and the WEBDA⁹ open cluster database maintained by J.-C. Mermilliod.

The position and shape of the MS of a cluster in its CMD depends on the cluster's distance, age, chemical composition, the evolutionary characteristics of the stars and the interstellar extinction between it and the Earth. These quantities can therefore, in principle, be inferred from the CMD of a cluster. The problem with this is that many of these parameters are significantly correlated. Additional difficulties are caused by

⁹<http://obswww.unige.ch/webda/>

the presence of stars which are not cluster members. These field stars can be both foreground and background objects. Unresolved binary stars will also appear in the CMD as single stars up to 0.7 mag brighter than single stars of the same colour (for binaries composed of two identical stars), or redder colours if the primary component has a significantly higher T_{eff} than the secondary star.

Attempts to derive the properties of open clusters have traditionally relied on fitting CMDs with isochrones by eye. This is statistically unacceptable (Taylor 2001) but remains a popular procedure due to the absence of a straightforward alternative. As the CMD morphology depends on many parameters which are correlated, most researchers assume reasonable defaults for some, for example tying helium abundance to metal abundance (as in most theoretical models from which isochrones are derived) and assuming no age spread, differential reddening or theoretical uncertainties in the isochrones used. The position of the clump of red giant stars is a useful piece of extra information in intermediate-age clusters, although the theoretical uncertainty in its position is significant (Daniel et al. 1994; Romaniello et al. 2000). Simultaneous analysis of two or more CMDs or colour-colour diagrams is subject to more minor correlations so allows the derivation of more accurate parameters (Tosi et al. 2004).

The presence of overshooting has a significant effect on the MS turn-off shape of intermediate-age open clusters. Studies of such objects consistently find that a moderate amount of overshooting is required (e.g., Chiosi 1998; Nordström, Andersen & Andersen 1997; Woo et al. 2003).

2 Analysis of detached eclipsing binaries

2.1 Observing detached eclipsing binaries

The study of dEBs requires high-quality data to give definitive results. The determination of accurate masses depends mainly on the analysis of high-resolution spectroscopic observations, and measurement of stellar radii requires accurate and extensive relative photometry. An additional complication is that both types of observations are needed at many orbital phases.

2.1.0.4 Photometry of dEBs

The observation of light curves for dEBs requires complete coverage of the light variation through both primary and secondary eclipse, plus regular observations outside eclipse to provide a reference light level and constrain effects such as reflection. The minimum requirements for a light curve to be definitive are discussed in section 2.4.2.

Using a telescope and CCD imager is a good way to obtain light curves of a dEB. During eclipses the dEB must be monitored continually by repeatedly imaging it and a comparison star. Differential photometry can then be performed on the images to obtain the light curve. It is advisable to observe light curves in several passbands to provide independent photometric datasets. This can be done by cycling continually through several passbands whilst observing but will obviously decrease the amount of data contained in each light curve. A balance must therefore be struck between obtaining several light curves and ensuring that each has sufficient data to be useful. The best approach depends on the length and depth of the eclipses of the dEB, its brightness and the passbands being used, on the amount of telescope time available, and on the observational efficiency achievable with the telescope and imager.

2.1.0.5 Spectroscopy of dEBs

Obtaining spectroscopy of dEBs is more interesting and time-efficient than observing light curves. The requirements for a definitive spectroscopic orbit are discussed in section 2.2.4, but mainly comprise regular observations throughout the orbital period of a dEB. As continual monitoring is not required, spectra can be obtained to determine the orbits of several dEBs at once. One observing run can therefore yield definitive orbits for many dEBs. The observing run must be long enough to cover most of the orbital phases of each dEB (see section 5.3 for the problems which occur when this was not possible) if good spectroscopic orbits are going to be obtained.

When acquiring spectroscopic observations of a dEB, it is a good idea to observe a spectrum when the RV separation of the two stars is minimal. This spectrum can be useful as a template spectrum when determining RVs by cross-correlation. It is also good practice to observe one spectrum with a very high signal to noise and a large RV separation between the two stars. This spectrum can then be analysed using spectral synthesis techniques to find more accurate T_{eff} s and rotational velocities for the stars.

2.2 Determination of spectroscopic orbits

2.2.1 Equations of spectroscopic orbits

A full derivation of the equations of motion of binary stars in an elliptical orbit is lengthy and readily available from other sources (for example Hilditch 2001). Therefore I shall quote the resulting equations which are of use to the study of spectroscopic binary stars. For these stars, the RVs of one or both components are observed at certain times, allowing the derivation of the mass function (for single-lined spectroscopic binaries) or the individual projected masses and stellar separation (for double-lined spectroscopic binaries).

Radial velocity (RV) as a function of time is given by:

$$V_r = K[\cos(\theta + \omega) + e \cos \omega] + V_\gamma \quad (2.1)$$

where θ is the orbital phase in radians, ω is the longitude of periastron, e is the orbital eccentricity, V_γ is the systemic velocity and the velocity semiamplitude K is

$$K = \frac{2\pi a \sin i}{P\sqrt{1-e^2}} \quad (2.2)$$

where a is the semimajor axis, i the inclination and P the period of the orbit.

From the definition of K we get the minimum masses of the stars:

$$M_{1,2} \sin^3 i = \frac{1}{2\pi G} (1-e^2)^{\frac{3}{2}} (K_1 + K_2)^2 K_{2,1} P \quad (2.3)$$

where G is the gravitational constant, and

$$a_{1,2} \sin i = \frac{\sqrt{1-e^2}}{2\pi} K_{1,2} P \quad (2.4)$$

$$a \sin i = a_1 \sin i + a_2 \sin i \quad (2.5)$$

Using the usual astrophysical units of solar masses, period in days and velocities in km s^{-1} , we obtain:

$$M_{1,2} \sin^3 i = 1.036149 \times 10^{-7} (1-e^2)^{\frac{3}{2}} (K_1 + K_2)^2 K_{2,1} P \quad (2.6)$$

where the value of the numerical constant has been recommended by the International Astronomical Union (Torres & Ribas 2002). Note that Andersen (1998) gives a different value of 1.036055×10^{-7} . We also get

$$a \sin i = 1.3751 \times 10^4 \frac{\sqrt{1-e^2}}{2\pi} (K_1 + K_2) P \quad (2.7)$$

where the projected separation, $a \sin i$, is in kilometres.

In the case of single-lined spectroscopic binaries we can get the mass function

$$f(M) = \frac{1}{2\pi G} (1-e^2)^{\frac{3}{2}} K^3 P = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} \quad (2.8)$$

where the factor $\frac{1}{2\pi G}$ has the numerical value discussed above in the equation 2.6. The significance of the mass function is that it provides an estimation of the mass of the secondary component of a single-lined spectroscopic binary.

2.2.2 The fundamental concept of radial velocity

The classical definition of RV is the component of the velocity of a star along the line of sight of the observer (e.g., Kaufmann 1994; Zeilik & Gregory 1998). Whilst this definition has the advantage of being simple, the observed spectroscopic RV of a star is different to its actual motion through space due to several physical effects. This has prompted the International Astronomical Union¹ to re-examine the fundamental concept of RV and provide a more precise definition (Lindgren & Dravins 2003).

There are several physical effects which cause observed spectroscopic RVs to differ from the actual RVs of celestial bodies (Lindgren & Dravins 2003):–

- Gravitational redshift is the increase in wavelength of photons caused by their escape from the gravitational potential of the star which emitted them. The term also encompasses the slight blueshift due to the photons falling into the gravitational potential well of the Sun and the Earth before being detected by observers. The gravitational redshift effect is of the order of 1 km s^{-1} for MS stars, increasing to 30 km s^{-1} for white dwarfs. It is usually unimportant because it affects all similar stars in a similar way, and is constant over long periods of time for individual stars. The velocity change due to gravitational redshift is given by the formula

$$V_{\text{grav}} = \frac{GM}{rc} \quad (2.9)$$

where G is the gravitational constant, M is the mass of the emitting body, r is the distance the photon is emitted from the centre of mass of the body and c is the speed of light.

- Convective blueshift is the decrease in wavelength caused by convective motions on the surfaces of stars of types F and later. These convective motions cause stellar surfaces to be divided into columns of rising and falling gas, visible as the

¹<http://www.iau.org/>

granulation effect on the surface of our Sun. The rising and falling components occupy roughly equal areas of a stellar surface but the convective velocities cause spectral lines to be blueshifted from rising columns and redshifted from falling columns. As the rising material is hotter, it is brighter, so it contributes more to the stellar flux, so the overall effect is a blueshift. This shift is of the order of 1 km s^{-1} for F stars, falling to 200 m s^{-1} for K stars. The magnitude of the effect is greater at shorter wavelengths but, again, is usually unimportant as its effects cause a constant RV offset for a specific star.

The above effects have recently become more important due to improvements in instrumentation, so a precision of 1 m s^{-1} is possible on bright stars, and due to the development of the concept of the astrometric RV. The analysis of this effect can provide accurate individual RVs of a group of stars with accurate trigonometrical parallaxes and the same motion in space. Astrometric RVs are not determined spectroscopically so are not subject to the difficulties and limitations given above (see Dravins, Lindegren & Madsen 1999 and subsequent works).

The total effect of convective blueshift and gravitational redshift was investigated by Pourbaix et al. (2002) for the components of the nearby visual binary α Centauri. The estimated difference between the two components, $215 \pm 8 \text{ m s}^{-1}$, is much smaller than that predicted by hydrodynamical model atmosphere calculations. This technique may provide a valuable constraint on theoretical model atmospheres in the future.

2.2.3 Radial velocity determination from observed spectra

There are two major difficulties in determining double-lined spectroscopic orbits from observations. The first problem is that the spectral lines of the secondary star, which is usually dimmer than the primary star, are diluted by the continuum emission of the primary star. It can be impossible to find signatures of the secondary component in spectra if the light ratio is very small. For a given mass ratio, the light ratio in the infrared is usually much closer to unity than the light ratio in the optical (Mazeh et

al. 1995) because cooler stars are redder.

The second problem is that the spectral lines of one star can be distorted by the presence of spectral lines due to a second star. This blending can cause the centres of the lines to be apparently displaced towards each other, lowering the masses derived from the spectra. This primarily affects hydrogen lines because they are much wider than metallic lines, but helium lines can also be affected. Whilst the measuring of individual spectral lines can be badly affected by this, more recent techniques for determining RVs from composite spectra are much more reliable.

2.2.3.1 Radial velocities from individual spectral lines

The traditional method of the determination of RVs from observed spectra involves the measurement of the wavelength centres of individual spectral lines, which are then compared with rest wavelengths found in either the laboratory or in high-resolution, high signal-to-noise stellar spectra. This method is ideally suited to the analysis of photographic plate spectra, where the plates are placed inside one of several different types of machine for interactive measurement of spectral line positions. Due to the small number of sharp (metallic) spectral lines exhibited by many early-type stars, this method is often competitive with more recent techniques of RV analysis of these stars, and has the advantages of simplicity and robustness.

One problem with the measurement of individual spectral lines is that the line centres may be displaced in wavelength by interference from other nearby lines – the blending effect (Petrie & Andrews 1966). If the interfering lines are from the same star then the blending effect will be constant and therefore easily dealt with. If, however, the interfering lines are from another star, in the case of composite spectra, the effects of blending can be very strong and difficult to quantify. Hilditch (1973) suggests that spectral lines should be used for RV determination only if the flux returns to the continuum level on both sides of the line. Andersen et al. (1987) found, during a study of V1143 Cygni using spectral lines measured from photographic spectra, that line blending can lower the derived RV difference in a double-lined spectrum without distorting

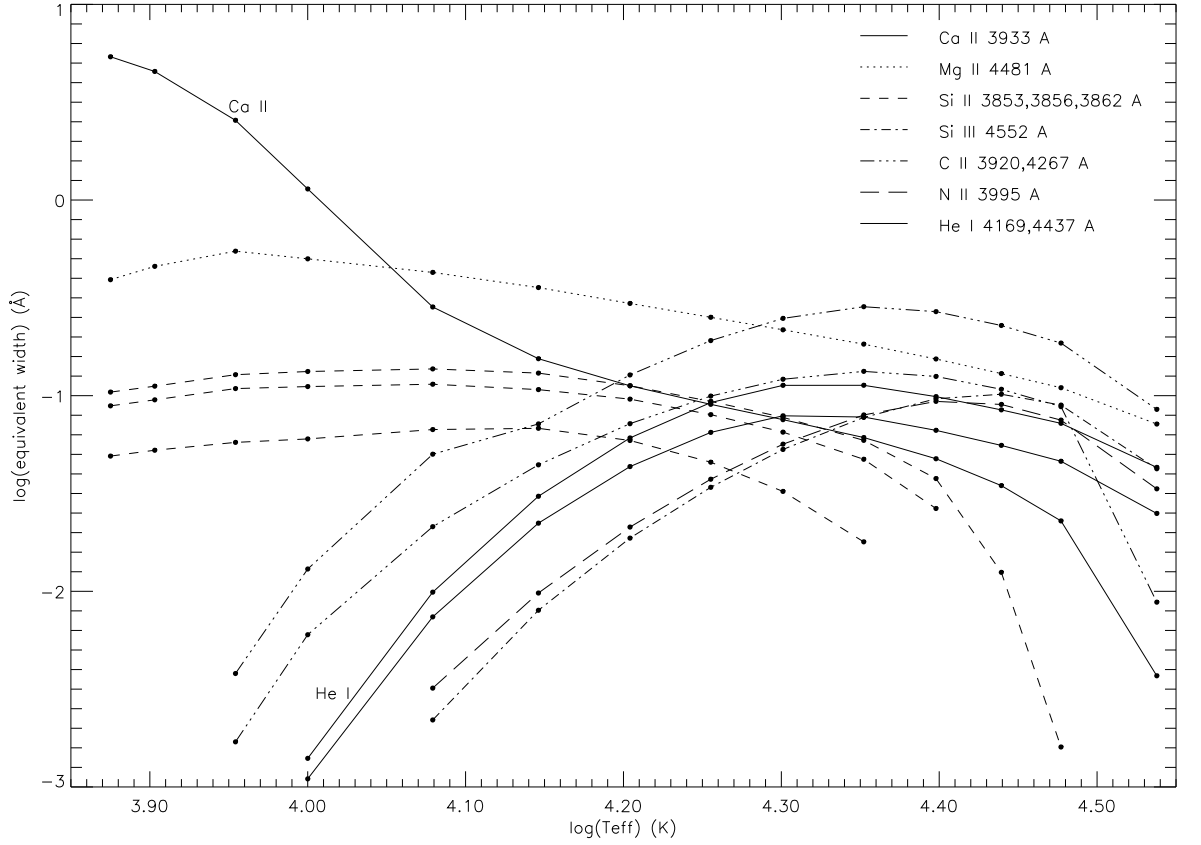


Figure 2.1: Variation of the equivalent widths, with T_{eff} , of the spectral lines given by Andersen (1975a) as good for deriving RVs of early-type EBs, with particular reference to CV Velorum ($\log T_{\text{eff}} = 4.26$ K). The data were generated using UCLSYN (section 1.4.3.2).

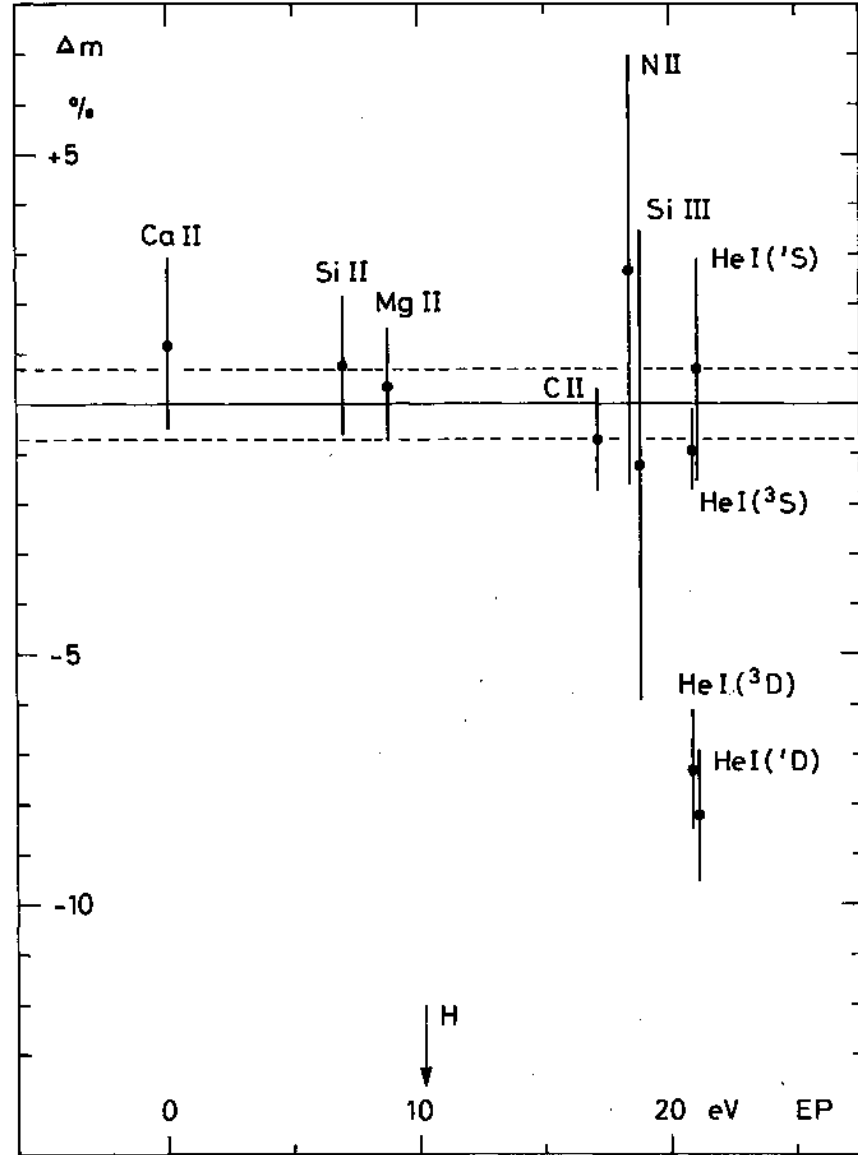


Figure 2.2: Percentage deviation of the masses of CV Velorum derived using the lines of individual ions, plotted against excitation potential. The reference masses are averages of the values for several of these ions. Taken from Andersen (1975a).

the shape of the spectroscopic orbit, so blending cannot necessarily be detected by analysing the residuals of a spectroscopic orbital fit. Andersen (1991) suggests that spectra of a high signal to noise ratio should be obtained so RVs can be measured from (weak) metal lines rather than (strong) helium or hydrogen lines.

Several researchers have investigated the best spectral lines for measurement of RVs and have generally found that hydrogen and helium lines should be avoided wherever possible. During the study of the EB PV Puppis (spectral type A8 V, $T_{\text{eff}} = 6920$ K), Vaz & Andersen (1984) found that the velocity semiamplitudes derived from analysis of hydrogen lines were 72% of those derived using sharp metal lines. Andersen (1975a) noted that the helium lines in the spectrum of CV Velorum (spectral type B2.5 V, $T_{\text{eff}} = 18300$ K), gave velocity semiamplitudes 8% smaller than those derived from sharp metallic lines.

Andersen (1975a) studied many blue spectra of CV Vel and suggested several spectral lines which are good for the determination of RVs in composite spectra. He noted that it was important to avoid hydrogen lines and diffuse helium lines (at wavelengths of 3819, 4009, 4026, 4143, 4388, 4471 Å) but that sharp helium lines at 3867, 4120, 4169, 4437, 4713 Å were reliable. Fig. 2.2 shows the masses derived for CV Vel from different spectral lines against the final adopted values. Mg II 4481 Å is the most reliable line despite it being a close triplet. Fig. 2.1 shows the equivalent widths of the spectral lines selected as good by Andersen for CV Vel, against T_{eff} . Note that the Mg II 4481 Å line is strong over a wide range of T_{eff} s, making it the best individual line for derivation of RVs in early-type stars (e.g., Popper 1980). For spectral types later than mid A, there is a profusion of spectral lines and the main problem faced in RV determination is the identification of lines which are not blended with neighbouring lines. For mid B to late O stars, there are several useful helium lines and a large number of weak, sharp O II lines in the blue spectral region. For stars earlier than late O, very few optical lines are visible, due to the generally fast rotation (Popper & Hill 1991) and the high ionisation causing most lines to be in the UV, and only helium lines are reliable. Table 2.1 gives several spectral lines, selected from the literature, which are considered to be reliable sources of RV information.

Table 2.1: Selected spectral lines indicated in the literature to be good for the determination of RVs in early-type stars. Only the earliest reference is given for each line.

Species	Wavelength (Å)	Reference
Si II	3853	Andersen (1975a)
Si II	3856	Andersen (1975a)
Si II	3862	Andersen (1975a)
He I (³ S)	3867	Andersen (1975a)
Fe I	3878.5	Andersen (1975b)
C II	3919	Andersen (1975a)
C II	3920	Andersen (1975a)
Ca II	3933	Andersen (1975a)
N II	3995	Andersen (1975a)
Fe I	4071.7	Andersen (1975b)
Si III	4089	Burkholder et al. (1997)
Si III	4116	Burkholder et al. (1997)
He I (³ S)	4120	Andersen (1975a)
Si II	4128.0	Popper (1982)
Si II	4130.9	Popper (1982)
Fe I	4143.6	Andersen (1975b)
He I (¹ S)	4169	Andersen (1975a)
Si IV	4212.4	Hensberge et al. (2000)
Sr II	4215.7	Andersen (1975b)
C II	4267	Andersen (1975a)
Fe II	4351.7	Andersen (1975b)
He I (¹ S)	4437	Andersen (1975a)
Mg II	4481	Andersen (1975a)
Ti II	4501.3	Andersen (1975b)
Fe II	4508.3	Andersen (1975b)
Si III	4552	Andersen (1975a)
Si III	4567	Popper & Guinan (1998)
Ti II	4572.0	Andersen (1975b)
Si III	4574	Popper & Guinan (1998)
O II	4591.0	Hensberge et al. (2000)
O II	4596.2	Hensberge et al. (2000)
Fe II	4583.8	Andersen (1975b)
N III	4634	Burkholder et al. (1997)
N III	4641	Burkholder et al. (1997)
C II	4650	Burkholder et al. (1997)
Si IV	4654.3	Hensberge et al. (2000)
O II	4661.6	Hensberge et al. (2000)
He I (³ S)	4713	Andersen (1975a)
Si II	6347.1	Zwahlen et al. (2004)
Si II	6371.4	Zwahlen et al. (2004)

2.2.3.2 Radial velocities from one-dimensional cross-correlation

The cross-correlation technique can be used to determine the RV shift of a star, or several stars if the observed spectra are composite, by comparison with a template spectrum. First introduced by Simkin (1974), the method was further developed by Tonry & Davis (1979). The cross-correlation function is

$$C_{f,g}(s) = \frac{\sum_n f(n)g(n-s)}{N\sigma_f\sigma_g} \quad (2.10)$$

where $f(n)$ is the observed spectrum, $g(n)$ is the template spectrum, s is a shift in velocity, $g(n-s)$ is a velocity-shifted template spectrum, N is the number of points in each spectrum, and the root-mean-squared values of the spectra are given by

$$\sigma_f^2 = \frac{1}{N} \sum_n f(n)^2 \quad (2.11)$$

$$\sigma_g^2 = \frac{1}{N} \sum_n g(n)^2 \quad (2.12)$$

The velocity shift between the observed and template spectra is estimated from the location, $\hat{s}$, of the maximum of the cross-correlation function $C_{f,g}$. The method of cross-correlation effectively involves the comparison between the observed spectrum and a velocity-shifted template spectrum for a range of velocity shifts, the derived RV difference being where the two spectra have best agreement.

In choosing the template spectrum it is important that it matches the observed spectrum as closely as possible. A close match is useful when studying single-lined spectra, but can be vital when analysing composite spectra. In this case, the spectral lines of each star will cause a local maximum in the cross-correlation function. If the maxima are well-separated in velocity, this causes no significant problem, but if the RV separation of the two stars is significantly less than the sum of their spectral line broadenings then the individual maxima in the cross-correlation function will become blended in a very similar way to individual spectral lines.

Template spectra can be observed spectra of e.g., standard stars. The advantage of using observed spectra is that the researcher is utilizing only observational data,

and so avoiding the use of any theoretical calculations. The disadvantages are that it takes telescope time to obtain template spectra, and the available templates may not be a very good match to the spectrum being analysed. An alternative possibility is to use synthetic spectra as templates. Whilst this means that careful steps must be taken to minimise the dependence of the result on theoretical calculations, it has the advantage that synthetic spectra are more readily available and are free of observational noise. Having no observational noise, the results will be more precise, and the synthetic spectrum can be carefully adjusted to best match the observed spectrum just by use of a desktop computer. However, systematic biases may occur if the synthetic spectrum has missing lines, or similar difficulties. Such problems are negligible for the analysis of relatively well-understood stars such as mid B-type to G-type dwarf stars.

The light ratios of double-lined binary systems can be found by comparison of the areas under the maxima of the cross-correlation function (e.g., Howarth et al. 1997). This is possible because these areas are approximately constant for different rotational velocities, but differences between the intrinsic stellar spectra can affect the area under the maxima of the cross-correlation function.

2.2.3.3 Radial velocities from two-dimensional cross-correlation

The main shortcoming of cross-correlation for measuring stellar RVs is that the cross-correlation function in composite spectra contains contributions from several stars, which may interfere with each other and bias the derived RVs. Zucker & Mazeh (1994) and Mazeh et al. (1995) extended the cross-correlation algorithm to explicitly allow for contaminating spectral lines from a second star. They called this two-dimensional cross-correlation algorithm TODCOR. The cross-correlation function is

$$R_{f,g_1,g_2}(s_1, s_2, \alpha) = \frac{\sum_n f(n)[g_1(n - s_1) + \alpha g_2(n - s_2)]}{N\sigma_f\sigma_g(s_1, s_2)} \quad (2.13)$$

where $g_1(n)$ and $g_2(n)$ are the template spectra, s_1 and s_2 are velocity shifts, α is the intensity ratio of the two stars which can be evaluated analytically, and

$$\sigma_g(s_1, s_2)^2 = \frac{1}{N} \sum_n [g_1(n - s_1) + \alpha g_2(n - s_2)]^2 \quad (2.14)$$

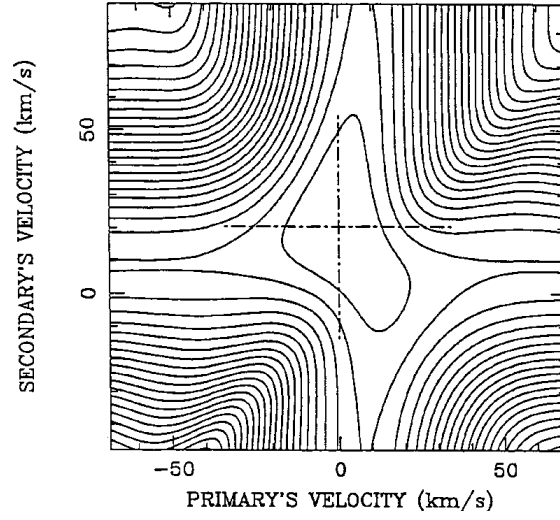


Figure 2.3: An example contour plot of the two-dimensional cross-correlation function around the global correlation maximum. The dashed lines are parallel to the axes and go through the maximum correlation value. Taken from Zucker & Mazeh (1994).

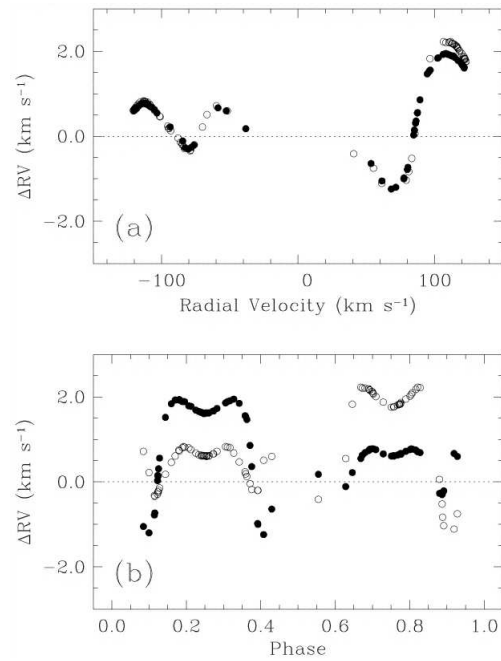


Figure 2.4: Systematic errors of the RVs derived by using TODCOR to analyse the M dwarf dEB YY Geminorum. The systematic error is shown as a function of RV and of orbital phase. Open circles refer to the primary star and filled circles to the secondary star. Taken from Torres & Ribas (2002).

This method effectively involves the simultaneous comparison between the observed spectrum and two template spectra, over a range of velocity shifts for each template spectrum. R_{f,g_1,g_2} is a two-dimensional function where the global maximum gives the RV shifts of both stars. Blending is much less important because two template spectra are fitted simultaneously, so lines which would otherwise contaminate the RV determination of the other star are explicitly dealt with (Latham et al. 1996). An example cross-correlation function is shown in Fig. 2.3.

The comments in the previous section on the choice of template spectra are equally valid for two-dimensional cross-correlation, but one important advantage of TODCOR is that the template spectra do not have to be the same – in fact it is helpful if they are not – so each template can be a close match to one of the two stars. This was not possible with one-dimensional cross-correlation where one template had to fit the spectra of all the stars in the spectrum.

One problem with this technique concerns the edges of the spectra. As the observed and template spectra are required to be the same length for cross-correlation, but a velocity shift is usually imposed, parts of one spectrum extend beyond the end of the other spectra. These parts do not contribute to the correlation function so can lower the overall correlation value, biasing the derived RVs. A simple compensation method is to taper the ends of each spectrum, but whilst this lowers the bias it cannot remove it entirely. Another method is to assess the systematic RV error by analysing synthetic spectra with known RVs and observational noise added. An example of systematic errors, which were removed from the individual velocities, is given in Fig. 2.4.

Zucker, Torres & Mazeh (1995) further extended TODCOR to the study of triple-lined stellar spectra where the correlation function is

$$R_{f,g_1,g_2,g_3}(s_1, s_2, s_3, \alpha, \beta) = \frac{\sum_n f(n)[g_1(n - s_1) + \alpha g_2(n - s_2) + \beta g_3(n - s_3)]}{N\sigma_f\sigma_g(s_1, s_2, s_3)} \quad (2.15)$$

This is effectively a three-dimensional function where three template spectra are simultaneously correlated against one observed spectrum. As such, it is quite expensive in

terms of computational time, and extensions to four or more templates would be prohibitively expensive. However, the stellar intensity ratios α and β can still be evaluated entirely analytically.

Zucker et al. (2003) have applied TODCOR to multi-order échelle spectroscopic observations. In this case, cross-correlation over the whole spectrum is problematic because of the gaps between individual orders, so orders were cross-correlated individually and the resulting functions combined, using the maximum-likelihood technique of Zucker (2003), to produce one function.

2.2.3.4 Radial velocities from spectral disentangling

The spectral disentangling technique can be used to find the individual spectra of a double-lined binary star from several observed spectra. The algorithm requires a set of observed spectra together with the RVs of both stars for each spectrum and outputs estimated individual disentangled spectra with a calculated residual of the fit. The RVs can be determined by minimising the residual value, either directly or by fitting a spectroscopic orbit. The algorithm was introduced by Simon & Sturm (1994) and applied to the high-mass EBs DH Cephei (Sturm & Simon 1994) and Y Cygni (Simon et al. 1994). The method was intended to help in the derivation of RVs when the spectral lines of one star were badly blended with those of the other star, and to create individual spectra which were suitable for spectroscopic analysis in the same way as single-lined spectra.

Hynes & Maxted (1998) investigated spectral disentangling and found that the quality of the results was dependent mainly on the total exposure time of the observed spectra, although Simon & Sturm (1994) suggest the minimum useful signal-to-noise ratio is 10. Hynes & Maxted were unable to find a robust method of estimating the errors in the derived RVs because the disentangling process is not strictly equivalent to least-squares minimisation. It is still not clear if disentangling can provide robust errors (P. F. L. Maxted, private communication), but Ilijić (2003) has pioneered the estimation of uncertainties by fitting spectroscopic orbits to observed spectra by dis-

entangling. The code `FDBINARY` (Ilijić 2003) calculates the best-fitting spectroscopic orbits for several data subsets where each subset contains $N - 1$ observed spectra, where N is the total number of spectra. This gives $N - 1$ estimations of the spectroscopic parameters, which can then be subjected to straightforward error analysis. This method has been used by Zwahlen et al. (2004) to determine a spectroscopic orbit in a double-lined binary system exhibiting severe blending of spectral lines.

An alternative approach to the use of singular value decomposition of matrix equations by Simon & Sturm (1994) is to use Fourier techniques as implemented in `KOREL` (Hadrava 1995). `KOREL` has been used in several studies, for example Hensberge, Pavlovski & Verschueren (2000).

2.2.4 Determination of spectroscopic orbits from observations

It is clear from the above discussion that determination of the gravitational masses of dEBs requires measurement of only the velocity semiamplitudes and the orbital inclination (Popper 1967). Under the assumption of a circular orbit, these quantities can be found using only four RVs measured from two spectra (e.g., Wilson 1941), but accurate and robust results require at least 25 RVs with individual uncertainties of 1 km s^{-1} (Andersen 1991). However, several complications exist:–

- The measured systemic velocities for the two stars may be different. This is an observational effect caused by (Popper & Hill 1991):–
 1. assumption that the orbit is circular when it has a small eccentricity,
 2. small differences in the spectral line profiles of the two stars,
 3. blending effects, where the spectral lines of one star cause the spectral line centres of the other star to shift slightly, particularly if the rotational velocities of the two stars are different (Popper 1974),
 4. small-number statistics,

5. stellar winds or gas streams modifying the spectral line profiles (the Barr effect; Barr 1908; Howarth 1993),
 6. the use of different spectral lines or regions for determination of the RVs of the two stars.
- The Rossiter effect causes asymmetric spectral line profiles, shifting the observed velocity centre away from the actual RV of the star. As most spectral line profiles depend mainly on rotational broadening, different parts of a star contribute to different parts of a spectral line. Therefore if one side of a star is not observed, for example during partial phases of eclipses, part of the spectral line profile is not present in observations, shifting the measured RV value. This effect was first noticed by Rossiter (1924). The Rossiter effect can be allowed for by solving spectroscopic and photometric observations simultaneously using, for example, the Wilson-Devinney code (section 2.4.1.2). In this case the information it holds on the sizes of the two stars can also be accessed.
 - When the exposure time of a spectroscopic observation becomes more than a few percent of the orbital period of the spectroscopic binary under study, the changes in RV of the two stars during the observation become important (Andersen 1975b). This orbital smearing can be corrected by adjusting each wavelength shift by (Lacy 1982)

$$\Delta\lambda = \frac{2\pi\lambda K}{c} \frac{t_{\text{exp}}}{P} \cos\theta \quad (2.16)$$

where t_{exp} is the exposure time in the same units as the period and θ is the orbital phase in radians. This shift must be applied to individual observations after a preliminary orbit has been calculated. An example of its use is in the study of the dEB CM Lacertae by Popper (1968). CM Lac has an orbital period of 1.6 days but exposure times of 150 minutes (6.5% of the period) were used for the spectroscopic observations.

- For RV work where the precision of an observation approaches 100 m s^{-1} , a level now routinely being passed by spectroscopic searches for extrasolar planets

(e.g., Butler et al. 1996), relativistic effects due to the position and motion of the Earth and Sun must be allowed for (Griffin et al. 1985).

- Spectroscopic orbital solutions often indicate an uncertainty, σ_e , in the orbital eccentricity, e , which is of the same order as the value itself. In this case the researcher must decide whether the orbit is circular, and the small eccentricity is a spurious effect caused by observational uncertainty, or that the orbit really is eccentric. Arias et al. (2002) point out that if $e/\sigma_e > 3.83$ then eccentricity is significant at the 5% level. Several studies have been devoted to the reanalysis of eccentric orbits which were previously assumed circular (e.g., Wilson 1970), and of circular orbits for which a spurious eccentricity was previously found (e.g., Lucy & Sweeney 1971). In the absence of consensus (as indicated by the last two references) it is up to the researcher to decide which procedure is appropriate for individual spectroscopic binaries under analysis.
- Fast apsidal motion (see section 1.7.2) can cause the orientation of the orbit (given by the longitude of periastron, ω) to change during a spectroscopic observing campaign. Whilst this can be incorporated into any analysis, the effect should be negligible in the vast majority of cases.
- The spectroscopic binary may be part of a hierarchical triple star system. This can cause a variation in the systemic velocity of the binary. The presence of the third star can be detected by observation of its spectral lines, light travel time effects (for an EB) or by the systemic velocity variation of the close binary.
- Reflection between the components of a close binary will tend to draw the light-centres of the two discs together and reduce the observed RV difference. This effect is significant for MS EBs only if the fractional sum of the radii is greater than 0.4 (Andersen 1975a), or when there is a large difference in luminosity between the two stars.
- The Struve-Sahade effect is that the secondary star tends to exhibit stronger lines when approaching the observer (Struve 1944; Penny, Gies & Bagnuolo

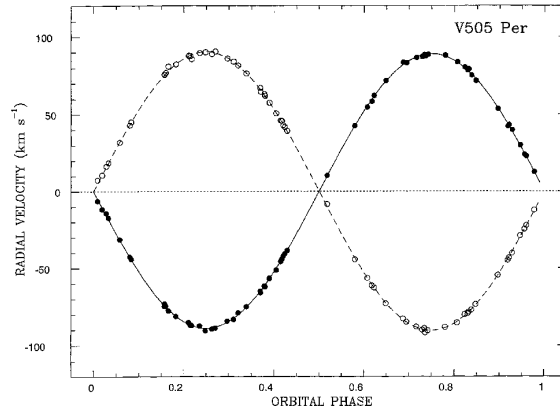


Figure 2.5: Example of a definitive spectroscopic orbit, for the dEB V505 Persei. Radial velocities were derived using one-dimensional cross-correlation of synthetic spectra against CCD spectra observed using an échelle spectrograph. Velocities for the primary star are shown by filled circles, and for the secondary star are shown using open circles. Taken from Marschall et al. (1997).

1999). It may result from interaction between the winds of the two stars (Arias et al. 2002).

An example spectroscopic orbit is shown in Fig. 2.5.

2.2.4.1 SBOP – Spectroscopic Binary Orbit Program

SBOP was written by P. B. Etzel² (2004) and is a modification of an earlier code by Wolfe, Horak & Storer (1967). The code fits single-lined or double-lined spectroscopic orbits to the observed RVs of a spectroscopic binary using one of several optimisation schemes based on differential corrections.

²<http://mintaka.sdsu.edu/faculty/etzel/>

2.2.5 Determination of rotational velocity from observations

The total broadening of metallic spectral lines can easily be measured using a Gaussian function (e.g., Abt, Levato & Grosso 2002). An alternative is to measure broadening from the cross-correlation function of the spectrum against a template, but this must be calibrated on stars with known rotational velocities, or using synthetic template spectra. However, broadening values determined from consideration of cross-correlation functions are better than those determined from individual spectral lines because they include contributions from all the lines and so are more precise (increased signal to noise) and accurate (they are insensitive to difficulties associated with individual spectral lines) (Hilditch 2001, p. 79).

The broadening due to the rotational velocity of the star, however, may be smaller than the total broadening. Additional broadening comes from microturbulence and macroturbulence, which are in principle separable from rotational broadening but in reality are highly degenerate. For most types of star the additional broadening is known to be negligible, from the study of dEBs which are rotationally synchronized, but for O stars and evolved B stars the contribution from macroturbulence can be much larger than the contribution from rotation (Trundle et al. 2004).

Popper (2000) used the measured rotational velocities for four late-type dEBs, and an assumption of synchronous rotation, to predict the stellar radii using

$$V_{\text{synch}} = 50.58 \frac{R}{R_{\odot}} \frac{\text{days}}{P} \quad \text{km s}^{-1} \quad (2.17)$$

where R is the stellar radius and P is the orbital period (Abt, Levato & Grosso 2002). This analysis is also possible in slightly eccentric orbits under the assumption of pseudosynchronous rotation (rotation velocity which is synchronous with orbital velocity at periastron). In this case the periastron rotational frequencies of the stars, ω_{peri} are related to the mean orbital frequency of the orbit, $\bar{\omega}_{\text{orbit}}$ by

$$\omega_{\text{peri}} = \frac{(1+e)^2}{(1-e^2)^{-3/2}} \bar{\omega}_{\text{orbit}} \quad (2.18)$$

(Griffin, Carquillat & Ginestet 2003).

2.3 Photometry

Photometry is the most fundamental of all observational tools used in astronomy (Crawford 1994). It allows us to find out what exists in our Galaxy and Universe and to classify them based on their brightness at different wavelengths. This classification relies on comparing the object being studied to objects with similar photometric characteristics for which much more is known.

Stellar parameters can be estimated from photometry, using calibrations based on the photometric properties of stars with known parameters. This allows researchers to estimate T_{eff} s and luminosities of other stars from comparison of their photometric indices with the indices of stars of known properties. Other properties, such as metal abundance and surface gravity, can also be found using calibrations reliant on stars with fundamental determinations of these properties.

2.3.1 Photometric systems

The first good photometric systems used wide-band passbands, to maximise the amount of detected light whilst still not being badly affected by chromatic effects such as atmospheric transmission. Broad-band systems, however, must be very well constructed to provide accurate and precise information about stars, and so often are not able to do so. This has led to the construction of intermediate-band systems, such as the Strömgren *uvby* and Geneva *UBB₁B₂VV₁G* passbands, which are much better suited to the classification of most types of stars than the broad-band *UBVR₁IKLMN* systems. Broad-band Johnson-style photometric systems are currently the most popular with observers, but intermediate-band systems have an important place in many research programmes and can be surprisingly successful at estimating stellar parameters.

The Asiago Database of Photometric Systems³ (Moro & Munari 2000) lists detailed information on the passbands and other characteristics of 167 optical, ultraviolet

³Also available on the internet at
<http://ulisse.pd.astro.it/Astro/ADPS/>

and infrared photometric systems, starting with the *UVBGR* system of Stebbins & Whitford (1943) and ending with the suggested passband systems of the GAIA astrometric satellite, along with brief descriptions of another 34 systems.

Intermediate band systems have many intrinsic advantages. Firstly, they are defined mainly by filters because the change in sensitivity of a light detector over 200 Å is usually negligible. Narrower filters can also be carefully targeted to measure the effects of individual features in the spectra of certain stars, resulting in easier and more accurate calibrations. However, using intermediate-band rather than broad-band systems is only advantageous if 1% photometric accuracy is achieved (Bessell 1979).

Mermilliod & Paunzen (2003) have studied the interagreement between different sets of photometry and photometric systems in the WEBDA open cluster database⁴. They conclude that the best photometry, in terms of agreement between different datasets, is photoelectric photometry in the Strömgren system and then the Johnson system (other intermediate-band systems were not considered). Intriguingly, CCD photometry is not as good as photoelectric photometry for both the Strömgren and Johnson systems, despite CCDs being better suited to photometry (R. Jeffries, 2005, private communication). This does suggest that the difficulties associated with photoelectric photometry – where only one star can be observed at any one time – means that researchers treat data reduction particularly carefully. Another possible difficulty is that different pixels on a CCD detector are used to observe light from different stars, whereas the same detector area is used for different stars when using a photoelectric photometer, so CCD accuracy is limited by flat-fielding errors.

2.3.1.1 Broad-band photometric systems

The most commonly used photometric system is *UBV* (ultraviolet, blue, visual) developed by Johnson & Morgan (1953) to aid in the classification of stars (Hilditch 2001, p. 186). The original system was defined using glass filters and photoelectric photome-

⁴Available on the internet at <http://obswww.unige.ch/webda/>

Table 2.2: Central wavelengths and bandwidths of broad-band filters. Data taken from Moro & Munari (2000).

Filter	Central wavelength (μm)	FWHM (μm)
<i>U</i>	0.36	0.04
<i>B</i>	0.44	0.10
<i>V</i>	0.55	0.08
<i>R</i> (Johnson)	0.70	0.21
<i>I</i> (Johnson)	0.90	0.22
<i>R</i> (Cousins)	0.67	0.15
<i>I</i> (Cousins)	0.81	0.11
<i>J</i>	1.25	0.3
<i>H</i>	1.62	0.2
<i>K</i>	2.2	0.6
<i>L</i>	3.4	0.9
<i>M</i>	5.0	1.1
<i>N</i>	10.2	6.0

ters. This system was subsequently extended to redder wavelengths with the $R_J I_J$ (Johnson red, Johnson infrared) passbands when more advanced photometers were developed. Alternative RI passbands have been defined by Cousins (1980), Kron & Smith (1951) and Eggen (1965). Bessell (1979) suggests that the Cousins passbands are the best broad-band red-light system, and provides transformation equations between the different systems.

The $UBVRI$ system has been extended to infrared wavelengths by Johnson (1966) with the passbands designated $JKLMN$, which are targeted at wavelength ranges where water vapour in the Earth's atmosphere does not attenuate photons significantly. $JHKL$ standard stars were published by Elias et al. (1982) and Bessell & Brett (1988) have revisited the $JKLMN$ system by Johnson and several alternative infrared broad-band systems (e.g., Glass 1973; Elias et al. 1982; Jones & Hyland 1982), and defined a homogenized system. Table 2.2 gives the central wavelengths of the broad-band passbands discussed above. The $J-K$ index is sensitive to metallicity,

but most infrared indices vary little for MS stars (Pinsonneault et al. 2003). The K passband is very insensitive to surface gravity and metallicity (Johnson 1966).

2.3.1.2 Broad-band photometric calibrations

$UBVRI$ photometry is not the best way to get individual stellar parameters, but the large light throughput of the filters causes them to remain popular with researchers. $B-V$ is sensitive to T_{eff} whereas $U-B$ is sensitive to T_{eff} and surface gravity (Phelps & Janes 1994). The B passband is also known to be sensitive to metallicity via flux redistribution due to line blanketing (Alonso, Arribas & Martínez-Roger 1996). However, for F, G and K stars $V-I$ is a good metallicity-independent T_{eff} indicator, and $R-I$ is useful for later-type stars (Alonso, Arribas & Martínez-Roger 1996).

The photometric index Q was introduced by Johnson & Morgan (1953) to provide a reddening-free estimator of T_{eff} :

$$Q = (U-B) - \frac{E_{U-B}}{E_{B-V}}(B-V) = (U-B) - 0.72(B-V) \quad (2.19)$$

where E_{X-Y} is the interstellar reddening effect in the colour index $X-Y$. The Q index can also be used to deredden colours using (Johnson 1958):

$$(B-V)_0 = 0.332Q \quad (2.20)$$

The ratio $\frac{E_{U-B}}{E_{B-V}}$ is empirically determined and depends on the properties of the interstellar matter which causes reddening (e.g., Reimann 1989). Barnes, Evans & Moffett (1978) investigated $UBVRI$ reddening using interferometrically measured angular diameters and found the relations

$$E_{U-B} = 0.75E_{B-V} \quad (2.21)$$

$$E_{V-R} = 0.75E_{B-V} \quad (2.22)$$

$$E_{R-I} = 0.76E_{B-V} \quad (2.23)$$

Moro & Munari give the total extinction in the $UBVRIJKL$ bands to be

$$A_U = 4.4E_{B-V} \quad (2.24)$$

$$A_B = 4.1E_{B-V} \quad (2.25)$$

$$A_V = 3.1E_{B-V} \quad (2.26)$$

$$A_R = 2.3E_{B-V} \quad (2.27)$$

$$A_I = 1.5E_{B-V} \quad (2.28)$$

$$A_J = 0.87E_{B-V} \quad (2.29)$$

$$A_K = 0.38E_{B-V} \quad (2.30)$$

$$A_L = 0.16E_{B-V} \quad (2.31)$$

where A_V is the total interstellar extinction in the V band.

Q is a useful T_{eff} indicator for hot stars, but the value of Q for MS stars with masses greater than $30 M_{\odot}$ is almost constant. Therefore higher-mass stars must be studied using spectroscopy (Massey & Johnson 1993). Massey, Waterhouse & DeGioia-Eastwood (2000) found theoretical relations between T_{eff} and Q , using Kurucz model atmospheres, for stars of luminosity classes I, III and V, respectively:

$$\log T_{\text{eff I}} = -0.9894 - 22.76738Q - 33.09637Q^2 - 16.19307Q^3 \quad (2.32)$$

$$\log T_{\text{eff III}} = 5.2618 - 3.42004Q - 2.93489Q^2 \quad (2.33)$$

$$\log T_{\text{eff V}} = 4.2622 - 0.64525Q - 1.09174Q^2 \quad (2.34)$$

2.3.1.3 Strömgren photometry

The Strömgren *uvby* system was defined by Strömgren (1963, 1966), and is designed to be used for the simultaneous determination of the parameters of early-type stars and the amount of interstellar reddening. The $H\beta$ index was defined independently by Crawford (1958) and Crawford & Mander (1966) and complements the *uvby* passbands very well. Table 2.3 gives the central wavelengths and widths of the passbands.

The main drawback of using the *uvby* β system is that the filters allow much less light through than broad-band filters; the original *uvby* passbands had *peak* transmission efficiencies of only about 50% (Crawford & Barnes 1970). The advantage is that

Table 2.3: Central wavelengths and spectral widths for the Strömgren-Crawford $uvby\beta$ photometric system (Strömgren 1963; Crawford & Mander 1966).

Filter	Central wavelength (μm)	FWHM (μm)
u	3500	300
v	4110	190
b	4670	180
y	5470	220
H β wide	4861	150
H β narrow	4861	30

the passbands are good at measuring particular features in early-type stellar spectra. The u passband measures flux density bluewards of the Balmer discontinuity, but does not extend to wavelengths short enough to be affected by water vapour in the Earth's atmosphere (Hilditch 2001, p. 192). The v passband is targeted at a part of the spectrum where iron lines are abundant so is sensitive to metallicity. The b and y passbands are intended to measure continuum flux and are sufficiently red to not be subject to line blanketing effects. The y passband has a very similar central wavelength to the Johnson V passband and is closely comparable. The β index, the ratio of intensities in the H β wide and H β narrow passbands, is useful because it is not affected by reddening so provides an unambiguous measurement of the strength of the H β line in stars.

The main Strömgren indices are the Balmer discontinuity index, m_1 , and the metal-line index, c_1 , given by

$$c_1 = (u-v) - (v-b) \quad (2.35)$$

$$m_1 = (v-b) - (b-y) \quad (2.36)$$

(Strömgren 1966), and the $b-y$ index is also commonly used. The dereddened indices are denoted by a subscripted 0, and c_0 and m_0 are given by (Strömgren 1966) as

$$c_0 = c_1 - 0.20E_{b-y} \quad (2.37)$$

$$m_0 = m_1 + 0.18E_{b-y} \quad (2.38)$$

c_0 is sensitive to surface gravity through its dependence on the Balmer discontinuity shape, but also has a slight sensitivity to rotational velocity (Crawford & Perry 1976; Gray, Napier & Winkler 2001). m_0 is sensitive to metal abundance and line blanketing effects but also is affected by convection in cool stars and by microturbulence (Smalley & Kupka 1997). $b-y$ is in general sensitive to T_{eff} , and β is in general sensitive to luminosity. However, the sensitivities of the different indices change significantly over T_{eff} , and different types of stars must be studied using different indices. The β index is also slightly affected by an interstellar absorption band at $4890 \pm 35 \text{ \AA}$ (Nissen 1976), has a minor dependence on rotation due to the narrow passband being only $30 \text{ \AA}$ wide (Crawford & Perry 1976; Relyea & Kurucz 1978), and is also affected by systemic velocity, although the effect is negligible for $V_\gamma \lesssim 200 \text{ km s}^{-1}$.

2.3.1.4 Strömgren photometric calibrations

The calibration of Strömgren (1966) is split into five groups of stars:–

1. For stars earlier than B9, c_0 and $u-b_0$ are excellent T_{eff} indicators and for a given T_{eff} the Balmer line strength gives the surface gravity and M_V .
2. For A0–A3 stars, which is where the Balmer line reaches its maximum strength, two indices are defined:

$$a_0 = (b-y) + 0.18[(u-b) - 1^m36] \quad (2.39)$$

$$r^* = (\beta + 2^m565) + 0.35c_0 \quad (2.40)$$

(with corrections in the equation for r^* given by Moon & Dworetzky 1984). The index a_0 is a good indicator of T_{eff} and is practically independent of surface gravity, whereas for a given a_0 , r^* is a good indicator of surface gravity.

3. For A4–F0 stars, T_{eff} is indicated by β , and c_0 gives surface gravity and M_V . The index m_0 indicates whether the star is chemically peculiar.

4. For F1–F9 stars, T_{eff} and surface gravity are given by β and c_0 , and the metallicity, $[\frac{\text{Fe}}{\text{H}}]$, can be determined to an accuracy of 0.1 dex using m_0 .
5. For G0–G5 stars, the β index is not useful due to the amount of contaminating metal lines around $\text{H}\beta$. It is suggested that the indices c_0 , m_0 and $b-y$ are good for parameter determination, but no calibration was given.

Crawford (1975, 1978, 1979, 1980) provided a detailed and careful calibration of the physical parameters of early-type stars, using $uvby\beta$ photometry obtained for about twelve nearby open clusters and some nearby stars. Crawford did not use information from spectral classifications, space motions, previous calibrations or theoretical calculations. Crawford (1975) investigated the F type stars. He gives relations for the reddening between the $uvby\beta$ photometric indices:

$$E_{b-y} \approx 0.73E_{B-V} \quad (2.41)$$

$$E_{m1} \approx -0.3E_{b-y} \quad (2.42)$$

$$E_{c1} \approx 0.2E_{b-y} \quad (2.43)$$

$$A_V = 3.2E_{B-V} \approx 4.3E_{b-y} \quad (2.44)$$

The calibration is tabulated and is valid for F2–G0 stars of luminosity classes III–V; in particular it is intended for stars with $2^m590 < \beta < 2^m720$. B stars with β in this range can be detected by their blue colour or lower m_0 values. F stars have significant line blanketing effects due to the profusion of metal lines in the blue part of the spectrum. The blanketing parameter is

$$\delta m_1 = m_1(\text{standard}) - m_1(\text{observed}) \quad (2.45)$$

and is a good indication of the metal abundances of A and F stars. Crawford (1978) investigated the B stars, the resulting calibration being valid for stars with $c_0 < 1.0$. Crawford (1979, 1980) calibrated the A stars, defined as those in between the previous two calibration validity ranges.

Moon & Dworetsky (1985) produced a calibration to find the T_{eff} s and surface gravities of B2–G0 stars. Their method was to determine the main functional form of the relationship using synthetic $uvby\beta$ values found from Kurucz model atmospheres (Relyea & Kurucz 1978). The synthetic $uvby\beta$ values were adjusted to bring them into agreement with observational data and the resulting calibration plotted as diagrammatical grids. The Moon & Dworetsky calibration has been transformed into a convenient FORTRAN program (called TEFFLOGG) by Moon (1985). A FORTRAN program for dereddening Strömgren photometry and then applying several calibrations, called UCBYBETA, has been written by Moon. Dworetsky & Moon (1986) extended their calibration to Am stars, and adjusted the calibration of surface gravities to include a slight dependence on metallicity.

Napiwotzki, Schönberner & Wenske (1993) investigated several calibrations for determination of T_{eff} and surface gravity for B, A and F stars. Their calibrating stars were those with good T_{eff} determinations in the literature, for which they also obtained spectra of hydrogen lines and derived surface gravities from fitting the $H\gamma$ profile with theoretical profiles. They recommended that the Moon & Dworetsky (1985) calibration be used, with a minor correction in the surface gravity calibration of

$$\log g = \log g_{\text{MoonDworetsky}} - 2.9406 + 0.7224 \log T_{\text{eff}} \quad (2.46)$$

Ribas et al. (1997) used empirical data for MS dEBs to provide a calibration of stellar mass, radius and surface gravity using $uvby\beta$ indices. The intention was to use one index sensitive to T_{eff} and one sensitive to evolutionary status, and the stars were split into early-type, intermediate, and late-type. The claimed accuracy is 5–8% in mass, 10–15% in radius and 0.08–0.10 dex in $\log g$ for MS stars with T_{eff} s between 7000 K and 20 000 K, but metal abundance is important for late-type stars.

2.4 Light curve analysis of detached eclipsing binary stars

The variation of the apparent brightness of an EB depends on the geometry of the system (which is generally taken to also include the direction it is viewed from), the variation of T_{eff} over the surfaces of the stars, the rotational velocities of the stars, and the characteristics of the mutual orbit of the two stars. Additional complications can arise from contaminating light, usually coming from a third star orbiting the EB, but possibly due to an entirely unrelated foreground or background star along the line of sight. Third light can also be contributed by gas streams or colliding winds produced by the components of the EB.

The analysis of the light variations during and outside eclipse is a relatively complex procedure due to the number of different effects which cause the light variation. The first useful method, also referred to as rectification, was introduced by Russell (1912a, 1912b) and first applied to the EBs Z Draconis and RT Persei (Russell & Shapley 1914). This method, based on calculations by hand, was extensively refined by researchers including Russell, Merrill and Kopal, who took it as far as could reasonably be achieved without the aid of computers (Wilson 1994).

In the late 1960s it was noticed that the increased sophistication of computers allowed the analysis of the light curves of EBs without many of the restrictions imposed by use of the Russell method. This led to three computer-based models for the simulation of EB light curves (in increasing order of sophistication): EBOP, WINK and the Wilson-Devinney code (WD). The initial releases of EBOP and WD were able to fit a model to observed data using the differential corrections minimisation algorithm. WINK was not, but this feature was always intended to be implemented and was afterwards quickly made available. EBOP and WINK approximate the surfaces of stars using geometrical shapes so are only applicable to stars which are detached and so close to the shapes used. WD is based on the Roche equipotential model so is able to model semidetached and contact binary stars, a fundamental advance on previous methods

for the analysis of the light curves of these types of variable star.

2.4.1 Models for the simulation of eclipsing binary light curves

Quantities are derived from the light curves of EBs by defining a model and adjusting the parameters of the model towards the best fit. The evaluation of the total brightness of an EB, as a function of orbital phase, is achieved by summing the light emitted by all parts of the surface which are visible to the observer, usually achieved by numerical integration calculations.

The simplest model of a dEB – uniformly-illuminated spheres moving in a circular or eccentric orbit – is analytically exactly solvable, but the inclusion of effects such as limb darkening and asphericity cause the analytical integration equations to become intractable. The models discussed below split the surface of each star into many small elements. The evaluation of the total light of the system then requires the summation of the light from each element which is visible to the observer, and the light emitted by each element depends on its area (elements are not of uniform area because the stars are not undistorted spheres). Limb darkening, gravity brightening and the reflection effect also affect the brightness of an element

The reflection effect arises because each star intercepts light emitted by its companion. This causes the sides of the star facing the companion to be hotter and brighter. Whilst effects such as limb darkening and gravity brightening are fairly easy to incorporate into a light curve model, a detailed treatment of the reflection effect – such as contained in WD – is complex and extremely expensive in terms of calculation time. All models therefore incorporate some simplification of this effect.

The choice of the parameters used to define the model – and to adjust towards the best solution – can be very important. Light curves depend on a large number of parameters which are significantly correlated. At best this means that many iterative adjustments are required to reach the least-square solution and, at worst, minor observational errors can cause large changes in the derived parameters. Possibly the most worrying aspect of this is that the formal errors of the fit can become hugely optimistic

in the presence of large parameter correlations and so lose all their significance. The estimation of uncertainties is dealt with below.

The procedure for solving a light curve is to choose an appropriate model and estimate a set of parameters for which the model gives a light curve as similar as possible to the observed data. The model is then iteratively refined to find the best-fitting least-squares solution parameter values.

2.4.1.1 EBOP – Eclipsing Binary Orbit Program

EBOP was written by Dr. P. B. Etzel for his Master's thesis and used to analyse light curves of the dEB WW Aurigae (Etzel 1975). Based on the simple Nelson-Davis-Etzel (NDE) model (Nelson & Davis 1972, and modifications by Etzel 1980), its main advantage is that it involves far fewer calculations than the WINK and WD models so is much faster to run on a computer. Details can also be found in Popper & Etzel (1981) and in Etzel (1981, 1993).

The geometric shape chosen to represent stars in the NDE model is the biaxial approximation of a triaxial ellipsoid (the two minor axes are the same length), although a quantity called oblateness is misleadingly assessed after the method of Binnendijk (1974) (P. B. Etzel, private communication). The three axes of the triaxial ellipsoid, a_3 , b_3 and c_3 , are given by

$$a_3 = r_A \left[1 + \frac{1}{6}(1 + 7q)r_A^3 \right] \quad (2.47)$$

$$b_3 = r_A \left[1 + \frac{1}{6}(1 - 2q)r_A^3 \right] \quad (2.48)$$

$$c_3 = r_A \left[1 - \frac{1}{6}(2 + 5q)r_A^3 \right] \quad (2.49)$$

where q is the mass ratio. To calculate these quantities for the secondary star, replace q with $\frac{1}{q}$. Set $b_3 = c_3$ and adopt oblateness $\epsilon = 1 - \frac{b_3}{a_3}$ (Binnendijk 1974) to give

$$b_2 = r(1 - \epsilon)^{1/3} \quad (2.50)$$

$$a_2 = \frac{b_2}{1 - \epsilon} = \frac{r}{(1 - \epsilon)^{2/3}} \quad (2.51)$$

The radii given by EBOP relate to a sphere of the same volume as the biaxial spheroid.

For partially-eclipsing systems with large oblatenesses, the orbital inclination can be underestimated because of the biaxial ellipsoids adopted to approximate stars. For V478 Cyg, which has $\langle \epsilon \rangle = 0.029$, the inclination is underestimated by 0.48° , several times its standard error (Popper & Etzel 1981). This effect was confirmed to exist by Andersen, Clausen & Giménez (1993).

The main philosophy of the EBOP code is to base the model, and the least-square fitting to observations, on parameters which are most closely related to the shape of light curves, and which are correlated as little as possible. This means that the adjustable parameters are

- $r_A = \frac{R_{rmA}}{a}$, the fractional radius of the primary star
- $k = \frac{r_B}{r_A}$, the ratio of the stellar radii (r_B is the radius of the secondary star)
- $J = \frac{J_B}{J_A}$, the surface brightness ratio where J_A and J_B are the central surface brightnesses of the primary and secondary star respectively
- L_3 , the amount of third light
- i , the orbital inclination
- $q = \frac{M_B}{M_A}$, the mass ratio
- u_A and u_B , the linear limb darkening coefficient for each star
- β_A and β_B , the gravity brightening exponent for each star
- $e \sin \omega$
- $e \cos \omega$

The orbital ephemeris, P and T_0 , are also required but must be fixed during least-squares fitting by differential corrections. Another two parameters, the outside-eclipse

light of the system and the phase difference between the midpoint of primary eclipse and phase zero, are also needed to place the light curve properly in phase space.

The quantities $e \sin \omega$ and $e \cos \omega$, rather than e and ω , have been chosen as model parameters because they tend to be better determined when the orbit is only slightly eccentric (Etzel 1993). To a first approximation, $e \cos \omega$ depends on the phase of midpoint of secondary eclipse and $e \sin \omega$ depends on the relative durations of the eclipses. More formally, and ignoring terms in eccentricity to powers greater than one,

$$e \cos \omega \approx \frac{\pi(\phi_{\text{Min II}} - 0.5)}{1 + \text{cosec}^2 i} \quad (2.52)$$

where $\phi_{\text{Min II}}$ is the phase difference between secondary minimum and the immediately preceding primary minimum (Güdür 1978). Zakirov (2001) gives the ratio of the durations of secondary and primary eclipses to be

$$\frac{\delta\phi_{\text{Min II}}}{\delta\phi_{\text{Min I}}} = \frac{1 + e \sin \omega}{1 - e \sin \omega} \quad (2.53)$$

$e \sin \omega$ is generally less well-determined than $e \cos \omega$, although the opposite situation exists in the calculation of spectroscopic orbits (Section 2.2).

Limb darkening is incorporated in EBOP using the linear law (Section 1.1.2.1, equation 1.7) – the simple nature of the NDE models means that more complex limb darkening laws are not needed. However, their inclusion is advantageous and has been implemented by Dr. Giménez and Dr. Díaz-Cordoves. Their revised version of EBOP also has a slightly improved geometrical basis and the ability to allow for apsidal motion (1.7.2), and was used by Giménez & Quintana (1992) in a study of V477 Cygni.

The reflection effect is dealt with in a very simple bolometric manner based on Binnendijk (1960) and is usually calculated from the geometry of the system being analysed. This approximation becomes less accurate when the T_{eff} s of the two stars are very different or vary significantly over the stellar surfaces, but in any case it is not recommended to use EBOP for systems with a significant reflection effect (Etzel 1980).

The proximity effects (reflection and asphericity) are not included in the calculation of the light lost during eclipse, so only well-detached systems, where the change in proximity effects throughout eclipse is negligible, can be studied.

Popper & Etzel (1981) find that the NDE model and the EBOP code are trustworthy for stars with oblateness $\epsilon < 0.04$. Beyond this point, biaxial ellipsoids are unable to satisfactorily approximate the shape of the distorted star. North & Zahn (2004) studied dEBs in the Magellanic Clouds using EBOP and WD. They found that for average fractional radii of 0.25 and 0.3, the radii derived using EBOP were 1% and 5% different, respectively, to the radii found using WD. These studies provide good estimates of the limits of applicability of EBOP. A study of the LMC dEB HV 2274 by Watson et al. (1992) found that the differences between an EBOP and WINK solution were minor for this system, for which $r_A + r_B \approx 0.5$.

2.4.1.2 The Wilson-Devinney (WD) code

The WD code (Wilson & Devinney 1971; Wilson 1993) is probably the most commonly used light curve analysis code, partly due to its much greater sophistication compared to EBOP. Rather than modelling the discs of stars using geometrical shapes, the components of a binary system are modelled in three dimensions using the Roche prescription of equipotential surfaces. This is implemented by defining points on the surface of the star, distributed approximately uniformly in a spherical coordinate system. The number of points is of the order of one thousand per star, although the WD code allows the user to choose the approximate amount.

Adoption of the Roche model for calculating the shapes and sizes of the stars being studied allows a very realistic approximation of the actual stellar shapes, and the WD code can accurately model not only semidetached but also contact binary systems. The radii of the stars are given by one value of the potential per star for detached and semidetached binaries, or one value of potential for the whole system in the case of contact binaries. Once this model has been implemented, it needs only minor adjustment for different stellar shapes. The model is fitted to observations using the method of differential corrections.

The model parameters are:–

- P and T_0

- e and ω
- i and q
- F_A, F_B , rotational velocities of the stars (in units of the synchronous value)
- Φ_A, Φ_B , the gravitational potentials of the stars
- $T_{\text{eff}A}, T_{\text{eff}B}$, the effective temperatures of the stars
- L_A, L_B , the luminosities of the stars
- $u_{1,A}, u_{1,B}, u_{2,A}, u_{2,B}$, the wavelength-dependent limb darkening coefficient(s) for each star
- $u_{\text{bolo},1,A}, u_{\text{bolo},1,B}, u_{\text{bolo},2,A}, u_{\text{bolo},2,B}$, the bolometric limb darkening coefficient(s) for each star
- β_A, β_B , the gravity brightening coefficient for each star
- w_A, w_B , the reflection coefficient (albedo) for each star
- λ , the effective wavelength of the passband used to observe the light curve
- NREF, the number of integration points per star

There are many additional control characters to choose between several solution options, and some other capabilities have also been implemented in more recent versions.

The stellar radii are calculated by WD for four different points on the surface of the star: at the pole, towards the companion star, and on the equator at 90° and 180° from the line joining the centres of the two stars.

The T_{eff} of one of the stars must be fixed at a previously known value as light curves do not contain enough information to directly fit for both T_{eff} s. Calculations involving T_{eff} s and the reflection effect can be performed using black-body physics or using the predictions of model atmospheres (Leung & Wilson 1977). The model atmospheres of Carbon & Gingerich (1969) are provided with the WD code but more

advanced Kurucz predictions have been added by Milone and co-workers (Kallrath et al. 1998) and used by several researchers. It is possible to link the luminosities (which here refer to the light contribution in the light curve under analysis, not the astrophysical definition of luminosity) to the T_{eff} of the stars (mode 0 in the WD code) but this is not advisable due to the inadequacies of the black-body or model-atmosphere calculations required (Wilson et al. 1972). Groenewegen & Salaris (2001) found that for the LMC close binary HV 2274 the adoption of different model atmosphere predictions did not significantly affect the derived radii but changed the T_{eff} ratio by 1.6%.

It is notable that the WD code has no provision for performing more than one iteration without human intervention, although the output files contain all the data needed for the researcher to apply the needed corrections to the parameters of the model. Wilson (1998) and Wilson & Van Hamme (2004) clearly state that this apparent shortcoming has been deliberately included to force researchers to pay careful attention to matters of convergence, and to the success of the WD model as a whole. Wilson & Woodward (1983) state that some researchers have been iterating until parameter corrections are small, whereas iteration must continue until corrections are negligible so as to get good error estimates.

Wilson & Devinney (1973) also modified the WD code to allow the simultaneous solution of several light curves. In this case the geometrical parameters such as potential and orbital inclination are common to all light curves, but each set of data has its own values of the wavelength-dependent parameters such as limb darkening coefficients.

Wilson (1979) extended the WD code to include the simultaneous solution of light and RV curves. The advantages of this approach have been covered in detail by Van Hamme & Wilson (1984). The main advantage is that common parameters such as the mass ratio (which can be well determined by the light curves of close and contact binaries) have one unique value, although it could be argued that the inconsistent values occasionally found by separate analysis suggest the existence of subtle physical effects and inadequacies of the method of analysis, and should therefore be noted and investigated. The extra information contained in subtle physical effects, such as the Rossiter effect (Section 2.2.4), can most easily be accessed by a simultaneous photo-

metric and spectroscopic solution. Note that it is important to get the observational errors correct for the two different types of data, and that when this is done it is found that the photometric data are generally more important due to the larger number of observations in a light curve compared to a RV curve. Wilson & Sofia (1976) have investigated the proximity effects on spectroscopic orbital solutions of close binaries.

The original treatment of reflection was elaborated upon by Wilson et al. (1972) but criticised by Wood (1973a). Wilson (1990) added a more detailed treatment of the reflection effect which is able to consider multiple reflections too. However, the detailed treatment of reflection had to be achieved by considering the light incident from each surface element on one star to each surface element on the other star, so can be very expensive in terms of computing time when analysing eccentric systems. This is because the reflection effect in eccentric EBs is dependent on orbital phase, so must be calculated once for every datapoint.

The 1993 version of the Wilson-Devinney code (generally referred to as WD93) is much faster than previous versions (Wilson 1998). Other advantages include the consideration of apsidal motion and a constant period change to the code, and the ability to fit for the parameters of several starspots. Whilst starspots were included in previous versions (defined by a position, area and relative surface brightness), their parameters could not be adjusted prior to WD93. The latest (WD2003) version of the program is capable of fitting many starspots simultaneously whereas WD93 and WD98 were only able to adjust two per iteration.

The original WD code used the linear limb darkening law; WD93 and later versions use the logarithmic and square-root laws too (Section 1.1.2.1).

2.4.1.3 Comparison between light curve codes

It is preferable to analyse a light curve with more than one light curve analysis code, to check that the models are reliable and there are no programming bugs. Some codes may have other advantages, such as speed. For example, EBOP is over twenty times faster than WD because of its simplicity (Popper & Etzel 1981), although care has been

taken to make WD quicker (Wilson 1998). It is a necessary but *not* sufficient condition that the parameters of a observed system are well known if two analysis codes agree on its parameters (Linnell 1984).

The Copenhagen research group usually analyses light curves using EBOP and the WINK code, which is based on triaxial ellipsoids (Wood 1971a, 1972), or WINK and WD. The results have always been essentially identical (e.g., Andersen, Clausen & Nordström 1984, 1990a, who used EBOP and WINK to analyse the dEBs VV Pyxidis and V1031 Orionis) except for a slight disagreement in the value of orbital inclination (Andersen, Clausen & Giménez 1993), which has been discussed in section 2.4.1.1. Popper (1980) also notes that EBOP and WINK agree very well.

2.4.1.4 Other light curve fitting codes

Linnell (1984) introduced a physical model based on numerical integration between points on a surface. This model is sophisticated enough for the analysis of contact binaries (Linnell 1986) and has been equipped with a simplex least-squares fitting routine (Kallrath & Linnell 1987), but has not proved popular with researchers. It is more complex than the WD code (Kallrath & Linnell 1987).

Dr. G. Hill has constructed the light curve model LIGHT, followed by LIGHT2 (Hill & Hutchings 1970; Hill 1979) and Dr. P. Hadrava has written FOTEL (Hadrava 1990, 1995), which models stars using triaxial ellipsoids and has the ability to make simultaneous photometric and spectroscopic solutions.

2.4.1.5 Least-squares fitting algorithms

Fitting a model to an observed light curve involves many parameters, some of which are quite correlated. An algorithm is required to navigate from a point in parameter space to the point where the best fit occurs. This problem can be visualised using a χ^2 surface in two dimensions, although it must be remembered there are usually far more dimensions to worry about and these cannot be easily visualised by the human

brain. The χ^2 surface is high at its edges and low towards the middle, where the best fit is found. Added to this large-scale form are many valleys, bumps and dips which are caused by the correlations between parameters, and observational errors.

All least-squares fitting algorithms navigate in steps (iterations) from the starting parameter values towards the best fit. There are, however, several problems. Large local gradients in the χ^2 surface can give a bad idea of the overall surface and cause excessive adjustments to be made to parameter values. Often this will result in values diverging to infinity and causing the solution to break down. If two parameters are strongly correlated, they will cause a deep valley in the χ^2 surface which can cause a large number of iterations until a good fit is found. The most worrying possibility, though, is that there are small dips in the χ^2 surface which can catch solutions on their way to the global minimum. These local minima can be difficult to detect and often give plausible results. In many cases it is difficult to be certain that a global, and not local, minimum has been reached, and also whether a local minimum has a position significantly different to the global minimum. Global search algorithms are not difficult to construct but can be impractically expensive in terms of computer time.

EBOP, WINK and WD are all capable of adjusting the parameters of their models to find the least-squares best fit to an observed light curve. They use differential corrections (Wyse 1939; Irwin 1947) to adjust the parameters from starting estimates to a final solution. This method estimates parameter adjustments from the local gradient of the χ^2 surface. It requires reasonably good initial conditions because it can diverge or settle in local minima (it is a local minimisation algorithm). Formal errors can be calculated for the final fitted parameter values.

The simplex algorithm (see Press et al. 1992, p.402, who have implemented the Nelder-Mead simplex algorithm in the subroutine AMOEBA) has been implemented in the WD code by Kallrath & Linnell (1987). As used by these authors it has some characteristics of a global search algorithm; it is certainly incapable of divergence but is still able to get trapped in local minima. One advantage is that it uses only χ^2 values, not the gradient of the χ^2 surface, so does not require the calculation of partial derivatives. This can cause it to be faster than the differential corrections process, but

it may often require more iterations so will be slower.

The Levenberg-Marquardt method (Press et al. 1992, p.678, who have implemented the method in the MRQMIN algorithm) is probably the most popular fitting algorithm at present. It was suggested by Levenberg (1944) and by Marquardt (1963), and utilises two minimisation algorithms simultaneously, one algorithm being slow and robust, the other fast and less reliable. The former method is used far from minimum, with a continuous switch towards the latter method close to the minimum. MRQMIN also has a provision for calculating formal errors of the fit. MRQMIN is still a local search algorithm and is capable of diverging.

There are many more least-squares fitting algorithms, such as singular value decomposition (Press et al. 1992, p.670), simulated annealing and genetic algorithms (Ford 2005) available to the interested researcher, but the three methods detailed above are quite adequate for fitting light curve models to observed data (Wilson 1994).

2.4.2 Solving light curves

Firstly a decent set of observations must be obtained. There are several requirements for a set of light curves to be definitive:–

- Good light curves in two or more passbands are needed (Andersen 1991), although I would suggest that data in three passbands should be the minimum requirement, preferably in intermediate band photometric systems such as Strömgren (Section 2.3.1.3), as the mean and standard deviation can be calculated for three or more estimates of one value.
- Both eclipses must be covered without any gaps in the phased data greater than a tenth of the total eclipse duration.
- The eclipses must contain at least one hundred datapoints with low observational errors. If limb darkening is to be studied then each observation must have an error of 0.005 mag or less (Popper 1984, 2000) for simple systems. More

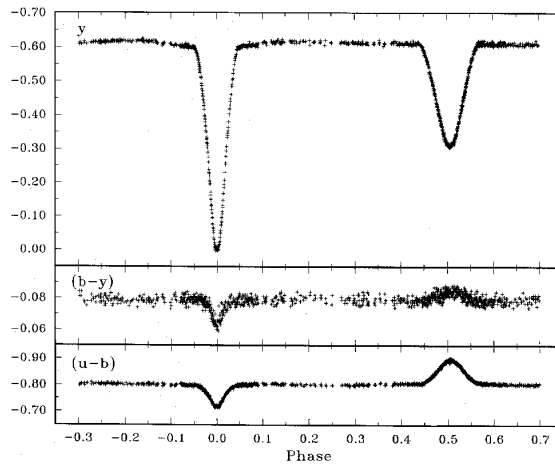


Figure 2.6: Example of a definitive light curve of a dEB. These data, of GG Lupi, were taken using a four-channel photoelectric photometer observing simultaneously in the Strömgren *uvby* passbands. Here the *y*-passband light curve is plotted and data from other passbands have been used to construct colour curves. Taken from Andersen, Clausen & Giménez (1993).

complicated dEBs will require better data. North, Studer & Kunzli (1997) suggest that meaningful results for limb darkening require five hundred points per eclipse, although my own experience suggests that this is quite conservative.

- Sufficient data must be available outside the eclipses to give an accurate reference brightness, to cover any outside-eclipse variation such as ellipticity and reflection effects, and to be sure that no significant complications could exist without being noticed. A minimum requirement is perhaps twenty accurate and well-spaced datapoints between each eclipse for an uncomplicated dEB.
- There are no significant night errors. If they are present then the results of analysing the light curves, which depend on observational errors being random, could be systematically wrong (Popper & Etzel 1981).

It appears that secondary eclipses are more sensitive to the variation of model parameters than primary eclipses (Popper 1986) although it is not clear why this should be so. The effect will be smaller for dEBs composed of similar stars than for those with very dissimilar components.

It is a good idea to have two different sets of observations obtained at different times and with different equipment, and possibly with different observers (Popper 1981, 1984). This can highlight difficulties such as night errors or data reduction errors. Also, if there are no problems, the uncertainties on the parameters will be reduced as there are more data available.

An example set of light curves, of the dEB GG Lupi, is given in Fig. 2.6. These light curves were observed using a four-channel photoelectric photometer observing simultaneously in the Strömgren u , v , b and y passbands.

2.4.2.1 Calculation of the orbital ephemeris

The first quantities to calculate are the orbital period and reference time of minimum (unless these quantities are going to be included in the overall fit using e.g., the WD code). For most dEBs it is entirely satisfactory to assemble times of minimum light, adopt a cycle number for each, and fit the data with a straight line. Many times of minima are available from the literature, particularly from the Information Bulletin of Variable Stars⁵, and the only point to be careful about is the quality of the data used and the method of determination.

Times of minima must be obtained from the observational data which are about to be analysed by least-squares. The traditional method for doing so was outlined by Kwee & van Woerden (1956). This requires the observational data to be resampled to constant time intervals. For a trial time of minimum (halfway between two resampled datapoints), one branch of the eclipse is reflected onto the other and the agreement is quantified. This is repeated for times midway between the preceding and proceeding

⁵<http://www.konkoly.hu/IBVS/IBVS.html>

pairs of datapoints and the amount of agreement is calculated. A parabola is then fitted to the three measures of agreement and the time of minimum found from the minimum of the parabola.

If the minima are asymmetric (due to the shape of the orbit) then the method of Kwee & van Woerden (1956) should be replaced by a parabola fitted directly to the data around the time of minimum (Giménez 1985). Alternatively, if the minima are symmetrical but not total, a straightforward Gaussian fit is usually quite acceptable and the uncertainty of the result is then easier to estimate.

Once times of minima have been found, a reference time is chosen. The choice is not important but it is best to choose an accurate time of primary minimum near the middle of the times covered by the data, as this will give lower uncertainties in the resulting reference time, T_0 . In the study of EBs the primary minimum is defined to be deeper than the secondary minimum, so in general refers to a transit of the star of lower surface brightness across the disc of the star of higher surface brightness. An approximate orbital period should be used to calculate how many orbits have occurred between each time of minimum and the reference time. This cycle number will be an integer for primary eclipses. A straight line is fitted to the cycle numbers and time of minima. The period and reference time are the parameters of the straight line.

The above technique runs into problems when the EB has an eccentric orbit. In this case the secondary minima will not in general occur halfway between the adjacent primary minima and the times of primary and secondary eclipse should be analysed separately. This runs into trouble when apsidal motion is present (section 1.7.2), which will cause the periods found from the primary and secondary minima to be different. In this case a full apsidal motion analysis must be done.

2.4.2.2 Initial conditions

Once the data have been assembled it is important to estimate a realistic set of initial parameter values to input into the least-squares fitting routine. Several parameters can be adopted directly from theory or previous observation. Theoretical limb darkening

coefficients have been tabulated by many authors (section 1.1.2.1) and gravity brightening exponents have expected values (section 1.1.3). The mass ratio can usually be fixed to a value available from a spectroscopic study of the dEB. The quantities just mentioned have only a minor impact on the light variation of a dEB except in specific circumstances, so any reasonable values can be used for a preliminary analysis.

Figures 2.7 and 2.8 display a set of model light curves generated using the EBOP code for sets of photometric parameters designed to illustrate the effect each parameter has on the light curve for typical dEBs. For convenience Table 2.4 contains the values of these parameters for each displayed light curve. All light curves have: a sum of the fractional radii, $r_A + r_B$, of 0.4 (towards the limit of capability of EBOP but chosen for display purposes); gravity brightening coefficients, β_A and β_B , of 1.0 (appropriate for radiative atmospheres); a mass ratio, q , of 1.0 (this parameter is unimportant for well-detached EBs); no third light, $L_3 = 0.0$ (the effect of third light is simply to reduce the total magnitude of variation without changing the shape of the light curve); and equal limb darkening coefficients for both stars, $u_A = u_B$.

Panels (a), (b) and (c) of Figure 2.7 each show three sets of parameters for typical MS dEBs, illustrating how the ratio of the radii (with a realistic adjustment to the surface brightness ratio also) changes. The orbital inclinations for the panels have been chosen to demonstrate total eclipses, deep eclipses and shallow eclipses. Figure 2.8 panel (a) shows how a change of orbital eccentricity affects a light curve, with the longitude of periastron chosen to be 90.0° so the secondary minimum is at phase 0.5 irrespective of the value of orbital eccentricity. Figure 2.8 panel (b) demonstrates how different values of the longitude of periastron change the phase of secondary minimum compared to the primary minimum (which has been put to phase 0.0 in all cases). Finally, Figure 2.8 panel (c) shows the change in a light curve brought about by a large change in limb darkening coefficients for both stars. The effect is very small, demonstrating that an exceptionally good set of observations is needed to make the limb darkening coefficients well determined.

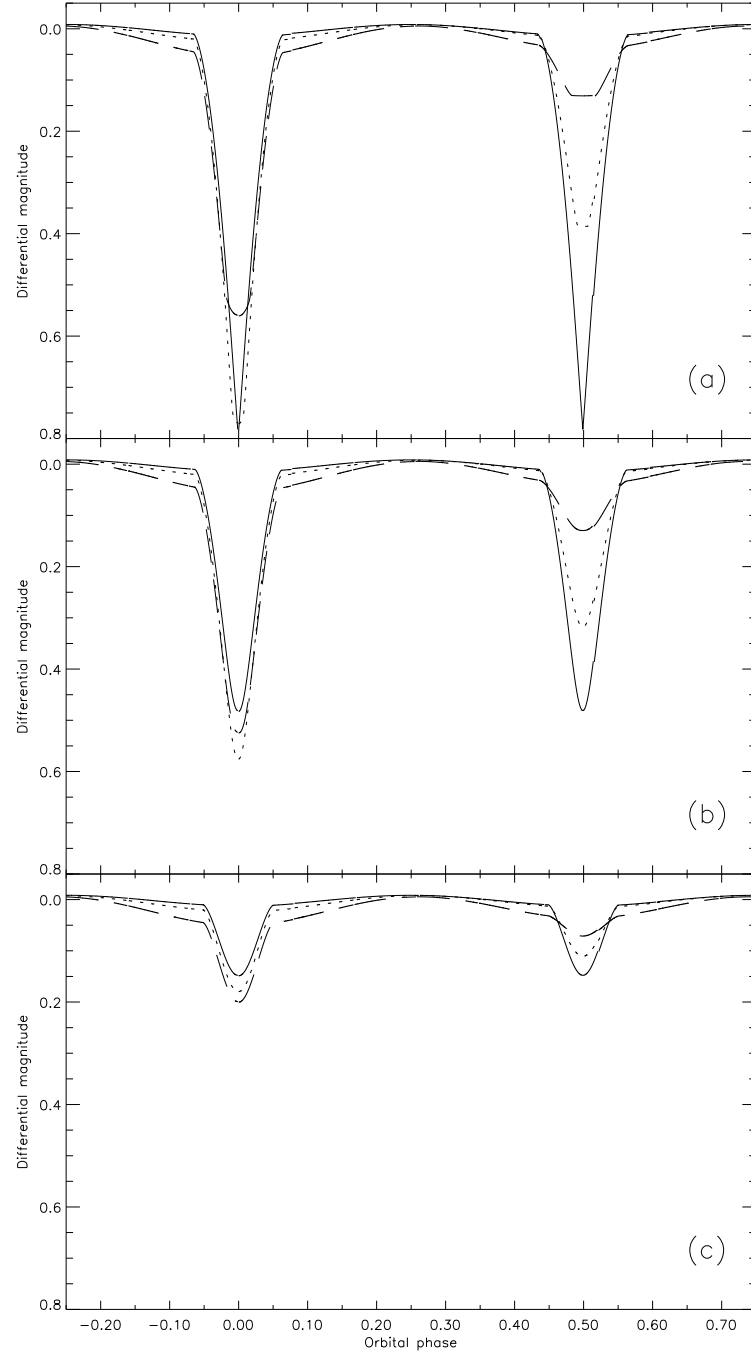


Figure 2.7: Representative light curves showing how orbital inclination affects the shape of light curves. The theoretical light curves were generated using the EBOP code (section 2.4.1.1). The parameters of the different models are given in Table 2.4. In each panel, curve 1 is shown with a solid line, curve 2 with a dotted line, and curve 3 with a dashed line.

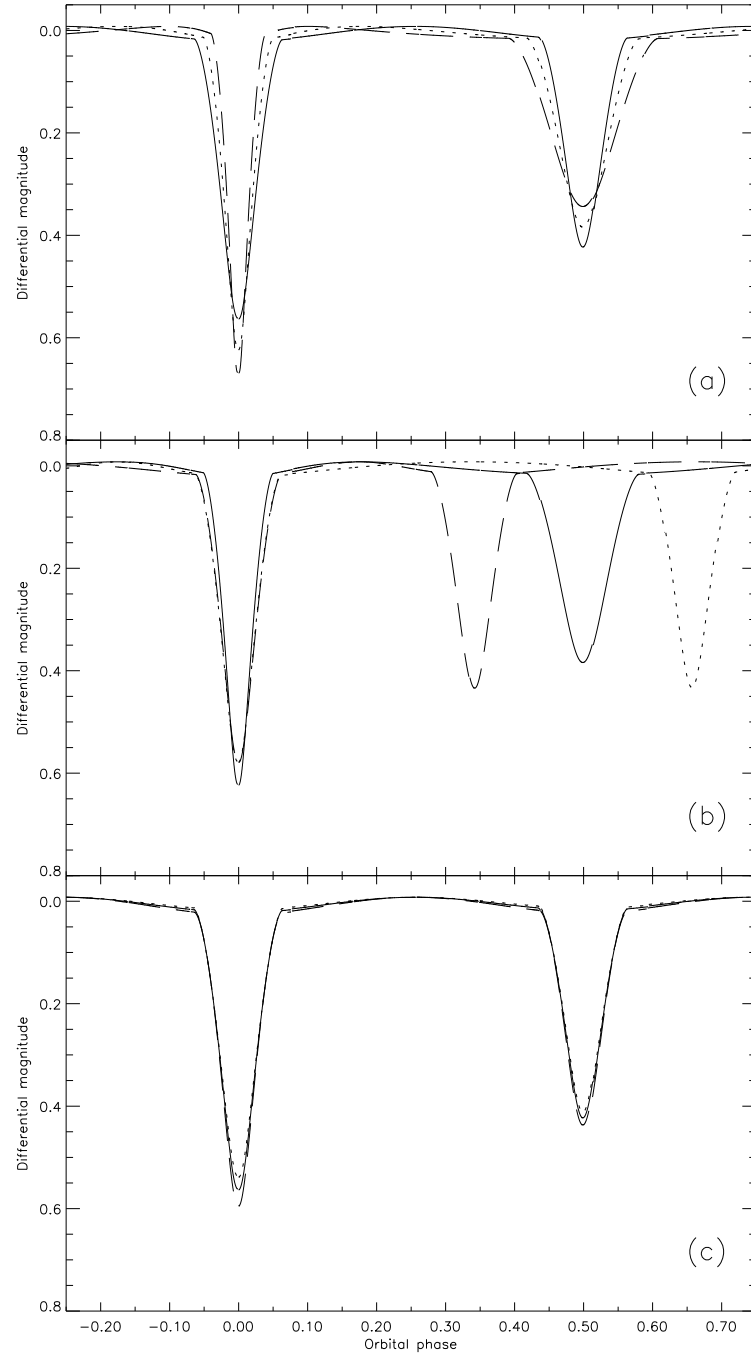


Figure 2.8: Representative light curves showing how orbital shape affects the shape of light curves. Symbols and references are as in Figure 2.7.

Table 2.4: Photometric parameters of the EBOP model light curves shown in Figures 2.7 and 2.8. Light curves are identified using the figure number, the panel and the light curve number. The parameters of interest to a particular panel are given in bold. All light curves have been generated using $r_A + r_B = 0.4$ (quite large but within the capability of EBOP), $\beta_A = 1.0$, $\beta_B = 1.0$, $q = 1.0$, $L_3 = 0.0$ and $u_A = u_B = u$.

Fig.	Panel	LC	k	i	J	u	e	ω
2.7	(a)	1	1.0	90.0	1.0	0.4	0.0	90.0
2.7	(a)	2	0.8	90.0	0.6	0.4	0.0	90.0
2.7	(a)	3	0.6	90.0	0.2	0.4	0.0	90.0
2.7	(b)	1	1.0	84.0	1.0	0.4	0.0	90.0
2.7	(b)	2	0.8	84.0	0.6	0.4	0.0	90.0
2.7	(b)	3	0.6	84.0	0.2	0.4	0.0	90.0
2.7	(c)	1	1.0	75.0	1.0	0.4	0.0	90.0
2.7	(c)	2	0.8	75.0	0.6	0.4	0.0	90.0
2.7	(c)	3	0.6	75.0	0.2	0.4	0.0	90.0
2.8	(a)	1	0.8	85.0	0.6	0.4	0.0	90.0
2.8	(a)	2	0.8	85.0	0.6	0.4	0.25	90.0
2.8	(a)	3	0.8	85.0	0.6	0.4	0.5	90.0
2.8	(b)	1	0.8	85.0	0.6	0.4	0.25	90.0
2.8	(b)	2	0.8	85.0	0.6	0.4	0.25	0.0
2.8	(b)	3	0.8	85.0	0.6	0.4	0.25	180.0
2.8	(c)	1	0.8	85.0	0.6	0.4	0.0	90.0
2.8	(c)	2	0.8	85.0	0.6	0.1	0.0	90.0
2.8	(c)	3	0.8	85.0	0.6	0.7	0.0	90.0

2.4.2.3 Parameter determinacy and correlations

Once a reasonable fit to the light curves under analysis has been found, the data can be fitted with a model using least-squares minimisation techniques. However, there are a number of well-known difficulties in the fitting of models to light curves of dEBs. Many of these relate to correlated parameters, although solutions exist. This means that choices must be made about which parameters to adjust freely, to fix to reasonable estimates, or to consider a variation of but not include in individual least-squares fits. A list of the problems follows.

The mass ratio becomes indeterminate in well-detached systems, so should be fixed at a spectroscopically-determined value or a good estimate (the latter possibility is allowable because the value of the mass ratio becomes unimportant). However, for close binaries which exhibit total eclipses, the mass ratio – and indeed the rotational velocity – may be found more easily from photometric data than from spectroscopic data (Wilson 1994; Fitzpatrick et al. 2003).

Investigating second-order effects such as limb darkening and gravity brightening is difficult except for certain types of light curves and very good data. Third-order phenomena, such as the effect of convection theory on limb darkening coefficients and gravity brightening exponents, are impossible to distinguish (Claret 2000a).

Third light can be very difficult to quantify in well-detached systems, and can be correlated with orbital inclination. Many researchers find no obvious trace of third light so arbitrarily set it to zero. This practice should be avoided when analysing good light curves. Either third light must be included as a free parameter, or an expected maximum possible value must be decided upon and the final parameter uncertainties modified to include a contribution due to this problem.

The light curves of close binary stars generally give better-determined values of the mass ratio, third light and of gravity brightening exponents. This can make them better distance indicators than well-detached binary stars (Harries, Hilditch & Howarth 2003; Lee 1997) but less good for studying the evolution of single stars as the influence of the binary companion on the evolution of each star is greater.

For dEBs composed of two very similar stars which don't exhibit total eclipses, the ratio of the radii can be very poorly determined (Popper 1984). In this case the sum of the radii is usually well-known but the individual radii are strongly correlated with each other, and the ratio of the radii is strongly correlated with the light ratio of the system. For some dEBs it may not be possible to break this degeneracy, but for others it can be solved by adopting a light ratio found spectroscopically.

The ratio of the radii may be correlated with $e \sin \omega$ (Clausen, Giménez & Scarfe 1986; Andersen & Clausen 1989; Clausen 1991; Barembaum & Etzel 1995) as both have a similar effect on the shape of the eclipses. This degeneracy can be broken by using results from a spectroscopic or apsidal motion analysis. Orbital eccentricity and periastron longitude can be also strongly correlated. This is the reason why EBOP and WINK solve for $e \cos \omega$ and $e \sin \omega$; these are better determined, particularly in systems with a small eccentricity.

Orbital inclination and the amount of third light can be correlated (Popper 1984).

2.4.2.4 Final parameter values

Once the data have been assembled, the orbital ephemeris found, estimated parameter values determined and the parameters to solve for selected, the light curve fitting algorithm can be unleashed. Usually several different choices of adjustable parameters are made and different solutions obtained, depending on the type of light curve being studied. Once a best solution has been selected and extended to each light curve (assuming they were not solved simultaneously), there exists a set of best-fitting values for each parameter. Whilst some parameters, for example the surface brightness ratio, depend on the passband used to obtain each light curve, other parameters, for example the stellar radii, are common between light curves. As several different determinations exist (one per light curve), the values can be compared to check that they are consistent. If they are, then the correct quantity to quote as a final result for each is the mean value. If uncertainties have been estimated (see below) then the weighted mean is the appropriate result to adopt.

When the ratio of the radii of the stars is poorly determined, it can be useful to constrain its value with a light ratio derived from spectroscopic observations (e.g., Andersen, Clausen & Nordström 1990a). On the MS, surface brightness decreases as stellar radius decreases, so a spectroscopic light ratio can provide a very accurate constraint on the ratio of the radii. An example of this is in the analysis of the dEB GG Orionis by Torres et al. (2000b). The B and V light curves for this dEB are shown in Figure 2.9; they exhibit a shape which makes the ratio of the radii relatively poorly determined. Figure 2.9 shows how a known light ratio (from spectroscopic observations) transforms directly into a constraint on the ratio of the radii for GG Orionis.

2.4.3 Uncertainties in the parameters

2.4.3.1 The problem

Uncertainties in the photometric parameters of a light curve fit have not generally been investigated as well as they should be. Whilst a result only has meaning if it is accompanied with a reasonable estimate of its uncertainty, this concept has been neglected by several researchers. The main cause of this is that all light curve analysis programs, as supplied, calculate formal uncertainties based on the final fit. Whilst these uncertainties have some value, they are generally very optimistic as they do not take proper account of the correlations between different parameters (Andersen 1991). Some researchers supply the formal errors of the fit as their final error estimates and subsequently cause difficulties, for example Schiller & Milone (1987) (see Pinsonneault et al. 2003) and Munari et al. (2004). Formal errors can be found, without discussion, in very recent works, for example Munari et al. (2004) and Stassun et al. (2004).

Popper (1984) provided an error analysis of the light curves of dEBs, using Monte Carlo simulations to estimate the sizes of errors. He found that no general rules exist to aid in the estimation of realistic uncertainties, but that robust uncertainties were generally no greater than three times the formal (internal) error of the fit. Popper found that the secondary eclipse is more sensitive to changes in model parameters than the

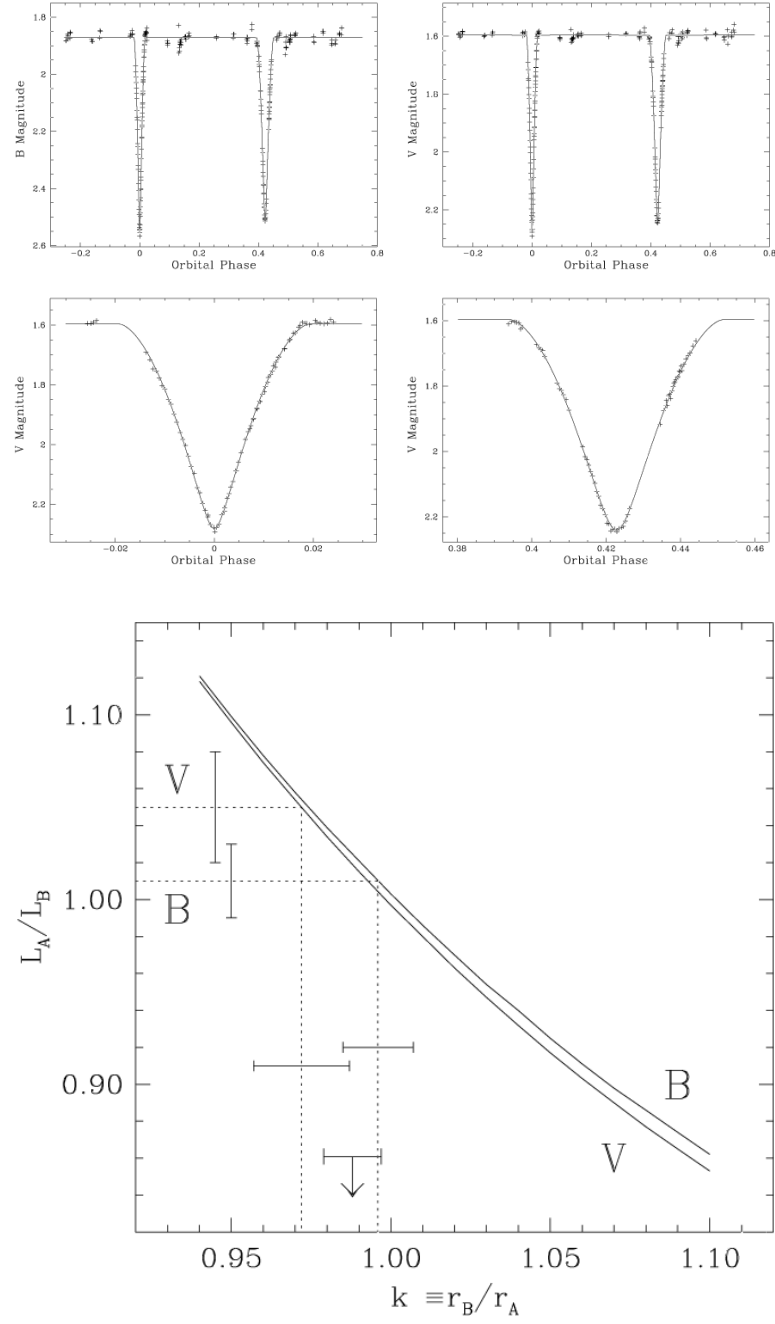


Figure 2.9: The B and V light curves of the dEB GG Orionis (top) and an illustration of the application of a spectroscopic light ratio in the determination of the ratio of its radii. A known light ratio (L_A/L_B) is used to find the corresponding ratio of the radii (r_B/r_A). Taken from Torres et al. (2000b).

primary eclipse. He also stated that analyses of the same dEB by different researchers tended to disagree by more than expected given the quoted errors. This occurs for two reasons: correlated errors in observational data (i.e., ‘night errors’) cause systematic errors in the derived parameters, and researchers have been quoting optimistic errors.

2.4.3.2 The solutions

The best way of estimating uncertainties is to observe many separate light curves, analyse them separately, and consider the values found for each parameter. Unfortunately, a sufficient number of light curves is not in general obtainable to provide an accurate estimation of the uncertainties. If only one or two light curves have been observed, this technique would provide no error estimates whatsoever.

One way of estimating reliable parameter uncertainties from light curves is, for each parameter, to fix it at several values, optimise the other parameters, and analyse the χ^2 of the resulting fits. This has been used by Hensberge, Pavlovski & Verschueren (2000) in their analysis of the high-mass dEB V578 Monocerotis. They found that the uncertainties they derived were roughly five times larger than the formal errors calculated by the WD93 code. They also considered the expected photometric errors and overall uncertainties and found that the systematic error, i.e., the difference between the two error estimates, was about twice as large as the random error for that study.

A full discussion of error analysis is given by Press et al. (1992, pp. 684–700). For the study of the light curves of dEBs, for which the model light curves provide a good representation of the actual light variation, the most reliable technique is Monte Carlo simulations. Once a best fit has been found, the model is evaluated at the actual points of observation. Random Gaussian scatter (to simulate observational errors) is then added and the resulting light curve is refitted. This process is repeated a large number of times and the spread of values of the derived parameters can then be analysed to determine robust uncertainties. Confidence intervals can then be constructed according to the requirements of the researcher. One problem with this process is that the confidence intervals refer to the point in parameter space where the initial best fit was

found, which is in general slightly different to the actual properties of the dEB being studied (Ford 2005). However, this bias is small and generally unimportant for the study of dEBs. A great advantage of Monte Carlo simulations is that study of the sets of parameter values which it provides can give an excellent idea of the relations and correlations between different parameters.

3 V615 Per and V618 Per in h Persei

The first two stars studied in this work are V615 Per and V618 Per, which are members of the well-studied young open cluster h Persei (NGC 869). V615 Per was selected as the primary target for this work by Dr. Maxted due to the importance of the cluster and the encouraging shape of the discovery light curve (deep and well separated eclipses). A few spectra and one night of photometry of V615 Per were obtained by Dr. Maxted in service mode using the William Herschel and Jakobus Kapteyn telescopes (ING, La Palma) before I began this work. Initial analyses of the photometry, taken to capture a primary minimum of V615 Per, serendipitously revealed an eclipse by V618 Per, which at that point did not have a reliable orbital period value. Spectroscopic observations were subsequently taken for both dEBs (see below for details) despite initial results for V618 Per at the telescope being quite disappointing.

3.1 V615 Per and V618 Per

V615 Per and V618 Per are two early-type dEBs which are members of the young open cluster h Persei (Table 3.1). V615 Per was noted to be variable, possibly of eclipsing nature, by Oosterhoff (1937) but the type of variation of both systems was formally established by Krzesiński, Pigulski & Kołaczkowski (1999, hereafter KPK99). These authors found two primary and two secondary eclipses in the light curve of V615 Per from over one hundred hours of *UBVI* observations. They estimated that the period is 13.7136 days. The eclipses are 0.6 and 0.4 mag deep. KPK99 observed one primary and one secondary eclipse of V618 Per approximately sixteen days apart, of depths 0.5 and 0.2 mag, respectively. This did not allow determination of the period, but KPK99 suggested a most likely period of 6.361 days, based on the width of the eclipses and assuming a circular orbit.

Observed photometric properties for both dEBs are given in Table 3.1. It is notable that the photometric spectral types for both dEBs, B8 and A3, and the orbital

Table 3.1: Identifications and combined photometric indices for V615 Per and V618 Per from various studies. The more recent Strömgren photometry of Capilla & Fabregat (2002) is not preferred because the data for V615 Per suggest it was in eclipse during some of their observations. All photometric parameters (including the spectral type determined from the Strömgren colours) refer to the combined system light.

* Calculated from the system magnitude in the V passband, the adopted cluster distance modulus and reddening (see section 3.1.1) and the canonical reddening law $A_V = 3.1E_{B-V}$.

References: (1) Oosterhoff (1937); (2) Keller et al. (2001); (3) Slesnick et al. (2002); (4) Capilla & Fabregat (2002); (5) Marco & Bernabeu (2001); (6) Uribe et al. (2002) based on proper motion and position.

	V615 Per	V618 Per	Ref
Oosterhoff number	Oo1021	Oo1147	1
Keller number	KGM 644	KGM 1901	2
Slesnick number	SHM 663	SHM 1965	3
α_{2000}	2 19 01.65	2 19 11.85	4
δ_{2000}	+57 07 19.2	+57 06 41.2	4
V	13.015	14.621	3
$B - V$	0.388	0.613	3
$U - B$	-0.101	0.286	3
$V - I$	0.370	0.661	2
$b - y$	0.351	0.473	5
m_1	-0.020	0.010	5
c_1	0.601	0.998	5
β	2.762	2.861	5
Photo. spectral type	B8	A3	5
M_V	-0.45	1.15	*
Membership prob.	0.96	—	6

periods, 13.7 and 6.4 days, indicate that all four stars are well separated from their companions, confirming that these stars have had negligible interaction during their MS lifetimes. This is important when comparing individual properties of dEBs to single-star theoretical models. The membership of both systems to the h Persei open cluster is also in little doubt. They fit onto the binary MS in all cluster CMDs, are situated on the sky in the cluster nucleus, and V615 Per has a measured proper motion which implies a membership probability of 0.96 (Uribe et al. 2002). Both dEBs were considered to be members of h Persei by van Maanen (1944) from a study of the proper motions of the stars in the region of h and χ Persei.

3.1.1 h Persei and χ Persei

The Perseus Double Cluster is a rich, young open cluster system relatively close to the Sun. This has made it an important and frequently used tool for studying the evolution of massive stars, and it is one of the most studied objects in the Northern Hemisphere. Many studies have been motivated by the disputed connection between h Persei and χ Persei. Their proximity to each other and the similar morphology of their photometric diagrams has led to the suggestion that the clusters are co-evolutionary, and in this sense perhaps unique in the Milky Way (see also Sandage 1958). The Double Cluster is also traditionally taken to be nucleus of the Perseus OB1 association (Humphreys 1978) although Slesnick et al. (2002) argue that it is impossible to be certain using current observational techniques.

The first detailed study was undertaken by Oosterhoff (1937), who used photographic photometry and very low-resolution photographic spectrophotometry to assign an “effective wavelength” to each star studied. Wildey (1964) conducted extensive photoelectric photometry of the general area and ascribed ages of 7, 17 and 60 Myr to the turn-off morphology of three perceived MSs in the cluster CMD. He also found ages of 6 Myr for PMS stars and at least 46 Myr for the faintest MS star observed. Schild (1965, 1967) claimed differences between the CMD morphology of the two clusters in the sense that h Persei was older than χ Persei and 0.3 mag more distant, even allowing

Table 3.2: Selected values of distance modulus, age and reddening taken from the literature. If one value is quoted for both clusters it is included in the table between the two relevant columns. Findings of differential reddening have not, in general, been indicated, and in such cases a best single reddening has been quoted.

* Reddening values in the Strömgren system have been converted to broad-band indices using $E_{B-V} \approx 1.37E_{b-y}$ (Crawford 1975).

† Converted from the Geneva photometric reddening index $E[B - V]$ by Waelkens et al. (1990) using $E_{B-V} \approx 0.86E[B - V]$.

Reference	Distance modulus		$\log \tau$ (years)		Reddening E_{B-V}	
	h Per	χ Per	h Per	χ Per	h Per	χ Per
Oosterhoff (1937)		11.51				
Bidelmann (1943)		11.42				
Johnson (1957)		11.76				
Wildey (1964)		11.9		6.78 – 7.78		
Schild (1967)	11.66	11.99	6.81	7.06		
Crawford et al. (1970)		11.4 $\pm$ 0.4			0.56 $\pm$ 0.03*	
Balona & Shobbrook (1984)		11.17 $\pm$ 0.09				
Tapia et al. (1984)					0.58 $\pm$ 0.03	0.59 $\pm$ 0.03
Liu et al. (1989)	11.74	11.73	7.26	6.48		
Waelkens et al. (1990)					0.56 $\pm$ 0.03 [†]	0.56 $\pm$ 0.06 [†]
Krzeciński et al. (1999)					0.52	
Marco & Bernabeu (2001)	11.56 $\pm$ 0.20	11.66 $\pm$ 0.20	6.8 – 7.0	7.15, 7.3	0.44 $\pm$ 0.02*	0.39 $\pm$ 0.05*
Keller (2001)		11.75 $\pm$ 0.05		7.10 $\pm$ 0.01	0.54 $\pm$ 0.02	
Uribe (2002)	11.42 $\pm$ 0.09	11.61 $\pm$ 0.06				
Slesnick (2002)		11.85 $\pm$ 0.05	7.10 $\pm$ 0.01	7.11 $\pm$ 0.01	0.57 $\pm$ 0.08	0.53 $\pm$ 0.08
Capilla & Fabregat (2002)		11.7 $\pm$ 0.1	7.10 $\pm$ 0.05		0.449–0.637*	0.545 $\pm$ 0.034*

for a 0.2 mag difference in extinction. He also noted that χ Persei contained many Be stars whilst h Persei did not, implying significant evolutionary differences.

Crawford, Glaspey & Perry (1970) observed the clusters in the Strömgren system and claimed there was no evidence that the clusters were not co-evolutionary. Waelkens et al. (1990) observed the cluster nuclei in the Geneva photometric system and confirmed the conclusions of Crawford et al. Intermediate-band systems have better procedures for individually dereddening single stars. This capability is important for clusters, such as h Persei, which display differential reddening. This may be the reason why intermediate-band photometric studies (before the year 2000) tend to find that h and χ Persei have common properties whereas broad-band studies do not.

Tapia et al. (1984) conducted *JHK* photometry and suggested that the variable reddening found in many previous studies may not be interstellar but intrinsic to the atmospheres of some B stars. They found no variation in extinction over the cluster but stated that a significant difference exists in the stellar contents of the two clusters, casting doubt on their co-evolutionary status.

There have been four recent photometric studies of the Double Cluster. Strömgren data were taken by Marco & Bernabeu (2001) who claimed that there were three distinct epochs of star formation: one of 6.3 to 10 Myr in h Persei, and two of 14 and 20 Myr in χ Persei. The distance moduli derived were consistent with a common distance.

Broadband observations were published by Keller et al. (2001) and Slesnick et al. (2002). Both studies found a common distance and age for h and χ Persei. Keller et al. claimed that Marco & Bernabeu had overinterpreted their data whilst Slesnick et al. claimed that Wildey (1964) did not sufficiently consider contamination by field stars, particularly background late-type giants.

Capilla & Fabregat (2002) undertook more extensive Strömgren photometry than Marco & Bernabeu and claim a common distance and age for h Persei and χ Persei. They also, like many previous studies, find strong differential reddening over h Persei and weaker, constant reddening over χ Persei. Comparison of their observed and dereddened photometric diagrams strongly implies that differences in reddening and membership selection have been the main cause of dispute over the relative and absolute

physical status of the two open clusters.

Table 3.2 lists selected published parameters of the two clusters. If the last four photometric studies are considered it can be seen that the values are converging towards a distance modulus of 11.70 ± 0.05 and an age of $\log \tau = 7.10 \pm 0.01$ (years). These values will be adopted for the purposes of discussion and model comparison below.

3.2 Observations

3.2.1 Spectroscopy

Spectroscopic observations were carried out over a fourteen-night observing run in 2002 October using the 2.5 m Isaac Newton Telescope (INT) on La Palma. Two of these nights were lost to bad weather but during the remaining twelve night complete spectroscopic observations were obtained for approximately ten dEBs. The 500 mm camera of the Intermediate Dispersion Spectrograph (IDS) was used with a holographic 2400 lines per millimetre grating, giving a reciprocal dispersion of $0.1 \text{ \AA}$ per pixel. All observations used the same grating, allowing us to avoid the increase in complexity and loss of time associated with changing gratings during the night.

The light detector was an EEV $4k \times 2k$ CCD which was binned by a factor of two in the slit direction to reduce readout time. Only the area of the CCD close to each spectrum was read out, which also helped to reduce readout time. Exposure times for V615 Per and V618 Per were 1800 seconds. The slit was set at the parallactic angle to avoid problems due to differential refraction. These targets were not close enough for spectra to be taken of both simultaneously. One arc lamp exposure was taken immediately before and after each science observation to provide wavelength calibration. Measurements of the full width half maximum (FWHM) of arc lines taken for wavelength calibration indicate that the resolution is about $0.2 \text{ \AA}$.

The main spectral window chosen for observation was 4230–4500 $\text{\AA}$. This contains the Mg II 4481 $\text{\AA}$ line which is known to be one of the best lines for RV work for early-

type stars (Andersen 1975; Kilian, Montenbruck & Nissen 1991). He I 4471 Å and H γ (4340 Å) are useful for determination of T_{eff} s and spectral types for such stars. One spectrum of V615 Per was observed at H β (4861 Å) to provide an additional T_{eff} indicator. Some spectra were taken a 4450–4710 Å for the first few nights before we changed our observing strategy slightly. An observing log is given in Table 3.3. The spectra of V615 Per have an average signal to noise ratio per pixel of approximately 50, whereas the signal to noise ratio for the spectra of V618 Per is approximately 15. Spectra were also obtained, using the same observational setup, of a wide range of standard stars for possible later use as template spectra.

Data reduction was undertaken using optimal extraction (Horne 1986; Marsh 1989) as implemented in the software tools PAMELA and MOLLY by T. Marsh¹ (Marsh 1989) in the software PAMELA² and MOLLY³.

3.2.2 Photometry

Observations in the *uvby* and β passbands were undertaken at the 1 m Jakobus Kapteyn Telescope (JKT), also on La Palma, during 2002 December and 2003 January using the SITe2 (2000 pixel)² CCD. The *uvby* β system was designed to provide accurate photometric parameters for early-type stars (section 2.3.1.3) and is useful in this case for its robust procedures concerning interstellar extinction, which is large and variable towards the Perseus Double Cluster.

The CCD was windowed, to image the centre of the cluster including both V615 Per and V618 Per, to reduce readout time and exposure times of 60–90 s were used depending on the atmospheric conditions and passband. Most observations were taken in the *b* and *y* passbands with the other passbands being used approximately every 1800 s. This observing strategy was intended to allow us to obtain good *by* light curves, for a full light curve analysis, whilst still obtaining enough *uv* β observations to

¹<http://www.warwick.ac.uk/staff/T.R.Marsh/index.html>~

²<http://www.warwick.ac.uk/staff/T.R.Marsh/pamela.tar.gz>

³<http://www.warwick.ac.uk/staff/T.R.Marsh/molly.tar.gz>

Table 3.3: Observing log for the spectroscopic observations of V615 Per and V618 Per.

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
V618 Per	323116	4450–4710	2452559.45217	1800	11/10/02	22:45:59
V618 Per	323117	4450–4710	2452559.47326	1800	11/10/02	23:16:21
V618 Per	323182	4450–4710	2452559.63107	1800	12/10/02	03:03:35
V618 Per	323183	4450–4710	2452559.65215	1800	12/10/02	03:33:57
V618 Per	323324	4450–4710	2452560.43535	1800	12/10/02	22:21:42
V618 Per	323367	4450–4710	2452560.58718	1800	13/10/02	02:00:20
V618 Per	323568	4230–4500	2452561.55650	1800	14/10/02	01:16:05
V618 Per	323592	4230–4500	2452561.63848	1800	14/10/02	03:14:08
V615 Per	323621	4230–4500	2452561.72838	1800	14/10/02	05:23:35
V615 Per	323735	4230–4500	2452562.45199	1800	14/10/02	22:45:33
V618 Per	323738	4230–4500	2452562.47587	1800	14/10/02	23:19:56
V615 Per	323757	4230–4500	2452562.52350	1800	15/10/02	00:28:31
V618 Per	323760	4230–4500	2452562.54707	1800	15/10/02	01:02:27
V615 Per	323777	4230–4500	2452562.63004	1800	15/10/02	03:01:56
V618 Per	323780	4230–4500	2452562.65363	1800	15/10/02	03:35:53
V618 Per	323793	4230–4500	2452562.71042	1800	15/10/02	04:57:40
V615 Per	323888	4230–4500	2452563.45589	1800	15/10/02	22:51:06
V618 Per	323891	4230–4500	2452563.47948	1800	15/10/02	23:25:04
V615 Per	323896	4230–4500	2452563.50735	1800	16/10/02	00:05:12
V618 Per	323908	4230–4500	2452563.54845	1800	16/10/02	01:04:23
V615 Per	323911	4230–4500	2452563.57542	1800	16/10/02	01:43:13
V618 Per	323914	4230–4500	2452563.59934	1800	16/10/02	02:17:39
V615 Per	323929	4230–4500	2452563.65758	1420	16/10/02	03:41:32
V618 Per	323933	4230–4500	2452563.67959	1800	16/10/02	04:13:13
V618 Per	323945	4230–4500	2452563.73442	1800	16/10/02	05:32:10
V615 Per	324051	4230–4500	2452564.40859	1800	16/10/02	21:42:56
V618 Per	324054	4230–4500	2452564.43240	1800	16/10/02	22:17:13
V615 Per	324071	4230–4500	2452564.51282	1800	17/10/02	00:13:01
V615 Per	324079	4230–4500	2452564.56024	1800	17/10/02	01:21:19

continued

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
V618 Per	324082	4230–4500	2452564.58390	1800	17/10/02	01:55:22
V615 Per	324133	4230–4500	2452564.65167	1800	17/10/02	03:32:57
V618 Per	324136	4230–4500	2452564.67586	1800	17/10/02	04:07:48
V615 Per	324139	4230–4500	2452564.69581	1270	17/10/02	04:36:31
V615 Per	324274	4230–4500	2452565.39948	1800	17/10/02	21:29:46
V618 Per	324277	4230–4500	2452565.42312	1800	17/10/02	22:03:49
V615 Per	324283	4230–4500	2452565.47128	1800	17/10/02	23:13:09
V618 Per	324286	4230–4500	2452565.49415	1800	17/10/02	23:46:05
V615 Per	324309	4230–4500	2452565.54775	1800	18/10/02	01:03:16
V618 Per	324313	4230–4500	2452565.57450	1500	18/10/02	01:41:47
V615 Per	324316	4230–4500	2452565.59591	1800	18/10/02	02:12:37
V618 Per	324319	4230–4500	2452565.62073	1800	18/10/02	02:48:21
V615 Per	324496	4230–4500	2452566.39114	1800	18/10/02	21:17:42
V618 Per	324499	4230–4500	2452566.41449	1800	18/10/02	21:51:20
V615 Per	324631	4230–4500	2452568.67580	1800	21/10/02	04:07:30
V618 Per	324637	4230–4500	2452568.70384	1800	21/10/02	04:47:53
V618 Per	324798	4230–4500	2452569.43018	1800	21/10/02	22:13:46
V615 Per	324834	4230–4500	2452569.51960	1800	22/10/02	00:22:32
V618 Per	324837	4230–4500	2452569.54261	1800	22/10/02	00:55:40
V615 Per	324886	4230–4500	2452569.61647	1800	22/10/02	02:42:01
V618 Per	324913	4230–4500	2452569.67614	1800	22/10/02	04:07:57
V615 Per	325175	4230–4500	2452570.49770	1800	22/10/02	23:50:57
V615 Per	325221	4230–4500	2452570.58729	1800	23/10/02	01:59:58
V615 Per	325270	4230–4500	2452570.69972	1800	23/10/02	04:41:51
V615 Per	325416	4230–4500	2452571.52125	1800	24/10/02	00:24:49
V615 Per	325419	4710–4970	2452571.54377	1800	24/10/02	00:57:15
V618 Per	325447	4230–4500	2452571.61565	1800	24/10/02	02:40:45
V618 Per	325447	4230–4500	2452571.61565	1800	24/10/02	02:40:45
V618 Per	325447	4230–4500	2452571.61565	1800	24/10/02	02:40:45
V618 Per	325450	4710–4970	2452571.63816	1800	24/10/02	03:13:10
V618 Per	325450	4710–4970	2452571.63816	1800	24/10/02	03:13:10
V618 Per	325689	4710–4970	2452572.61489	1800	25/10/02	02:39:37

allow the flux ratios in these passbands to be found using the geometry of the system as found from the *by* light curves.

The bias level on the CCD images was removed by subtracting the median value of the overscan region in each image as there was no significant structure in the bias images taken during the observing run. Sky flat-field images were taken during evening and morning twilight (weather permitting) for each passband. These were combined into master flat-field images for each night by clipped-mean averaging. CCD science images were flat-fielded by dividing by the relevant master flat fields. Attempts were made to keep the images from individual stars within the same pixel over the observing run but these were generally unsuccessful due to autoguiding errors and problems, and minor changes in observing strategy.

Optimal photometry (Naylor 1998) was initially used to find differential-magnitude light curves for V615 Per and V618 Per using the Starlink PHOTOM routine (Eaton, Draper & Allen 1999⁴). These attempts met with only limited success due to small charge-transfer problems causing trailing in the CCD images. This trailing moved a significant proportion of the counts of a stellar image to pixels with much lower weights. Aperture photometry was used instead as it was not significantly affected by the trailing, because all pixels are given equal weights, and field crowding was not a problem. Aperture radii of six pixels were found to give the best results. However, light curves from some nights exhibit significant night errors, and whilst the internal precision of the data are good, observations on different nights fail to agree on the outside-eclipse brightness of the system by about 0.05 mag. This is not due to intrinsic variability: such an effect is not present in the discovery light curves from KPK99 but was noticed in other data obtained on the same observing run as V615 Per and V618 Per.

A nonlinearity was also found in the CCD images. This was quantified by Dr. P. Maxted by fitting a polynomial to the magnitudes of the stars on each image compared to the magnitudes of the same stars on a reference image. Removal of the nonlinearity effects halved the night errors but the light curves of V615 Per remain unsuitable for

⁴<http://www.starlink.rl.ac.uk/star/docs/sun45.htx/sun45.html>

model fitting. The data for V618 Per seem to be less affected, and the effect can be minimised by offsetting light curves from different nights by small amounts (of the order of 0.01 mag). Complete light curves in the *by* passbands were obtained for both targets, along with some photometry outside the eclipses.

h Persei contains many stars which can be used as secondary standards in the Strömgren-Crawford *uvby β* system (Crawford et al. 1970). Several of these were observed simultaneously with V615 Per and V618 Per but no attempt was made to calibrate the light curves of the dEBs because the night errors discussed above make the photometry unreliable.

The KPK99 discovery light curves contain a total coverage of 24, 57, 103 and 10 hours of observation in the broad-band *U*, *B*, *V* and *I* passband respectively. Although observations are somewhat sparse during eclipses of V615 Per and V618 Per, the light curves are of sufficient quality for an approximate determination of the stellar radii and orbital inclination of V615 Per. They also show no sign of any stellar brightness variation apart from the eclipses.

Supplementary *BVI* service data were taken with the JKT on 2001 September 16 to capture part of a primary eclipse of V615 Per. This also serendipitously captured a descending branch of a primary eclipse of V618 Per. These data were combined with the KPK99 discovery light curves. This slight inhomogeneity of the *BVI* data should be negligible, and comparable to the inhomogeneity introduced by KPK99 by the use of two different observatories in their search for variable stars.

3.3 Period determination

3.3.1 V615 Per

KPK99 observed descending and ascending branches of two primary and two secondary eclipses and determined a period for V615 Per of 13.7136(1) days. As no observations overlapped during eclipse, and no actual light minima were observed, it was not possible

to determine the period of light variation without assuming a certain shape for both primary and secondary eclipses. This difficulty resulted in them underestimating the width of the eclipses and their ephemeris disagrees slightly with our own.

The times of mid-eclipse are reproduced in Table 3.4 for those eclipses for which an actual light minimum was observed. The *BVI* light curves obtained as service data were fitted with Gaussian functions to determine times of minimum and their formal errors. Gaussian functions provide a very good representation of the eclipse shapes of this system as the eclipses are deep but not total, and the orbit has only a very small eccentricity so eclipses will be symmetric about their centre. The night errors in the Strömgren *b* and *y* light curves may cause asymmetry and so bias the results derived by fitting a Gaussian function, so only the central parts of the eclipse were fitted.

A straight line fitted by least squares to the times of minima, using the most accurately determined time of eclipse as cycle zero, showed larger $O - C$ (observed minus calculated) values than expected. Inspection of phased light curves indicated that the correct period had to be 13.71390 days so the times of minimum for cycle 36.0 are earlier than expected. We have not been able to discover the reason for this. A straight line fit to the remaining times of minima gives the ephemeris

$$\text{Min I} = \text{HJD } 2\,452\,169.6821(5) + 13.71390(2) \times E \quad (3.1)$$

The quoted uncertainties are standard errors – this convention will be used throughout the following study. The observed minus calculated ($O - C$) curve is shown in Fig. 3.1.

3.3.2 V618 Per

KPK99 observed one primary and one secondary eclipse of V618Per, separated by approximately 16 days. The light curves each cover just over half of one minimum but are very sparse. V618Per was also in eclipse during the service observations of the eclipse of V615Per and just over half of one primary eclipse was observed in *BVI*. Our *b* and *y* light curves covered most of one primary and most of one secondary eclipse

Table 3.4: Times of minima and $O - C$ values determined for V615 Per from data taken with the JKT. Cycle zero was chosen to be the eclipse with the best-defined time of minimum. The times of minimum for cycle 36.0 are incorrect by approximately five minutes and were not used to determine the period. They are included here for completeness.

[†] All times are given as (HJD - 2 400 000).

[‡] The quoted error is the formal error of the Gaussian fit.

Source		Cycle	T_0 (HJD) [†]	Error [‡]	$O - C$
2001 Sep	<i>B</i>	0	52169.68218	0.00085	0.00008
2001 Sep	<i>V</i>	0	52169.68200	0.00088	-0.00010
2001 Sep	<i>I</i>	0	52169.68218	0.00078	0.00008
2002 Dec	<i>b</i>	32.5	52615.38403	0.00034	0.00018
2002 Dec	<i>y</i>	32.5	52615.38316	0.00031	-0.00069
2003 Jan	<i>b</i>	35.5	52656.52649	0.00054	0.00094
2003 Jan	<i>y</i>	35.5	52656.52509	0.00050	-0.00046
2003 Jan	<i>b</i>	36	52663.37978	0.00032	-0.00272
2003 Jan	<i>y</i>	36	52663.37950	0.00027	-0.00300

Table 3.5: Times of minima and $O - C$ values determed for V618 Per. Cycle zero was chosen to be the eclipse with the best-defined time of minimum. Only times of primary minima were used to determine the final ephemeris.

[†] All times are given as (HJD - 2 400 000).

[‡] The quoted error is the formal error of the Gaussian fit.

Source		Cycle	T_0 (HJD) [†]	Error [‡]	$O - C$
KPK99	<i>B</i>	-398	50081.4483	0.0069	-0.0022
KPK99	<i>V</i>	-398	50081.4523	0.0021	0.0019
KPK99	<i>B</i>	-395.5	50097.3528	0.0057	-0.0144
KPK99	<i>V</i>	-395.5	50097.3541	0.0041	-0.0132
2001 Sep	<i>B</i>	-70	52169.7212	0.0015	-0.0056
2001 Sep	<i>V</i>	-70	52169.7260	0.0016	-0.0008
2001 Sep	<i>I</i>	-70	52169.7232	0.0027	-0.0036
2002 Dec	<i>b</i>	0	52615.3953	0.0007	-0.0003
2002 Dec	<i>y</i>	0	52615.3958	0.0010	0.0003
2003 Jan	<i>b</i>	5.5	52650.4161	0.0019	0.0039
2003 Jan	<i>y</i>	5.5	52650.4135	0.0013	0.0012

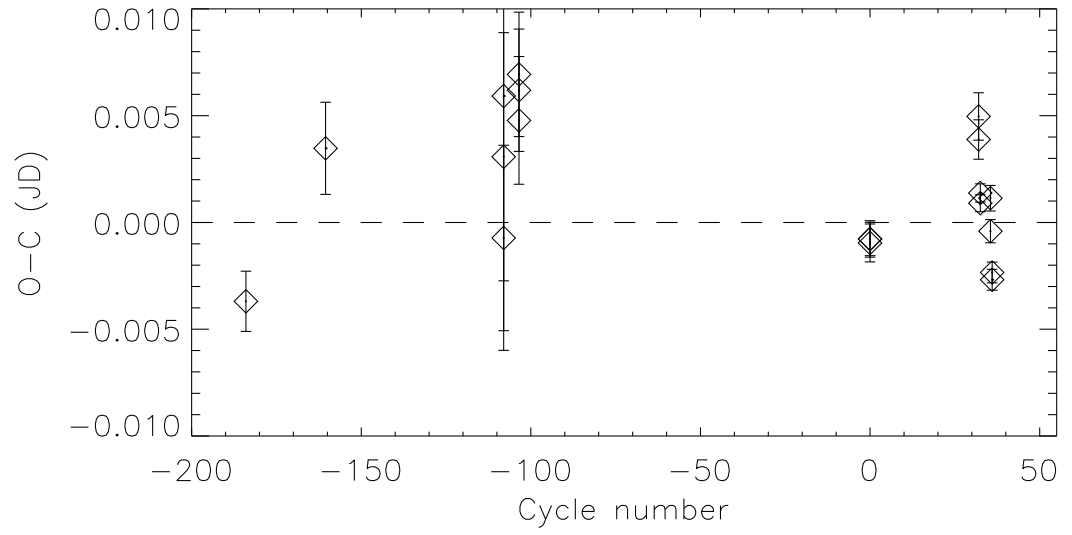


Figure 3.1: Observed minus calculated ($O - C$) curve for V615 Per.

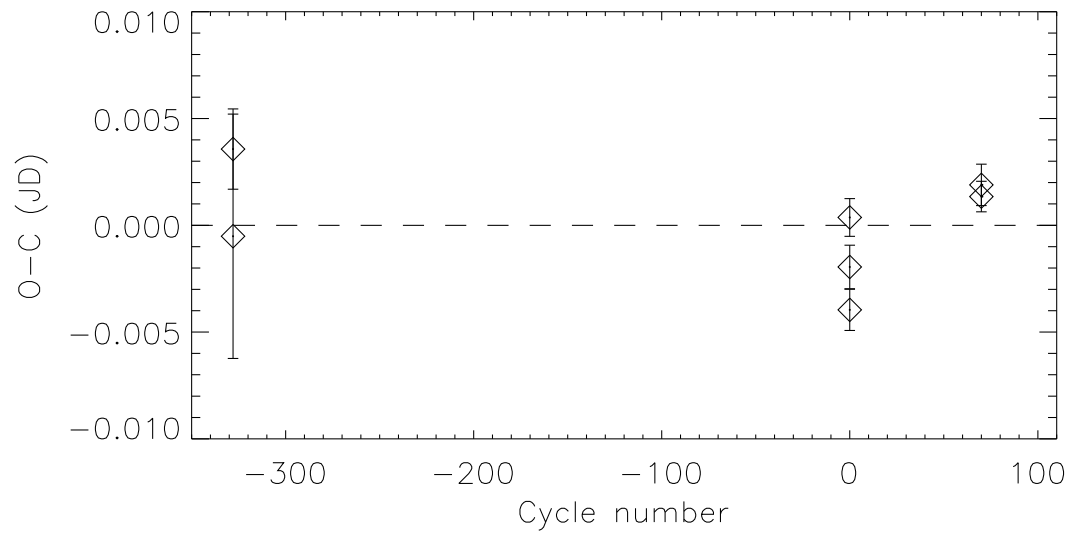


Figure 3.2: ($O - C$) curve for V616 Per.

but the small depth of the secondary eclipse means that the primary eclipse has a better-defined minimum.

All eclipses were fitted with Gaussian functions. Eclipse widths were held fixed to the width of the best observed eclipse and the uncertainty generated by this has been added in quadrature with the formal errors of the Gaussian fit. The times of mid-eclipse are reproduced in Table 3.5. Adopting a period found using a linear least-squares fit to the primary minima and a timebase corresponding to the best-defined light minimum observed (in two passbands) gives the ephemeris

$$\text{Min I} = \text{HJD } 2\,452\,615.3955(3) + 6.366696(4) \times E \quad (3.2)$$

The secondary minima contain fewer datapoints and are shallower than the primary minima, but give a period similar to that derived from the primary eclipses. The $O - C$ curve is shown in Fig. 3.2.

3.4 Spectral disentangling

Spectral disentangling (section 2.2.3.4) requires a spectroscopic orbit to calculate the RVs for the stars in each observed spectrum so a preliminary orbit was derived for V615 Per. Gaussian functions were fitted to the Mg II 4481 Å spectral lines using MOLLY and a spectroscopic orbit was fitted to the resulting RVs using SBOP (section 2.2.4.1). The results are consistent with a circular orbit (the eccentricity value found is smaller than its standard error) so a final solution was made with no eccentricity. The velocity semiamplitudes are $K_A = 75.9 \pm 0.8 \text{ km s}^{-1}$ and $K_B = 95.9 \pm 0.7 \text{ km s}^{-1}$.

The Simon & Sturm (1994) algorithm was used to produce disentangled spectra of the components of V615 Per. The resulting spectra show significant variation in continuum level over the observed wavelength range. This is easily removed by polynomial fitting over small spectral windows but cannot cope with the shapes of broad lines, so the disentangled spectra have unreliable Hγ 4340 Å profiles. The individual spectra are shown in two spectral windows in Figure 3.3.

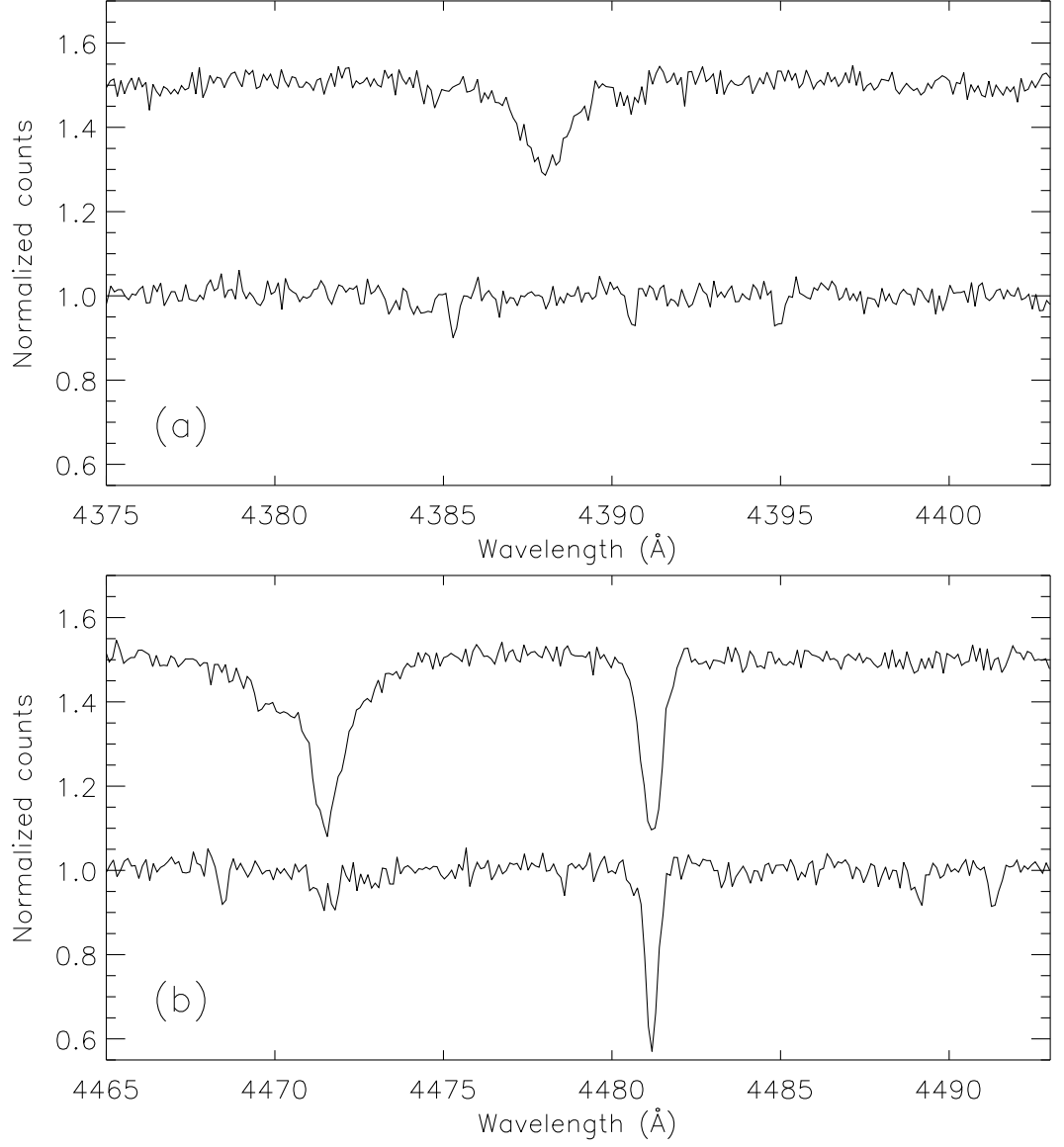


Figure 3.3: Disentangled spectra for V615 Per. Two spectral windows are shown, with the primary spectrum offset by +0.5 for clarity. Panel (a) contains the He I 4388 Å line in the primary spectrum and several sharp weak secondary lines. Panel (b) contains the He I 4471 Å and the Mg II 4481 Å lines from which most RV and T_{eff} information were derived.

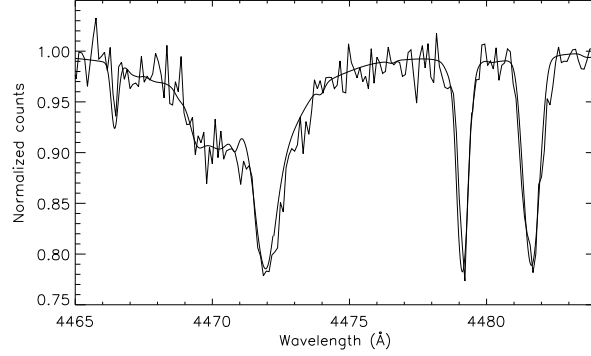


Figure 3.4: Representation of the best-fitting synthetic composite spectrum of V615 Per. The thick line shows the average of the last four spectra observed on HJD 2452564. This has less noise than one spectrum and the orbital smearing of the spectral lines is approximately 3 km s^{-1} for both stars. The spectral window containing the He I 4471 Å and Mg II 4481 Å lines is shown and the primary lines are redward of the secondary lines.

A preliminary spectroscopic orbit of $K_A = 68.2 \text{ km s}^{-1}$ and $K_B = 108.4 \text{ km s}^{-1}$ was found for V618 Per by disentangling the observed spectra over a grid of K_A and K_B values to find where the residuals of the fit were the lowest.

3.5 Spectral synthesis

The work in this section was undertaken by Dr. B. Smalley and is included here for completeness.

T_{eff} s and projected rotational velocities were derived for V615 Per by comparing the observed and disentangled spectra with synthetic spectra calculated using UCLSYN (section 1.4.3.2). The spectra were rotationally broadened as necessary and instrumental broadening was applied to match the resolution of the observations.

V615 Per was spectroscopically analysed using the binary star mode (BINSYN) within UCLSYN. A value of $\log g = 4.4$ was adopted for both components, based on

preliminary analyses. Microturbulence velocities of 0 km s^{-1} and 2 km s^{-1} were assumed for the primary and secondary, respectively. Properties of the two components were obtained by fitting to the observations using the least-square differences, which also enabled a monochromatic light ratio to be obtained. Figure 3.4 shows the best-fitting synthetic composite spectrum overplotted on an observed coadded spectrum.

The T_{eff} and rotational velocity derived for the primary are $15000 \pm 500 \text{ K}$ and $V \sin i = 28 \pm 5 \text{ km s}^{-1}$ respectively. For the secondary these values are $11000 \pm 500 \text{ K}$ and $8 \pm 5 \text{ km s}^{-1}$ respectively. The relative contributions of the stars to the total system light at a wavelength of $4481 \text{ \AA}$ are 0.65 ± 0.03 and 0.35 ∓ 0.03 for the primary and secondary respectively. These results are robust against small changes in metallicity but rely on the helium abundance being roughly solar.

The spectra of V618 Per are of much lower signal to noise ratio so a wide range of parameters provided acceptable fits. Microturbulence velocities of 0 km s^{-1} and surface gravities of $\log g = 4.4$ were assumed for both components. The T_{eff} s and rotational velocities found are $11000 \pm 1000 \text{ K}$ and $10 \pm 5 \text{ km s}^{-1}$ for the primary and $8000 \pm 1000 \text{ K}$ and $10 \pm 5 \text{ km s}^{-1}$ for the secondary. The relative contributions of the stars to the total system light are 0.7 ± 0.1 and 0.3 ∓ 0.1 .

3.6 Spectroscopic orbits

The two-dimensional cross-correlation algorithm TODCOR (section 2.2.3.3) was used to derive RVs for both dEBs. Synthetic spectra were generated using UCLSYN for the T_{eff} s and rotational velocities found in the spectral synthesis analysis (section 3.5) and used as templates for all four stars.

3.6.1 V615 Per

Several template spectra were generated with UCLSYN for different values of T_{eff} , rotational velocity and microturbulence velocity. These were used in the TODCOR analysis

Table 3.6: RVs and $O-C$ values (in km s^{-1}) for V615 Per calculated using TODCOR. Weights were derived from the amount of light collected in that observation and were used in the SBOP analysis.

HJD – 2 400 000	Primary velocity	$O-C$	Secondary velocity	$O-C$	Wt
52561.7284	–3.4	–1.3	–91.6	2.2	1.5
52562.4520	17.6	1.6	–118.6	–0.5	1.2
52562.5235	20.1	2.7	–118.6	1.5	1.0
52562.6300	22.5	3.0	–122.9	0.0	0.9
52563.4559	28.5	–1.4	–142.0	–4.0	0.9
52563.5074	27.9	–2.3	–136.5	2.0	0.9
52563.5754	31.7	1.2	–143.8	–4.6	0.5
52563.6576	32.3	1.5	–141.4	–1.7	0.2
52564.4086	21.0	–7.8	–137.1	1.8	1.1
52564.5128	27.6	–0.2	–138.9	–1.0	1.2
52564.5602	30.0	2.6	–137.4	–0.0	1.1
52564.6517	28.8	2.5	–136.2	0.0	1.2
52564.6958	30.3	4.5	–139.2	–3.6	0.5
52565.3995	8.9	–4.4	–120.3	0.7	1.1
52565.4713	12.0	0.4	–118.9	0.1	1.1
52565.5478	11.4	1.7	–117.4	–0.6	0.8
52565.5959	9.7	1.1	–115.4	0.0	0.6
52566.3911	–16.7	–2.6	–78.9	8.6	0.3
52568.6758	–82.4	6.6	6.5	–2.5	0.6
52569.5196	–109.6	–1.0	36.3	0.7	1.0
52569.6165	–115.7	–5.4	35.5	–2.5	0.1
52570.4977	–120.0	–0.7	51.4	–0.1	1.7
52570.5873	–118.0	1.5	54.3	2.2	1.3
52570.6997	–119.2	0.5	52.3	–0.2	1.2
52571.5213	–122.6	–8.0	39.3	–8.6	0.4

Table 3.7: RVs and $O-C$ values (in km s^{-1}) for V618 Per calculated using TODCOR. Weights were derived from the amount of light collected in that observation and were used in the SBOP analysis.

[†] RVs rejected from SBOP fit (see text for details).

HJD – 2 400 000	Primary velocity	$O-C$	Second. velocity	$O-C$	Wt
52559.4522	–113.4	1.4	63.9	2.9	1.4
52559.4733	–115.6	–0.5	66.9	5.4	1.3
52559.6311	–117.6	–1.0	63.9	0.2	1.0
52559.6522	–118.1	–1.5	62.0	–1.8	1.0
52560.4354	–104.8	–6.9	28.2	–7.5	0.7
52560.5872	–93.6	–3.6	25.5	1.5	0.4
52561.5565	–22.5	2.4	–78.6	–5.0	1.5
52561.6385	–22.7	–3.5	–81.2	0.7	1.8
52562.4759	23.6	1.2	–144.7	–0.3	1.3
52562.5471	24.8	0.6	–147.6	–0.6	1.2
52562.6536	22.2	–4.1	–148.4	1.6	1.5
52562.7104	28.2	1.3	–152.8	–1.7	1.5
52563.4795	16.1	0.9	–133.1	0.4	1.1
52563.5485	14.0	1.8	–129.0	0.1	0.8
52563.5993	10.3	0.4	–123.5	2.1	1.0
52563.6796	6.2	0.2	–120.0	–0.3	0.9
52563.7344	4.2	1.0	–116.3	–0.8	0.8
52564.4324	–43.0	–0.7	–48.8	–1.3	1.0
52564.5839	–50.8	2.3	–35.8	–4.5	1.6
52564.6759	–53.8	5.8	–18.4	3.3	1.4
52565.4231	–103.6	–0.4	43.9	0.3	1.6
52565.4942	–103.0	3.0	48.5	0.7	1.2
52565.5745	–109.9	–1.1	49.1	–2.9	0.5
52565.6207	–109.3	0.9	52.6	–1.6	0.7
52566.4145	–159.1 [†]	–47.0	56.6	–0.5	0.4
52568.7038	30.0 [†]	11.9	–133.1	4.7	0.2
52569.4302	23.6	–2.9	–149.6	0.9	1.1
52569.5426	66.2 [†]	41.6	–145.5	2.1	1.0
52569.6761	21.6	0.4	–140.0	2.4	0.9
52571.6157	–98.1	–3.1	–29.7 [†]	–61.3	1.0

Table 3.8: Final spectroscopic orbit for both dEBs using SBOP to fit RVs derived from TODCOR. All symbols have their usual meanings and those parameters held fixed in the SBOP analysis are indicated. All quoted uncertainties include errors arising from spectral template mismatch, added in quadrature. The ephemeris timebase T_0 refers to the time of minimum light of a primary eclipse.

	V615 Per A	V615 Per B	V618 Per A	V618 Per B
Period (days)	13.71390 (fixed)		6.366696 (fixed)	
Ephemeris T_0 (HJD)	2 452 169.6821 (fixed)		2 452 615.3955 (fixed)	
Velocity semiamplitude K (km s^{-1})	75.44 ± 0.82	96.71 ± 0.62	72.28 ± 0.82	108.16 ± 0.69
Systemic velocity (km s^{-1})	-44.27 ± 0.73	-44.08 ± 0.54	-44.42 ± 0.82	-44.29 ± 0.51
Orbital eccentricity	0.0 (fixed)		0.0 (fixed)	
$M \sin^3 i$ ($M_\odot$)	4.072 ± 0.055	3.177 ± 0.051	2.323 ± 0.031	1.552 ± 0.025
$a \sin i$ ($R_\odot$)	46.64 ± 0.28		22.70 ± 0.13	
Mass ratio q	0.7801 ± 0.0098		0.6682 ± 0.0087	

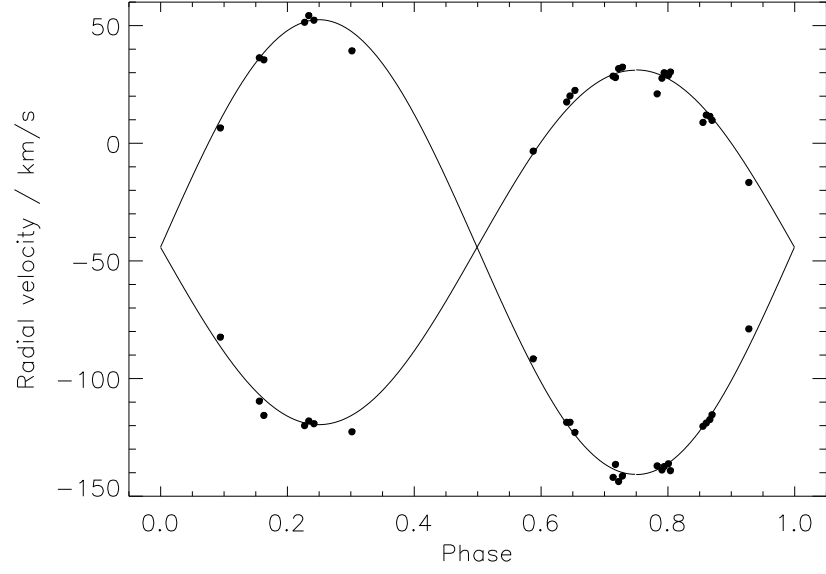


Figure 3.5: Spectroscopic orbit for V615 Per from an SBOP fit to RVs from TODCOR.

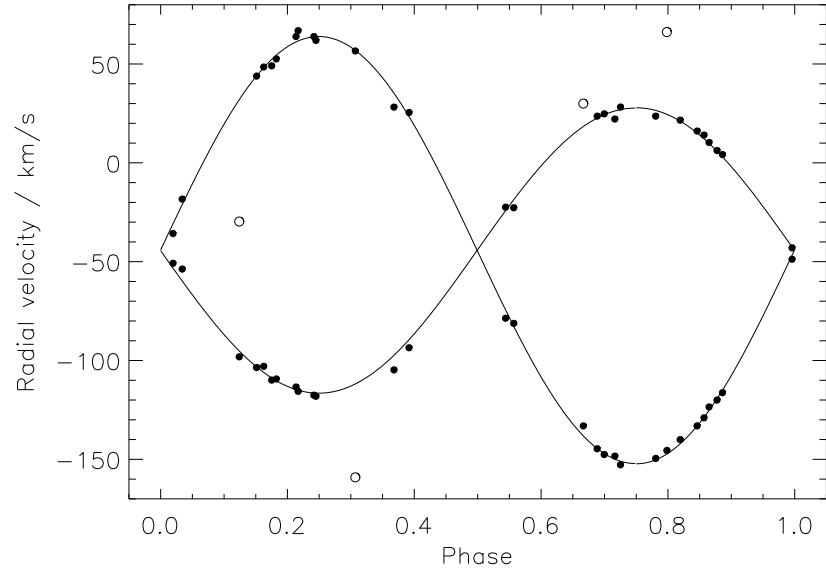


Figure 3.6: Spectroscopic orbit for V618 Per from the TODCOR analysis. Filled circles indicate RVs included in the SBOP fit and open circles indicate rejected RVs.

to derive several spectroscopic orbits for V615 Per. Additional orbits were derived after varying the size of a mask positioned over the broad $H\gamma$ 4340 Å line. The variation of velocity semiamplitudes resulting from changes in T_{eff} , rotational velocity, microturbulence velocity and mask size are 0.1, 0.05, 0.05 and 0.01 km s⁻¹, respectively. These represent estimates of the systematic errors incurred during the radial velocity analysis and were added in quadrature to the final velocity semiamplitudes, which were found using the best-fitting template spectra produced by Dr. B. Smalley.

The RVs have been reproduced in Table 3.6 and separate orbits have been fitted to the stars with SBOP (section 2.2.4.1); the systemic velocities of the two stars were not forced to be equal (see section 2.2.4 for some reasons for this). Analysis of the light curves of V615 Per suggest that its orbital eccentricity is very small so circular orbits were fitted; a small eccentricity does not significantly affect the results. The final spectroscopic orbit is plotted in Figure 3.5 and its parameters are given in Table 3.8.

3.6.2 V618 Per

The analysis of V618 Per was more complicated as the T_{eff} s and rotational velocities of the component stars are more uncertain. For this reason TODCOR was run on combinations of synthetic spectra with $\log g = 4.4$. Template spectra were generated for a wide range of microturbulence velocities, rotational velocities and T_{eff} s. Each combination of these was used as templates in TODCOR and the resulting RVs for each star were fitted using SBOP with an external automatic outlier rejection. The template spectra corresponding to the lowest residuals in the spectroscopic orbits have T_{eff} s of 11000 K and 8000 K, rotational velocities of 10 km s⁻¹, and microturbulence velocities of 2 km s⁻¹ and 0 km s⁻¹ for primary and secondary star, respectively. Systematic errors in the velocity semiamplitudes were estimated as with V615 Per and amount to 0.2, 0.2 and 0.3 km s⁻¹ for the primary star and to 0.1, 0.2 and 0.0 km s⁻¹ for the secondary star, for T_{eff} , rotational velocity and microturbulence velocity respectively.

Final RVs from TODCOR are given in Table 3.7 and points rejected from the SBOP analysis are indicated. These points were rejected as their $O - C$ values were

large compared to the other datapoints. Circular orbits were fitted as SBOP showed negligible orbital eccentricity; a small eccentricity does not significantly affect the final results. Template mismatch errors were added in quadrature and the final quantities are shown in Table 3.8.

3.6.3 The radial velocity of h Persei

All four stars under investigation have a systemic velocity around -44.2 km s^{-1} , consistent with the radial velocities for h Per in the literature of -43 km s^{-1} (Oosterhoff 1937), -41.9 km s^{-1} (Bidelman 1943), -40.0 km s^{-1} (Hron 1987) and -44.8 and -46.8 km s^{-1} (Liu, Janes & Bania 1989, 1991). The systemic velocity of h Persei can be redetermined from the measured systemic velocities of the two dEBs, using a weighted average over the four stars, to be $44.2 \pm 0.3 \text{ km s}^{-1}$. This figure is based on only two stellar systems so the precision of its determination is greater than the accuracy with which it gives the cluster velocity. The good agreement with literature values for the systemic velocity of h Persei suggests that V615 Per and V618 Per are almost certainly members.

3.7 Light curve analysis

3.7.1 JKTEBOP

The EBOP light curve modelling code (section 2.4.1.1) has been adopted to analyse the light curves of well-detached dEBs. As the EBOP code has some shortcomings (particularly in the input of data, output of results, and calculation of only formal errors) it has been significantly modified from its original form. The input and output subroutines were replaced by entirely new versions.

EBOP fits a light curve model to observation using the method of differential corrections, which can diverge if the initial estimates of the parameters of the model are significantly different from those suggested by the light curve being solved. To avoid this, the downhill simplex optimisation method of Nelder & Mead, as implemented in

the subroutine AMOEBA by Press et al. (1992, p.402), was adopted as the solution method of the light curves and the method of differential corrections was removed. The resulting program is called JKTEBOP and contains only the LIGHT subroutine of the original EBOP code. The LIGHT subroutine contains the light curve model used and provides the brightness of the dEB given a set of physical parameters and an orbital phase, so is the heart of the EBOP code.

3.7.2 V615 Per

V615 Per is well suited to a photometric analysis using JKTEBOP because the orbital separation of the stars is much greater than the sum of their radii. The eclipses are deep but not total and it is known that in such cases the ratio of the radii of the two stars can be relatively poorly constrained (e.g., Clausen et al. 2003), particularly when the component stars are sufficiently well separated to have no discernable reflection effect. This causes the ratios of the radii and of the surface brightnesses of the components to be significantly correlated. This correlation can be alleviated in the case of V615 Per because a spectroscopic light ratio has been obtained (section 3.5).

The KPK99 light curves and the JKT service data were combined and phased using the ephemeris derived in section 3.3.1 and the resulting data investigated using JKTEBOP. The *UBVI* light curves are shown in Figure 3.7. There is no photometric or spectroscopic indication of extra light from a third star close to V615 Per, and light curve solutions were consistent with this, so third light was fixed at zero. The secondary eclipse cannot be fitted properly without a small amount of orbital eccentricity, so the quantities $e \cos \omega$ and $e \sin \omega$ were allowed to vary in all solutions, where e is the orbital eccentricity and ω the longitude of periastron of the binary orbit. passband-specific linear limb darkening coefficients were taken from van Hamme (1993), gravity darkening exponents β_1 were fixed at 1.0 (Claret 1998) and the mass ratio was fixed at the spectroscopic value.

Solutions were made for many different values of the ratio of the radii and for the *BVI* light curves separately. The residuals of the fit were almost the same for ratios of

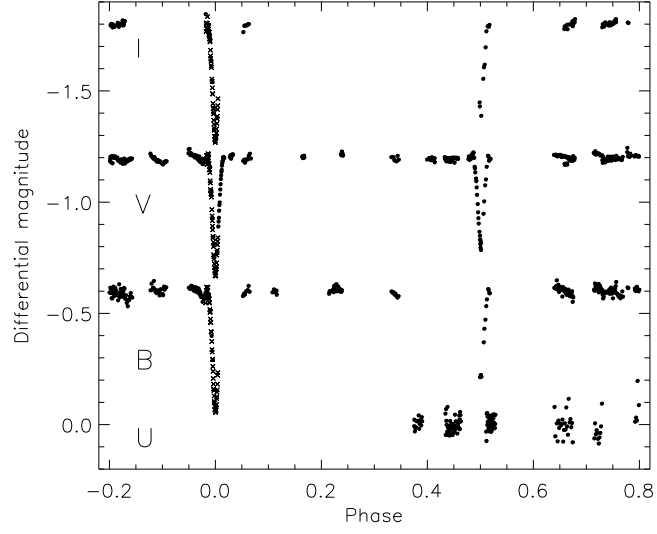


Figure 3.7: Phased broad-band light curves for V615 Per. KPK99 data are represented by filled circles and our JKT service data around the primary eclipse is shown using crosses. Light curves in the B , V and I passbands are offset by -0.6 , -1.2 and -1.8 mag, respectively.

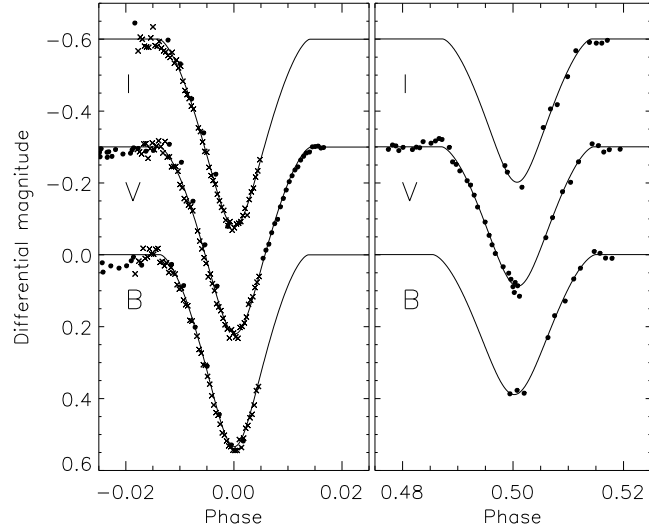


Figure 3.8: The best JKTEBOP model light curve fits to the BVI light curves of V615 Per. KPK99 data are represented by filled circles and our JKT service data around the primary eclipse is shown using crosses. Light curves in the V and I passbands are offset by -0.3 and -0.6 mag, respectively.

Table 3.9: Parameters of the light curve fits for V615 Per. The final values and uncertainties represent confidence intervals. It is not possible to be more precise about the meaning of the quoted uncertainties due to the systematic change of parameter values with wavelength (see text for details). Super- and sub-scripted errors represent the effects of changing the spectroscopic light ratio by its own uncertainty.

	B	V	I	Adopted value
Limb darkening coefficient u_A	0.348	0.301	0.189	
Limb darkening coefficient u_B	0.440	0.380	0.238	
Light ratio ($\frac{L_B}{L_A}$)	$0.512^{+0.071}_{-0.064}$	$0.555^{+0.077}_{-0.070}$	$0.594^{+0.082}_{-0.075}$	
Ratio of the radii (k)	$0.796^{+0.064}_{-0.056}$	$0.840^{+0.057}_{-0.058}$	$0.864^{+0.067}_{-0.059}$	0.833 ± 0.058
Fractional radius of the primary star (r_A)	$0.0509^{+0.0013}_{-0.0013}$	$0.0488^{+0.0013}_{-0.0013}$	$0.0475^{+0.0019}_{-0.0013}$	0.0491 ± 0.0030
Fractional radius of the secondary star (r_B)	$0.0405^{+0.0013}_{-0.0017}$	$0.0410^{+0.0016}_{-0.0019}$	$0.0410^{+0.0013}_{-0.0017}$	0.0408 ± 0.0020
Central surface brightness ratio (J)	$0.838^{+0.003}_{-0.012}$	$0.811^{+0.003}_{-0.006}$	$0.810^{+0.003}_{-0.004}$	
Orbital inclination (i) (degrees)	$88.83^{+0.10}_{-0.19}$	$88.76^{+0.07}_{-0.12}$	$88.81^{+0.10}_{-0.05}$	88.80 ± 0.20
Orbital eccentricity (e)	$0.0396^{+0.0012}_{-0.0011}$	$0.0105^{+0.0022}_{-0.0037}$	$0.0189^{+0.0054}_{-0.0013}$	0.02 ± 0.02

the radii between 0.7 and 1.1. The light ratio at 4481 Å found with spectral synthesis was converted to values for the *BVI* passbands using ATLAS9 fluxes convolved with filter and CCD efficiency functions. Filter transmission functions and the quantum efficiency function of the SITe2 CCD used to observe our JKT service data were taken from the Isaac Newton Group website⁵.

Corresponding values of the ratio of the radii were derived and used to determine the individual stellar radii, the surface brightness ratio and the orbital inclination. The best JKTEBOP fits are shown in Figure 3.8. The results for each light curve together with the adopted values are given in Table 3.9. The upper and lower bounds quoted for individual quantities show the effect of changing the light ratio within the errors quoted. The adopted results include this source of error and a contribution from other error sources, for example the period used to phase the light curves.

It is notable that the primary radius and the ratio of the radii (but not the secondary radius) show a systematic variation with wavelength. Eclipse depths are known to depend on wavelength when a dEB contains stars of different T_{eff} s and therefore colours, but such a variation is not noticeable in the current low-quality light curves. This inconsistency should be resolved when better light curves are obtained.

3.7.3 V618 Per

V618 Per is a more difficult case to analyse using our present light curves. Its spectroscopic light ratio is uncertain. Its eclipses are also less deep than V615 Per, causing a strong degeneracy between the ratios of the radii and the surface brightnesses of the two stars. All known light curves are shown in Figure 3.9. The KPK99 *B* light curve suggests a slight reflection effect outside eclipse but this is not present in the *V* light curve. Our Strömgren data from 2002 December and 2003 January suffer less from night errors than the data for V615 Per. In the absence of high quality light curves, the night errors have been compensated for with slight offsets for different nights, and

⁵<http://www.ing.iac.es/Astronomy/astronomy.html>

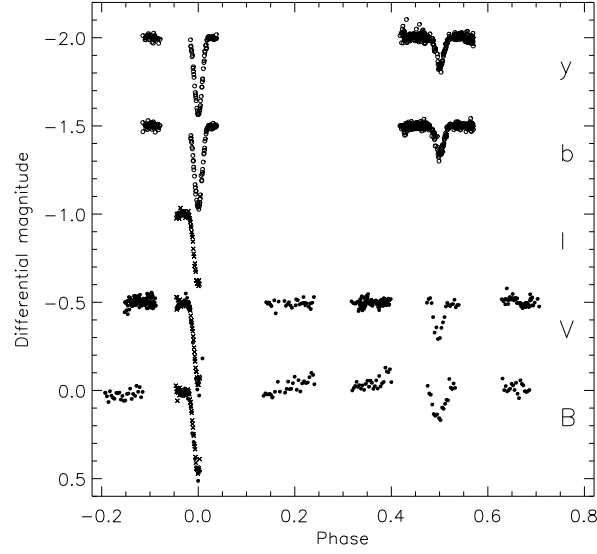


Figure 3.9: Phased light curves for V618 Per. KPK99 data are represented by filled circles and our JKT service data around the primary eclipse is shown using crosses. Strömgren data are represented by open circles. Light curves in V , I , and Strömgren b and y passbands are offset by -0.5 , -1.0 , -1.5 and -2.0 mag, respectively.

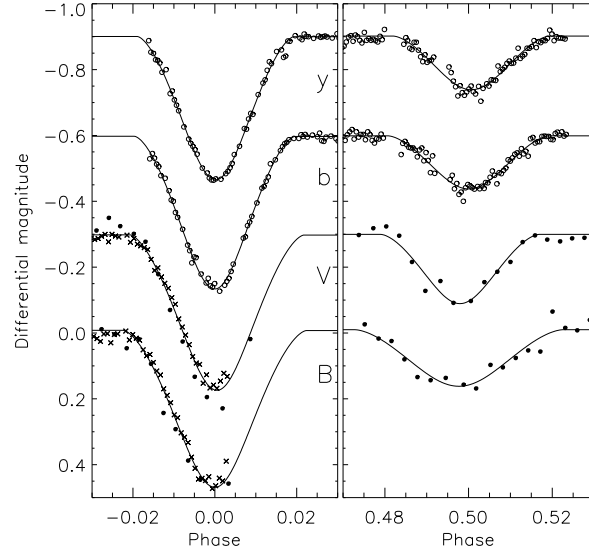


Figure 3.10: The best JKTEBOP model light curve fits to the light curves of V618 Per. KPK99 data are represented by filled circles and our JKT service data around the primary eclipse is shown using crosses. Strömgren data are represented by open circles. Light curves in V , b and y passbands are offset by -0.3 , -0.6 and -1.2 mag, respectively.

Table 3.10: Parameters of the light curve fits for V618 Per using JKTEBOP. Parameter designations are as in Table 3.9. The uncertainties are approximately 1σ confidence intervals (see text for details).

	B	V	b	y	Adopted
u_A	0.440	0.380	0.432	0.379	
u_B	0.575	0.509	0.571	0.509	
r_A	0.0876	0.0737	0.0733	0.0715	0.072 ± 0.003
k	0.831	0.816	0.804	0.800	0.802 ± 0.010
r_B	0.0728	0.0601	0.0590	0.0572	0.058 ± 0.003
J	0.480	0.474	0.407	0.429	
i ($^\circ$)	86.4	87.4	87.1	87.1	87.1 ± 0.5
e	0.0646	0.0658	0.00462	0.0119	0.01 ± 0.01
ω ($^\circ$)	266.9	266.6	276.6	274.2	275 ± 2

the data have been included in the analysis with JKTEBOP along with the BV KPK99 and JKT service light curves. Passband-specific linear limb darkening coefficients were taken from van Hamme (1993), gravity darkening exponents β_1 were fixed at 1.0 (Claret 1998) and the mass ratio was fixed at the spectroscopic value. Third light was set to zero; there is no photometric or spectroscopic indication of contaminating light.

Table 3.10 gives the best-fitting parameters for the $BVby$ light curves of V618 Per using JKTEBOP. Figure 3.10 shows that there is disagreement between the eclipse depths in B and V between the JKT service data (crosses) and the KPK99 light curves (filled circles). Combined with the sparseness of the data during secondary eclipse, this renders the BV light curve solutions unreliable. The adopted photometric parameters of V618 Per (Table 3.10) have therefore been taken from the b and y light curve solutions with a significant uncertainty added to account for degeneracy between the ratio of the radii and the primary fractional radius. Figure 3.10 shows the JKTEBOP model fits to primary and secondary eclipses.

3.8 Absolute dimensions and comparison with stellar models

Table 3.11 contains the absolute dimensions and radiative properties of V615 Per and V618 Per calculated from the results of spectroscopic, photometric and spectral synthesis analyses. An important check is whether the surface gravity values of the stars are consistent. Except for the primary component of V615 Per, they are all close to the expected values for ZAMS stars. V615 Per A is the most massive star being studied here and has a marginally lower surface gravity consistent with slight evolution away from the ZAMS.

3.8.1 Stellar and orbital rotation

The timescales of rotational synchronisation and orbital circularisation are large for both dEBs. The formulae of Zahn (section 1.7.1.1) give the timescales for convective-envelope stars, which are much lower than those for radiative-envelope stars like the components of V615 Per and V618 Per. The convective-envelope timescales of rotational synchronisation are ~ 500 Myr and ~ 25 Myr, for V615 Per and V618 Per respectively. For orbital circularisation they are ~ 1200 Gyr and ~ 21 Gyr.

All four stars considered here are slow rotators compared to nearby single B-type stars (Abt, Levato & Grosso 2002); only V615 Per A has a measured rotational velocity greater than the synchronous value. Both orbits show negligible eccentricity. For both V615 Per and V618 Per the timescales are much greater than their age so the rotational velocities and orbital characteristics of the stars will not have changed significantly during their MS lifetime. Therefore their slow rotation must be primordial in nature (see Valtonen 1998); their closeness to having circular orbits must also be primordial (Zahn & Bouchet 1989).

Table 3.11: Absolute dimensions of the dEBs V615 Per and V618 Per in the open cluster h Persei.

* Calculated using the combined system magnitudes in the V passband, light ratios found using the V (V615 Per) and y (V618 Per) passband light curves, the assumed cluster distance modulus and reddening and the canonical reddening law $A_V = 3.1E_{B-V}$.

† Calculated using the theoretical T_{eff} – bolometric correction calibration of Bessell, Castelli & Plez (1998).

	V615 Per A	V615 Per B	V618 Per A	V618 Per B
Cluster age $\log \tau$ (years)				
Cluster distance modulus				
Period (days)	13.71390 $\pm$ 0.00002		7.10 $\pm$ 0.01	
Mass ratio	0.7801 $\pm$ 0.0098		11.70 $\pm$ 0.05	
Mass ($M_{\odot}$)	4.075 $\pm$ 0.055	3.179 $\pm$ 0.051	6.366696 $\pm$ 0.000004	
Radius ($R_{\odot}$)	2.291 $\pm$ 0.141	1.903 $\pm$ 0.094	0.6682 $\pm$ 0.0087	
Surface gravity $\log g$ (cm s^{-2})	4.328 $\pm$ 0.059	4.381 $\pm$ 0.050	2.332 $\pm$ 0.031	1.558 $\pm$ 0.025
Effective temperature (K)	15 000 $\pm$ 500	11 000 $\pm$ 500	1.636 $\pm$ 0.069	1.318 $\pm$ 0.069
Luminosity $\log (L/L_{\odot})$	2.37 $\pm$ 0.08	1.67 $\pm$ 0.09	4.378 $\pm$ 0.042	4.391 $\pm$ 0.052
Absolute visual magnitude*	0.15 $\pm$ 0.14	1.09 $\pm$ 0.13	11 000 $\pm$ 1000	8 000 $\pm$ 1000
Distance (pc) [†]	2070 $\pm$ 210		1.54 $\pm$ 0.16	0.81 $\pm$ 0.22
Rotational velocity (km s^{-1})	28 $\pm$ 5	8 $\pm$ 5	1.35 $\pm$ 0.22	2.72 $\pm$ 0.51
Synchronous rotational velocity (km s^{-1})	8.45 $\pm$ 0.52	7.02 $\pm$ 0.35	2250 $\pm$ 290	
Systemic velocity (km s^{-1})	−44.27 $\pm$ 0.73	−44.08 $\pm$ 0.54	10 $\pm$ 5	10 $\pm$ 5
			13.01 $\pm$ 0.55	10.48 $\pm$ 0.55
			−44.42 $\pm$ 0.82	−44.29 $\pm$ 0.52

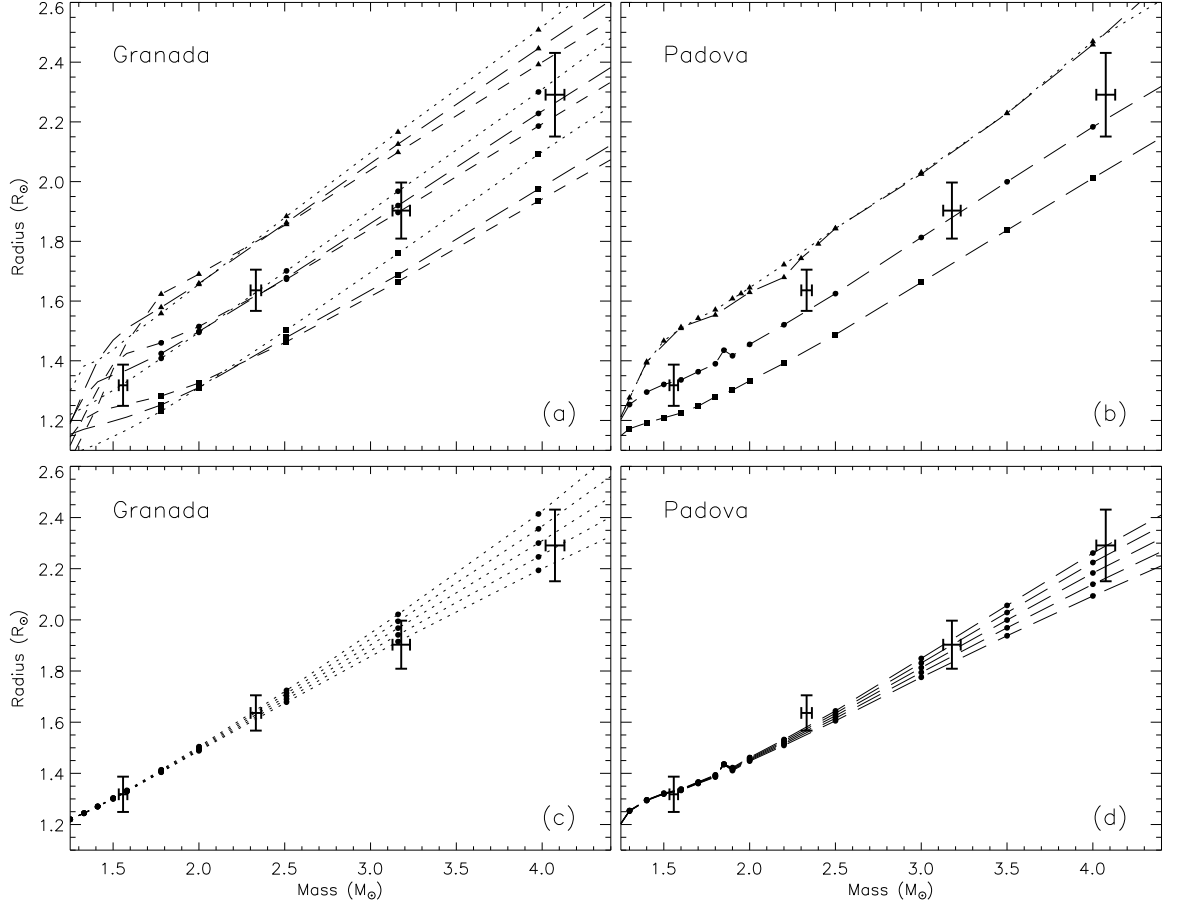


Figure 3.11: Comparison of stellar evolutionary models to the masses and radii of the stars of V615 Per and V618 Per for two different sets of theoretical models. (a) Granada models plotted for metal abundances of $Z = 0.004$ (squares), $Z = 0.01$ (circles) and $Z = 0.02$ (triangles). Each metal abundance is available with three hydrogen abundances (see text) which are plotted with long dashes for standard, short dashes for lower, and dots for higher hydrogen abundance. For clarity, symbols are shown only for masses above $1.75 M_{\odot}$. (b) Padova models plotted for metal abundances (bottom to top) $Z = 0.004$, $Z = 0.008$, and $Z = 0.019$. The $Z = 0.019$ track with no convective overshooting is shown using a dotted line. (c) Granada models with $(X, Z) = (0.63, 0.01)$ plotted for ages (bottom to top) of 3, 8, 13, 18 and 23 Myr. (d) Padova models with $(Y, Z) = (0.25, 0.008)$ plotted for ages (bottom to top) of 3, 8, 13, 18 and 23 Myr.

3.8.2 Stellar model fits

The physical parameters of the four stars in V615 Per and V618 Per have been compared to two different sets of stellar models (section 1.3.2), the Granada models (section 1.3.2.1) and the Padova models (section 1.3.2.3). The two sets of models have been plotted in the mass–radius plane, with the two dEBs, for three metal abundances and for an age of 13 Myr ($\log \tau = 7.11$) in Figure 3.11(a)(b). A best fit is obtained using the Granada models with $(X, Z) = (0.63, 0.01)$, although panel (b) suggests the Padova models would fit equally well for the same $Z = 0.01$. Panels (c) and (d) of Figure 3.11 show the best-fitting evolutionary models for the Granada and Padova sets, for ages of $\log \tau = 6.47, 6.90, 7.11, 7.26, 7.36$ (years).

3.9 Discussion

Absolute dimensions have been derived for two early-type dEBs in the young open cluster h Persei. Spectral synthesis has given the T_{eff} s and rotational velocities of both systems. The negligibly eccentric orbits and low rotational velocities of all four stars supports the ‘delayed break-up’ route of binary star formation (Tohline 2002). In this scenario a protostellar core embedded in a molecular cloud contracts towards the ZAMS. It accretes material with a high specific angular momentum from the surrounding cloud, and spins up whilst losing gravitational potential energy. When the ratio of rotational energy to the absolute value of gravitational energy, β , reaches about 0.27 (Lebovitz 1974, 1984), the core deforms into an ellipsoidal shape. From this it forms a ‘dumbbell’ shape which splits to form a binary system with a circular orbit and low rotational velocities. Other examples exist of young long-period spectroscopic binaries with circular orbits, e.g., #363 in NGC 3532 (González & Lapasset 2002).

The four stars exhibit a good spread of masses and radii which should provide an excellent test of stellar evolutionary models. The radii could not be determined from the current light curves with great accuracy, so the analysis has been restricted

to a determination of the bulk metal abundance of the h Persei cluster to be $Z \approx 0.01$. The existence of Galactic disc low-metallicity young B stars is already known (e.g., GG Lupi; Andersen, Clausen & Giménez 1993).

The chemical composition of h and χ Persei has been investigated many times with somewhat conflicting results. Nissen (1976) found the helium abundance of h Persei to be significantly lower than that of field stars, based on narrow-band photometry of twelve ZAMS and slightly evolved B stars. This conclusion was supported by the spectroscopic observations of Wolff & Heasley (1985). However, from high-resolution spectroscopic abundance analyses of four stars, Lennon, Brown & Dufton (1988) and Dufton et al. (1990) found that the helium abundance was normal and suggested that the surface gravities derived by Nissen (1976) were too low. Dufton et al. also found that h and χ Persei have approximately solar metal abundances. This conclusion was supported by Smartt & Rolleston (1997), but Vrancken et al. (2000) find that the abundances of various metals are between 0.3 and 0.5 dex below solar from abundance analyses of eight early B-type giant stars.

The above results refer to empirical determinations of the mean photospheric abundances of helium and several light metals. Our derivation of the cluster metallicity, $Z \approx 0.01$, has been found by comparison with theoretical stellar evolutionary models and refers to the overall metal abundance in the interiors of the stars analysed. This quantity is directly relevant to the fitting of theoretical isochrones to the positions of stars in observed CMDs of the h and χ Persei open clusters.

The four recent photometric studies of h and χ Persei have not included the effects of non-solar metallicity in their analyses; whilst Marco & Bernabeu (2001) and Slesnick et al. (2002) assumed a metallicity of $Z = 0.02$, the works of Capilla & Fabregat (2002) and Keller et al. (2001) make no mention of metallicity. If all analyses used a solar metallicity, the derived age and distance modulus of h and χ Persei could be systematically incorrect. This possibility also is increased by the dependence on one set of model isochrones; three of the four works used the stellar models of the Geneva Group (section 1.3.2.2), although Keller et al. used the previous generation of Padova models (section 1.3.2.3). Once more accurate radii for the four stars studied here are

obtained, a reanalysis of η and χ Persei should be undertaken to ensure that reliable parameters are known.

The four recent photometric analyses suggest the age of the cluster is $\log \tau = 7.10 \pm 0.05$ (see section 3.1.1) and the positions of the stars of V615 Per and V618 Per in the mass–radius plane are consistent with this. The traditional degeneracy between age, metal abundance and helium abundance (e.g., Thompson et al. 2001) could be broken with better light curves for the two dEBs. In that case the large range of masses of the four stars would allow the less massive stars to set the metallicity and the more massive stars to set the age of the cluster, with information on the helium abundance contained in the slope of the observational line in the mass–radius plane. Such an analysis would benefit from better sampled grids of stellar models and a greater choice of helium abundance and degree of overshooting (see section 1.3.3).

The distance to the dEBs, and so η Persei, can be found using bolometric corrections (see section 1.6.3.1). Using the bolometric corrections of Bessell, Castelli & Plez (1998) gives distance moduli 11.58 ± 0.21 and 11.76 ± 0.26 mag for V615 Per and V618 Per respectively (Table 3.11). The weighted mean of these quantities is 11.65 ± 0.16 mag, which is in excellent agreement with previous determinations in the literature (section 3.1.1).

Definitive light curves of the dEBs should give radii to accuracies of between one and two per cent and the individual brightnesses of the component stars in the observed passbands. This will allow more discriminate testing of stellar evolutionary models and the construction of a cluster HR diagram with accurate mass and radius determinations for four individual stars. If the light curves are observed in the *VRIJHK* passbands, accurate surface brightnesses of the stars could be derived (section 1.1.1.5) and an accurate distance found to each dEB (see also section 6.6).

4 V453 Cyg in the open cluster NGC 6871

Two observing runs were undertaken to obtain photometry of the dEBs studied in this thesis, totalling 29 nights on the Jakobus Kapteyn Telescope (ING, La Palma). Useful datasets were obtained only for V615Per and V618Per due to bad weather and technical problems (including dome shutters frozen closed), and these suffer from several problems (see section 3.2.2). Due to these problems there were only a limited number of dEBs for which I had sufficient data for a useful study. V453Cyg was clearly a good choice among these systems as we were able to obtain extensive spectroscopy, there are good-quality published light curves, and the system itself is very interesting. V453Cyg exhibits total eclipses, allowing the fractional radii to be found to good accuracy, apsidal motion, allowing the internal structure of the component stars to be investigated, and is composed of two somewhat evolved and dissimilar early-type high-mass stars.

4.1 V453 Cyg

V453Cyg is a high-mass dEB with an orbital period of 3.89 days. Its membership of the young open cluster NGC 6871 means that its age and distance can be found independently. The primary component of V453Cyg is approaching the terminal-age main sequence (TAMS) and its large radius causes the eclipses to be total, allowing a very accurate determination of the radii of both stars. Table 4.1 contains identifications and some photometric properties of the system.

The eclipsing nature of V453Cyg was discovered by Wachmann (1939) and an early spectroscopic orbit was calculated by Pearce (1941). A period study by Cohen (1971) provided a determination of the orbital longitude of periastron, ω , inconsistent with that derived by Pearce. In a period study by Wachmann (1973) this was correctly interpreted as apsidal motion (section 1.7.2). Wachmann derived an apsidal period of $U = 71$ years using measurements of the time differences between several groups

Table 4.1: Astrophysical parameters for V453 Cygni system.

References: (1) Cannon & Pickering (1923); (2) Argelander (1903); (3) Høg et al. (1998); (4) Hoag et al. (1961); (5) Popper (1980); (6) Zakirov (1992); (7) Cohen (1969); (8) Reimann (1989).

	V453 Cygni	Reference
Henry Draper number	HD 227696	1
Bonner Durchmusterung	BD +35°3964	2
Hoag number	NGC 6871 13	3
α_{2000}	20 06 34.967	4
δ_{2000}	+35 44 26.28	4
Spectral type	B 0.4 IV + B 0.7 IV	5
V	8.285	6
$B - V$	+0.179	6
$U - B$	-0.061	6
$V - R$	+0.254	6
β	2.590	7,8

of adjacent primary and secondary eclipses. A more recent period study, including parabolic and periodic terms, was undertaken by Rafert (1982).

Excellent photoelectric UBV light curves were observed by Wachmann (1974) and analysed using the Russell-Merrill method (section 2.4.1). Wachmann's work contains a plot of the light curves adjusted for the effects of orbital eccentricity and apsidal motion using parameters updated from that of his previous work, but the data themselves have so far been unobtainable. It is possible that they are in an unlabelled file in the IAU Variable Star Archives (P. D. Hingley, 2003, private communication; Breger 1988), but no record of them exists at Hamburg Observatory, where the light curves were observed (A. Reiners, 2003, private communication).

Cohen (1974) published complete photoelectric UBV light curves which contain fewer datapoints and more observational scatter than those of Wachmann (1974). He analysed these using the Russell-Merrill method but stated that his observations were not definitive. They have since been analysed by Cester et al. (1978) using the light

Table 4.2: Published spectroscopic orbits of V453 Cygni. BMM97 originally fitted their data with a circular orbit. We have refitted their RVs with an eccentric orbit to increase the accuracy of our determination of the apsidal motion (see section 4.3 for details). A colon after a number indicates that it is uncertain. Quantities without quoted errors or a colon were not determined by that investigation. When quantities are given separately for each star we have quoted a weighted mean of the two values. Symbols have their usual meanings. Times are written as (HJD – 2 400 000).

* The reference time, T_0 , refers to a time of periastron passage, not a time of minimum light.

	Pearce (1941)	Abt, Levy and Gandet (1972)	Popper and Hill (1991)	Simon and Sturm (1994)	BMM97	BMM97 (this solution)
P (days)	3.87972	3.8890	3.8898128	3.88982309	3.8898128	3.889825
T_0 (HJD)	$30231.0843 \pm 0.0543^*$	40495.027^*	39340.099	36811.7296	$48141.82 \pm 0.01^*$	$48500.64 \pm 0.66^*$
K_A (km s^{-1})	181.8 ± 1.13	152:	171 ± 1.5	171.7 ± 2.9	173.2 ± 1.3	173.7 ± 1.4
K_B (km s^{-1})	237.4 ± 2.78		222 ± 2.5	223.1 ± 2.9	213.6 ± 3.0	212.4 ± 3.4
e	0.07 ± 0.007	0.05:	0.0	0.0	0.0	0.011 ± 0.015
ω (degrees)	175.2 ± 5.06	99:				88.6 ± 6.0
V_γ (km s^{-1})	-15.0 ± 0.94	-22.7:	-14	-7:	-17.6 ± 1.0	-18.0 ± 1.6

curve analysis code WINK (Wood 1972). This is the only previous photometric study of V453 Cyg to use modern techniques.

A recent investigation using photoelectric *UBVRI* light curves has been published by Zakirov (1992). He analysed his light curves using the “direct machine method of Lavrov (1993)”, which is based on rectification. The results of the four photometric analyses of V453 Cyg are substantially in agreement about the basic photometric parameters of the system.

Recent spectroscopic orbits have been published by Popper & Hill (1991), Simon & Sturm (1994) and Burkholder, Massey & Morrell (1997, hereafter BMM97). These results are collected in Table 4.2. Simon & Sturm used seven spectra to demonstrate their spectral disentangling algorithm (section 2.2.3.4) as the total secondary eclipse of V453 Cyg allowed them to directly compare their disentangled primary spectrum with a spectrum observed during secondary eclipse. BMM97 derived a good spectroscopic orbit from 25 spectra of a high signal to noise ratio and compared the dEB to models to investigate the discrepancy at higher masses between models and observations (which has since been resolved; Hilditch 2004). The rotational velocities of the components of V453 Cyg were determined by Olson (1984) to be $107 \pm 9 \text{ km s}^{-1}$ and $97 \pm 20 \text{ km s}^{-1}$ for the primary and secondary stars respectively. A preliminary single-lined spectroscopic orbit was also given by Abt, Levy & Gandet (1972).

An abundance analysis of V453 Cyg was undertaken by Daflon et al. (2001) using both LTE and non-LTE calculations. The results suggest that V453 Cyg has a slightly sub-solar metallicity. Daflon et al. do not state whether they analysed the spectral lines of the primary or of the secondary component (although as the system undergoes total eclipses there are times when its spectrum comes entirely from the primary star). These authors derived a T_{eff} of 29 100 K using the Q parameter based on *UBV* magnitudes (Johnson 1957), and a surface gravity of $\log g = 4.45 \text{ (cm s}^{-2}\text{)}$ from profile fitting of the $\text{H}\gamma$ 4340 Å spectral line. Both values are substantially larger than expected and generally inconsistent with previous analyses. Their surface gravity value is in fact somewhat larger than theoretically predicted even for the ZAMS.

4.1.1 NGC 6871

The open cluster NGC 6871 is a concentration of bright OB stars which forms the nucleus of the Cyg OB3 association (Garmany & Stencel 1992). This makes it an important object for the study of the evolution of high-mass stars. The cluster itself has been studied photometrically several times but its sparse nature means determination of its physical parameters is difficult.

UBV photometry of the 30 brightest stars was published by Hoag et al. (1961). Crawford, Barnes & Warren (1974) observed 11 stars using Strömgren *uvby* passbands and 24 stars using $H\beta$ passbands (section 2.3.1.3), finding significantly variable reddening and a distance modulus of 11.5 mag. This *uvby* β photometry was extended to 40 stars by Reimann (1989), who found reddening E_{b-y} with a mean value of 0.348 mag and an intracluster variation of about 0.1 mag. His derived distance modulus of 11.94 ± 0.08 mag and age of 12 Myr are both greater than previous literature values.

Massey, Johnson & DeGioia-Eastwood (1995) conducted extensive *UBV* CCD photometry of 1955 stars in the area of Cyg OB3. Their values of distance modulus, 11.65 ± 0.07 , and of reddening, $E_{B-V} = 0.46 \pm 0.03$ mag with individual values between 0.04 and 1.11 mag, agree well with previous determinations. They find an age of 2 to 5 Myr for stars with spectral types earlier than B0 but give evidence for a significant spread of stellar ages in the cluster. Whilst the highest-mass unevolved cluster members have MS lifetimes of 4 to 5 Myr, NGC 6871 contains evolved $15 M_{\odot}$ stars despite their MS lifetimes being of the order of 11 Myr.

4.2 Observations

Spectroscopic observations were carried out during the same observing run and using the same observational and data reduction techniques as for V615 Per and V618 Per (section 3.2.1). The spectral windows chosen for observation were again 4450–4715 Å (31 spectra) and 4230–4500 Å (12 spectra). Additional spectra were observed around

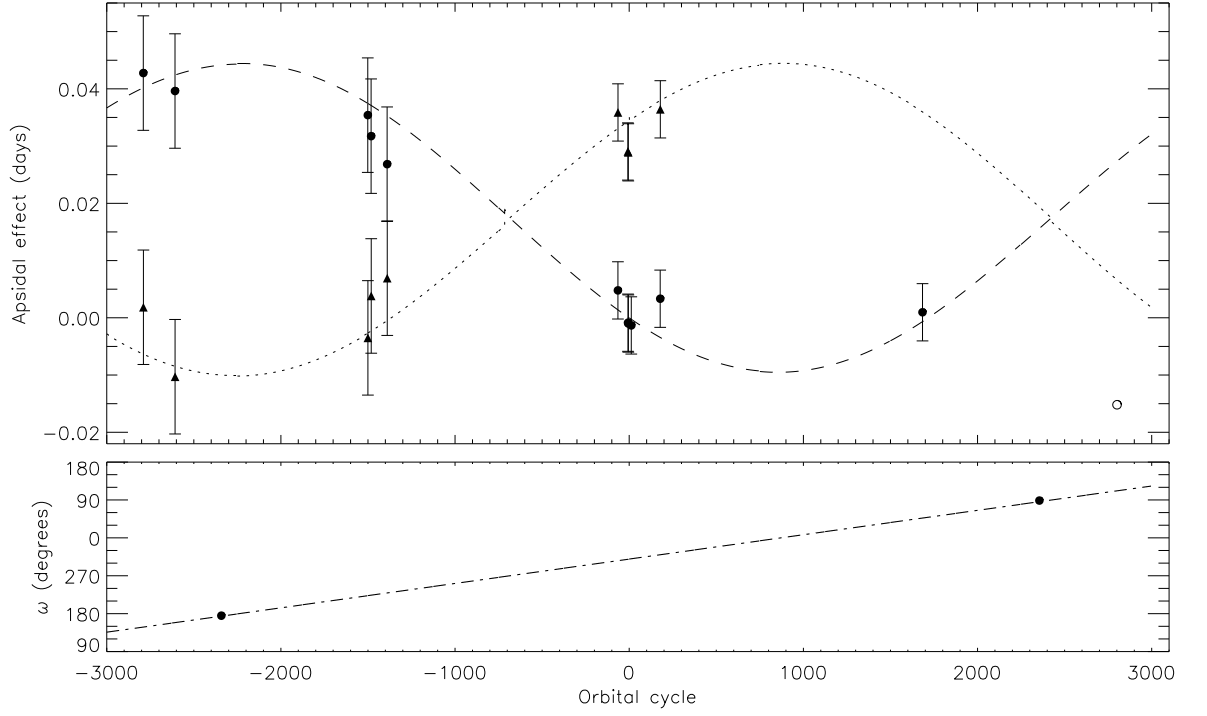


Figure 4.1: Representation of the best-fitting apsidal motion parameters. The upper panel shows the observed times of primary (circles) and secondary (triangles) minima, minus the expected times given by a linear ephemeris, compared to the best-fitting curves of primary (dashed) and secondary (dotted) minima. The open circle represents the rejected time of minimum of Bíró et al. (1998). The lower panel shows the spectroscopic longitudes of periastron, ω , and the change of ω over orbital cycle. Errors have only been shown if they are larger than the corresponding symbol.

H β (4861 Å) to provide an additional T_{eff} indicator for spectral analysis. The signal to noise ratio per pixel of the observed spectra is between 100 and 450. An observing log is given in Table 4.3.

Table 4.3: Observing log for the spectroscopic observations of V453 Cyg

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
V453 Cyg	323080	4210–4480	2452559.32558	180	11/10/02	19:46:31
V453 Cyg	323081	4210–4480	2452559.32791	180	11/10/02	19:49:52
V453 Cyg	323082	4210–4480	2452559.33024	180	11/10/02	19:53:14
V453 Cyg	323085	4450–4710	2452559.33775	180	11/10/02	20:04:02
V453 Cyg	323086	4450–4710	2452559.34009	180	11/10/02	20:07:24
V453 Cyg	323285	4450–4710	2452560.31872	180	12/10/02	19:36:43
V453 Cyg	323286	4450–4710	2452560.32104	180	12/10/02	19:40:03
V453 Cyg	323287	4450–4710	2452560.32335	180	12/10/02	19:43:23
V453 Cyg	323288	4450–4710	2452560.32567	180	12/10/02	19:46:43
V453 Cyg	323289	4450–4710	2452560.32799	180	12/10/02	19:50:04
V453 Cyg	323317	4450–4710	2452560.40911	180	12/10/02	21:46:53
V453 Cyg	323318	4450–4710	2452560.41142	180	12/10/02	21:50:13
V453 Cyg	323319	4450–4710	2452560.41375	180	12/10/02	21:53:33
V453 Cyg	323320	4450–4710	2452560.41606	180	12/10/02	21:56:53
V453 Cyg	323321	4450–4710	2452560.41837	180	12/10/02	22:00:13
V453 Cyg	323479	4450–4710	2452561.30395	180	13/10/02	19:15:31
V453 Cyg	323480	4450–4710	2452561.30627	180	13/10/02	19:18:52
V453 Cyg	323481	4450–4710	2452561.30859	180	13/10/02	19:22:12
V453 Cyg	323482	4450–4710	2452561.31091	180	13/10/02	19:25:32
V453 Cyg	323483	4450–4710	2452561.31322	180	13/10/02	19:28:52
V453 Cyg	323689	4450–4710	2452562.32195	180	14/10/02	19:41:31
V453 Cyg	323690	4450–4710	2452562.32426	180	14/10/02	19:44:51
V453 Cyg	323691	4450–4710	2452562.32657	180	14/10/02	19:48:10
V453 Cyg	323744	4450–4710	2452562.49486	300	14/10/02	23:50:31
V453 Cyg	323747	4230–4500	2452562.50016	300	14/10/02	23:58:09
V453 Cyg	323861	4710–4970	2452563.30557	300	15/10/02	19:18:00
V453 Cyg	323864	4450–4710	2452563.31216	300	15/10/02	19:27:30
V453 Cyg	323867	4230–4500	2452563.31850	300	15/10/02	19:36:38
V453 Cyg	324026	4450–4710	2452564.31111	600	16/10/02	19:26:04
V453 Cyg	324029	4230–4500	2452564.31988	600	16/10/02	19:38:41
V453 Cyg	324247	4450–4710	2452565.32035	600	17/10/02	19:39:26
V453 Cyg	324252	4710–4970	2452565.33054	600	17/10/02	19:54:07
V453 Cyg	324255	4230–4500	2452565.33945	600	17/10/02	20:06:57
V453 Cyg	324455	4230–4500	2452566.29608	300	18/10/02	19:04:34
V453 Cyg	324458	4450–4710	2452566.30176	300	18/10/02	19:12:45
V453 Cyg	324462	4230–4500	2452566.31116	300	18/10/02	19:26:17
V453 Cyg	324465	4450–4710	2452566.31654	300	18/10/02	19:34:02
V453 Cyg	324505	4230–4500	2452566.43565	600	18/10/02	22:25:34
V453 Cyg	324581	4230–4500	2452568.34709	600	20/10/02	20:18:11
V453 Cyg	324584	4450–4710	2452568.35585	600	20/10/02	20:30:48
V453 Cyg	324753	4450–4710	2452569.30022	600	21/10/02	19:10:46
V453 Cyg	324757	4230–4500	2452569.31004	600	21/10/02	19:24:55
V453 Cyg	324771	4450–4710	2452569.33166	600	21/10/02	19:56:03
V453 Cyg	324774	4230–4500	2452569.34031	600	21/10/02	20:08:30
V453 Cyg	325126	4230–4500	2452570.34581	600	22/10/02	20:16:30
V453 Cyg	325129	4450–4710	2452570.35446	600	22/10/02	20:28:58
V453 Cyg	325348	4450–4710	2452571.30720	600	23/10/02	19:20:59
V453 Cyg	325351	4710–4970	2452571.31595	600	23/10/02	19:33:35
V453 Cyg	325354	4230–4500	2452571.32458	600	23/10/02	19:46:00

Table 4.4: Times of minimum light of V453 Cyg taken from the literature. The $O-C$ values refer to the difference between the observed and calculated values.

* Rejected from the fit due to a large $O-C$ value.

References: (1) Wachmann (1973) photographic, (2) Wachmann (1973) photoelectric, (3) Cohen (1971) photoelectric, (4) R. Diethelm (see text) photoelectric, (5) Bíró et al. (1998) CCD.

Cycle number	Minimum time (HJD-2 400 000)	Adopted error	$O-C$	Ref.
-2790.0	28487.531	0.01	0.0026	1
-2789.5	28489.435	0.01	0.0008	1
-2608.0	29195.476	0.01	-0.0028	1
-2607.5	29197.371	0.01	-0.0018	1
-1501.0	33501.508	0.01	-0.0020	1
-1500.5	33503.414	0.01	-0.0008	1
-1482.0	33575.411	0.01	-0.0054	1
-1481.5	33577.328	0.01	0.0061	1
-1390.0	33933.270	0.01	-0.0084	1
-1389.5	33935.195	0.01	0.0074	1
-65.0	39087.266	0.005	0.0039	2
-64.5	39089.242	0.005	0.0028	2
-7.5	39310.9552	0.005	-0.0052	2
-7.0	39312.8702	0.005	-0.0010	3
-5.5	39318.7347	0.005	-0.0054	3
-5.0	39320.6497	0.005	-0.0011	3
12.0	39386.7764	0.005	-0.0010	3
178.0	40032.492	0.005	0.0068	2
178.5	40034.470	0.005	-0.0015	2
1684.0	45890.5660	0.005	0.0016	4
2801.0	50235.4843*	0.005	-0.0426	5

Table 4.5: Spectroscopic data used in the apsidal motion analysis.

References: (1) Pearce (1941), (2) BMM97 (our solution).

Cycle number	Eccentricity	$O-C$ (e)	ω (degrees)	$O-C$ (ω)	Ref.
-2342.0	0.070 ± 0.007	0.048	175.2 ± 5.1	1.0	1
2355.0	0.011 ± 0.015	0.011	88.6 ± 6.0	2.6	2

Table 4.6: apsidal motion parameters for V453 Cyg. To illustrate the final result we have also included the best-fitting parameters using only photometric data, and the results of Wachmann (1974). Times are written as (HJD – 2 400 000).

	Wachmann (1974)	This paper (photometric only)	This paper (final results)
Anomalistic period P (days)		3.890426 ± 0.000073	3.890450 ± 0.000017
Reference minimum time T_0	36811.7296	39340.1011 ± 0.0022	39340.0998 ± 0.0019
Orbital inclination i (degrees)		88.0 (fixed)	88.0 (fixed)
Orbital eccentricity e	0.02	0.019 ± 0.002	0.022 ± 0.002
Periastron longitude at T_0 , ω_0 (degrees)	309.2	313.2 ± 6.7	309.7 ± 3.1
Apsidal motion rate $\dot{\omega}$ (degrees P_s^{-1})	0.0539	0.0556 ± 0.0068	0.0579 ± 0.0016
Sidereal period P_s (days)	3.88982309	3.889824 ± 0.000082	3.889825 ± 0.000018
Apsidal motion period U (years)	71	68.9 ± 8.5	66.4 ± 1.8

4.3 Period determination and apsidal motion

A period study by Wachmann (1973, 1974) indicated fast apsidal motion with a period of $U = 71$ yr from eighteen times of minimum light covering almost three thousand orbital cycles. Photographic minima exist dating back to the year 1902 but they are not of sufficient quality to improve the apsidal motion analysis (Ashbrook, unpublished, but tabulated in Cohen 1971). Zakirov (1992) states that his observations are not consistent with Wachmann’s apsidal motion period.

Times of minima for inclusion in the apsidal motion analysis were taken from Cohen (1971), Wachmann (1973), R. Diethelm¹, and Bíró et al. (1998). The method of Lacy (1992) was adopted to solve the apsidal motion equations (section 1.7.2). Our implementation of this method (APSMOT) uses subroutines very generously supplied by D. Holmgren (see Holmgren & Wolf 1996).

It was immediately clear that the more recent times of minima were not in full agreement with each other or with the times of minima used by Wachmann (1973). In such cases, independent information is needed to decide which published observations are reliable and which are discrepant. For this reason we added to our APSMOT code the ability to include spectroscopic determinations of eccentricity, e , and longitude of periastron, ω , in the overall fit.

Values of e and ω were taken from the spectroscopic studies by Pearce (1941) and BMM97. The RV observations of BMM97 were originally fitted with a circular spectroscopic orbit so we have reanalysed the velocities (Table 4.2) using SBOP (section 2.2.4.1 to determine e and ω , and assigned a cycle number corresponding to the approximate midpoint of the observations. There is a large correlation between ω and the ephemeris reference time, T_0 , which makes both values somewhat uncertain. The solution of the secondary velocities did not converge without fixing the value of T_0 to that of the primary star’s spectroscopic orbit.

The spectroscopic data of Pearce (1941) and BMM97 allowed us to identify the

¹Eclipsing Binaries Minima Database at <http://www.oa.uj.edu.pl/ktt/index.html>

time of minimum of Bíró et al. (1998) as being in disagreement with the other photometric data. This datapoint has been rejected from the apsidal motion solution, and the other data were assigned appropriate uncertainties. These data are given in Table 4.4 and Table 4.5 along with the assigned uncertainties and the $O - C$ values. The final solution is plotted against the data in Figure 4.1 and given in Table 4.6, where it is compared to the solution of Wachmann (1974) and to a solution without the inclusion of spectroscopic data.

The time of minimum of Bíró et al. (1998) has been confirmed by reanalysis and another unpublished time of minimum (I. Bíró, 2004, private communication). If they are correct, they may indicate the existence of another effect on the times of minima, for example the light-time effect. Further data are needed to investigate this possibility.

4.4 Spectral synthesis

The work in this section was undertaken by Dr. B. Smalley and is included here for completeness.

The observed spectra were fitted to synthetic spectra, by the method of least squares, to derive the T_{eff} s of the components of V453 Cyg. Synthetic spectra were calculated using UCLSYN (section 1.4.3.2) and rotationally broadened as necessary. Instrumental broadening was applied to match the resolution of the observations.

The primary star was analysed using a spectrum obtained during a total secondary eclipse. For the secondary star we used the spectra at quadrature to measure the lines and corrected them for dilution effects. The equivalent widths of the helium lines (Table 4.7) were used to obtain T_{eff} s by ensuring ionisation balance between He I and He II, for assumed values of surface gravity and microturbulence velocity.

Using the surface gravities found from the spectroscopic and photometric analyses in this work, we find $T_{\text{eff}} = 26\,600 \pm 500$ K for the primary star and $T_{\text{eff}} = 25\,500 \pm 800$ K for the secondary star. These parameters imply that the atmospheres of these stars are helium-rich by about 0.25 dex compared to solar. Further support for these T_{eff} values

is given by the $H\gamma$ 4340 Å profiles; $H\gamma$ profiles with higher T_{eff} and $\log g$ (as found by Daflon et al. 2001) are too broad to fit the observations.

Using the *uvby* β photometry from Hauck & Mermilliod (1998) and the UVBYBETA and TEFFLOGG codes of Moon (1985), we have obtained de-reddened photometry and fits to the grids of Moon & Dworetsky (1984). Values of $T_{\text{eff}} = 26\,710 \pm 800$ K and $\log g = 3.78 \pm 0.07$ were obtained, in excellent agreement with the parameters of the primary star, which produces most of the light of the system, obtained in this work.

Daflon et al. (2001) adopted $T_{\text{eff}} = 29\,100$ K, $\log g = 4.45$ (cm s^{-2}) and $\xi_t = 12 \text{ km s}^{-1}$ in their detailed analysis of V453 Cyg. The above ionisation balance analysis gives a $T_{\text{eff}} = 29\,200$ K for their $\log g$ and ξ_t . While this is in agreement with the T_{eff} they adopted, their value of $\log g$ is not supported by our absolute stellar dimensions, our $H\gamma$ profile fitting and the *uvby* β photometry, so we prefer cooler T_{eff} s for the components of V453 Cyg.

4.5 Spectroscopic orbits

RVs of the two stars were derived from the observed spectra using TODCOR (section 2.2.3.3). Whilst UCLSYN synthetic spectra were used as templates of V615 Per and V618 Per (section 3.6), here the spectra obtained around the midpoint of secondary eclipse can be used. As the eclipses are total, this contains only light from the primary star (and a negligible amount of contaminating light – see section 4.6). This template was used for both stars due to the similarity of their spectral characteristics, and allows the avoidance of possible systematic errors due to the use of theoretical spectra. Best-fitting synthetic spectra were also generated using UCLSYN and independent spectroscopic orbits were obtained to check our observed-template solution and determine the systemic velocities of the stars.

TODCOR was unreliable when the velocity separation of the stars was significantly lower than the combined rotational velocities of the stars. For this reason the best spectra were selected by eye, leaving sixteen covering the wavelength region of 4450–

Table 4.7: Equivalent widths of helium lines in the spectra of V453 Cyg. These are true equivalent widths per individual star, after corrections for dilution due to the spectra being composite.

Species	Wavelength of line ($\text{\AA}$)	Equivalent widths ($\text{\AA}$)	
		primary star	secondary star
He I	4387.93	0.661	0.890
He I	4437.55	0.089	...
He I	4471.50	0.992	1.18
He II	4685.70	0.139	0.057

Table 4.8: RVs and $O-C$ values (in km s^{-1}) for V453 Cyg calculated using TODCOR. Weights are given in column “Wt” and were derived from the amount of light collected in that observation and were used in the SBOP analysis.

HJD – 2 400 000	Primary velocity	$O-C$	Secondary velocity	$O-C$	Wt
52562.5002	–188.9	–0.4	201.3	–9.3	1.0
52564.3199	142.9	–4.0	–218.9	4.2	1.0
52566.2961	–181.4	1.8	200.7	–3.0	0.7
52566.3112	–191.0	–6.7	204.5	–0.7	0.7
52566.4356	–191.5	–1.9	190.6	–21.4	1.2
52568.3471	150.8	–5.2	–227.8	7.0	1.0
52560.3187	137.6	3.5	–206.5	0.1	1.0
52560.3210	136.0	1.5	–211.7	–4.6	1.0
52560.3233	133.0	–1.8	–217.9	–10.5	1.0
52560.3257	133.8	–1.3	–218.2	–10.3	1.0
52560.3280	131.1	–4.3	–211.7	–3.4	1.1
52560.4091	142.5	–2.4	–219.0	1.4	0.8
52560.4114	146.4	1.2	–221.5	–0.7	0.8
52560.4137	149.3	4.0	–225.9	–4.8	0.8
52560.4161	144.7	–0.9	–212.8	8.6	0.8
52560.4184	151.3	5.4	–225.9	–4.2	0.8
52562.4949	–184.9	3.4	219.0	8.6	1.2
52564.3111	147.4	1.3	–217.4	4.7	1.4
52566.3018	–189.0	–5.4	216.5	12.3	1.1
52566.3165	–183.5	1.1	216.5	10.9	1.0
52568.3558	160.0	3.7	–227.5	7.7	1.3
52570.3545	–184.4	5.4	216.5	4.4	1.3

Table 4.9: Parameters of the spectroscopic orbit derived from V453 Cyg using TODCOR only on narrow lines. The systemic velocities were derived using TODCOR and synthetic template spectra.

	Primary	Secondary
Orbital period P (days)	3.889825 (fixed)	
Reference time T_0 (HJD)	39340.6765 (fixed)	
Eccentricity e	0.022 (fixed)	
Periastron longitude ω ($^\circ$)	140.1 (fixed)	
Semi-amplitude K (km s^{-1})	173.7 ± 0.8	224.6 ± 2.0
Systemic velocity (km s^{-1})	-13.1 ± 0.3	-16.2 ± 1.8
Mass ratio q	0.773 ± 0.008	
$a \sin i$ ($R_\odot$)	30.59 ± 0.17	
$M \sin^3 i$ ($M_\odot$)	14.35 ± 0.20	11.10 ± 0.13

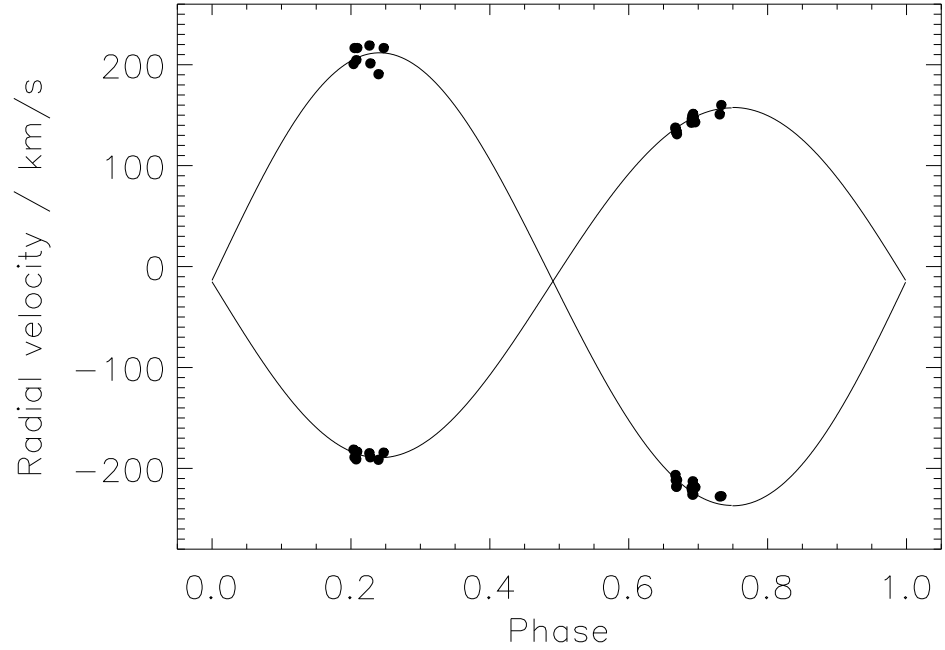


Figure 4.2: Spectroscopic orbit for V453 Cyg from an SBOP fit to RVs from TODCOR.

4715 Å and six spectra covering 4230–4500 Å. The hydrogen and helium lines at 4340 Å, 4471 Å and 4686 Å were masked to avoid significant errors due to the blending of broad spectral lines (Andersen 1975). The resulting RVs are given in Table 4.8 and an orbit was fitted to each star using SBOP (section 2.2.4.1). The orbital period, ephemeris time of reference, eccentricity and ω were fixed at values derived from the apsidal motion analysis. The results of this analysis are given in Table 4.9 and the final spectroscopic orbit is plotted in Figure 4.2.

Our velocity semiamplitudes are slightly larger, although in general consistent with, those found in the recent spectroscopic analyses of Popper & Hill (1991), Simon & Sturm (1994) and BMM97. This effect is probably because our RVs have been derived using only metal lines, whereas previous studies have relied mainly on helium lines. The effect of neglecting orbital eccentricity, however, is negligible, as can be seen from the two orbital solutions of the BMM97 RVs in Table 4.2.

4.6 Light curve analysis

We have analysed the *UBV* light curves taken from the work of Cohen (1974). As discussed in section 4.1, these observations are not definitive, but for this totally eclipsing system they are able to provide accurate values of the individual stellar radii. We have used the JKTEBOP code (section 3.7.1). The calculated oblatenesses of the best-fitting model for V453 Cyg are within the limits of reliability for the EBOP code (Popper & Etzel 1981). Difficulties were experienced with convergence to a best fit during the preliminary light curve solutions, so JKTEBOP was modified to use the Levenberg-Marquardt minimisation algorithm MRQMIN (Press et al. 1992, p. 678).

The light curves were phased with the sidereal period. Passband-specific linear limb darkening coefficients (LDCs) were taken from Van Hamme (1993), gravity darkening exponents β_1 were fixed at 1.0 (Claret 1998) and the mass ratio was fixed at the spectroscopic value. After an initial solution was obtained, datapoints which showed residuals of more than 3σ were rejected. The omission of these observations has not

Table 4.10: Theoretical limb darkening coefficients (LDCs) for stars similar to the components of V453 Cyg, compared to constraints from analysis of the light curves. A subscripted A or B denotes the LDC of the primary star or secondary star respectively. Passband designations are given in brackets.

Reference	$u_A(U)$	$u_B(U)$	$u_A(B)$	$u_B(B)$	$u_A(V)$	$u_B(V)$
Klinglesmith & Sobieski (1970)	0.340	0.321	0.317	0.302	0.268	0.249
Wade & Rucinski (1985)	0.327	0.318	0.292	0.279	0.274	0.248
van Hamme (1993)	0.324	0.296	0.318	0.287	0.260	0.247
Díaz-Cordovés, Claret & Giménez (1995)	0.374	0.357	0.376	0.357	0.334	0.316
Claret (1998)	0.370	0.360	0.365	0.358	0.320	0.309
Claret (2000)	0.423	0.384	0.420	0.381	0.374	0.334
Largest LDCs which fit the light curves well	0.5		0.4		0.35	

Table 4.11: Results of the light curve analysis for V453 Cygni. The adopted values are the weighted means of the values determined from the individual light curves.

	U	B	V	Adopted
Total number of datapoints	538	540	540	1618
Number used in solution	531	532	534	1597
Linear limb darkening coefficient u_A	0.324	0.318	0.260	
Linear limb darkening coefficient u_B	0.296	0.287	0.247	
Primary radius (a)	0.2788 ± 0.0021	0.2800 ± 0.0014	0.2793 ± 0.0014	0.2795 ± 0.0009
Secondary radius (a)	0.1781 ± 0.0039	0.1785 ± 0.0030	0.1811 ± 0.0029	0.1794 ± 0.0018
Ratio of the radii	0.648 ± 0.016	0.637 ± 0.013	0.649 ± 0.012	0.644 ± 0.008
Orbital inclination (degrees)	89.9 ± 1.3	88.2 ± 1.2	89.0 ± 1.1	89.0 ± 0.7
Surface brightness ratio	0.938 ± 0.014	0.953 ± 0.010	0.948 ± 0.015	
Light ratio	0.381 ± 0.022	0.375 ± 0.016	0.384 ± 0.015	
Third light (fraction of total light)	0.079 ± 0.025	0.068 ± 0.024	0.089 ± 0.022	

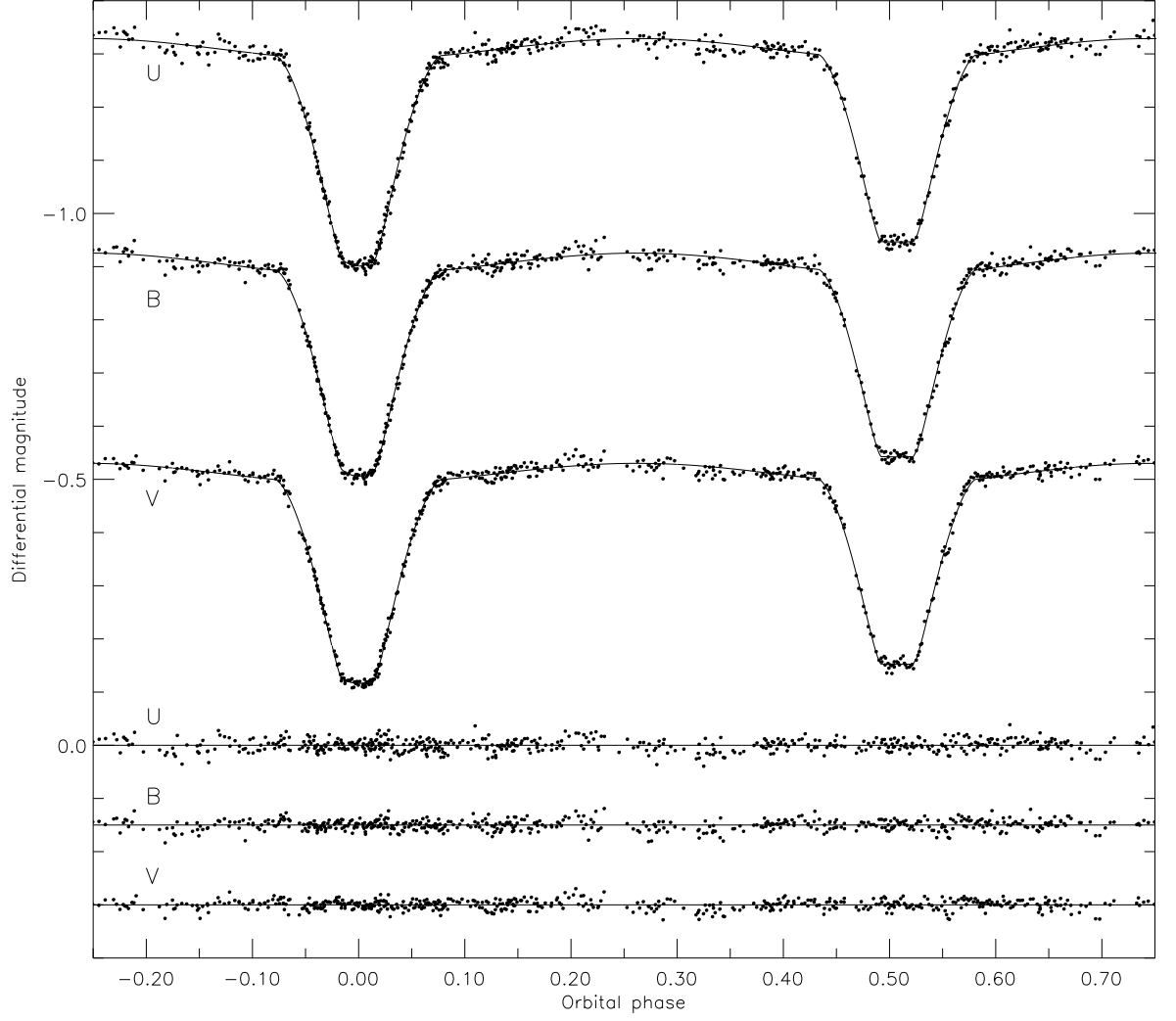


Figure 4.3: Observed phased light curves of V453 Cyg with the best-fitting JKTEBOP model light curves. The lower three curves show the residuals of the JKTEBOP fits. For clarity the B and V residuals are offset by $+0.15$ and $+0.3$ magnitudes, respectively.

affected the derived parameter values but has lowered their uncertainties.

Initial investigation suggested that there is a small amount of third light, L_3 , but acceptable solutions can be found without this effect. However, solutions with $L_3 \neq 0$ fit slightly better than solutions with $L_3 = 0$ for all three light curves, so third light has been included in all final solutions. If third light is neglected, a ratio of the radii lower by about 0.04 is required to reproduce the observed eclipse depths. The effect on the derived stellar radii is an increase in R_A by about 1% and a decrease in R_B by about 5%. These adjustments would bring our photometric solution into agreement with previous light curve analyses, which have all neglected third light and therefore may be systematically wrong.

Table 4.10 shows several theoretical determinations of linear LDCs for stars having similar T_{eff} s and surface gravities to the stars of V453 Cyg. We have evaluated the effect of a change in LDCs on the parameters of the photometric solution. Table 4.10 suggests that there is a variation of about 0.05 between different investigations of LDCs, so we perturbed the van Hamme (1993) values by this amount and refitted the light curves. The resulting errors have been added to the quoted uncertainties in Table 4.11 but are significant only for the surface brightness ratios. We have also determined the upper values of the LDCs for which light curve fits are not notably worse than our best fits, assuming the same LDC for both stars.

The best-fitting light curves are compared to the observations in Figure 4.3. The residuals of the fit are also shown, and some minor systematic trends are noticeable. Whilst the EBOP light curve model is adequate to fit the current photometric data, definitive light curves may require a more sophisticated treatment such as that contained in the Wilson-Devinney code (section 2.4.1.2), which has a better representation of limb darkening and the reflection effect. V453 Cyg is a good system for the determination of observational LDCs due to the long totality of its primary eclipse. The Cohen (1974) light curves are not of sufficient quality to determine LDCs here; definitive light curves will be required.

4.6.1 Error analysis

Whilst the MRQMIN minimisation algorithm in JKTEBOP allows calculation of the formal errors of the adjusted light curve parameters, it is known that these uncertainties are very optimistic when some parameters are significantly correlated (section 2.4.3). Correlations are generally small for systems which exhibit total eclipses, except for systems with third light. Orbital inclination and third light can be strongly anticorrelated as both have a significant dependence on the depth of the eclipses. Robust estimation of uncertainties must include an assessment of parameter correlations for the physical characteristics of the system under investigation.

We have used a Monte Carlo algorithm to evaluate the uncertainties and correlations of the parameters derived from the light curve analysis. After the best fit was determined for each light curve, a synthetic light curve was evaluated at the phases of observation of the real light curve. We added observational noise of the same magnitude as the real light curve and refitted the synthetic light curve. This process was undertaken ten thousand times for each observed light curve.

It is important to understand what information these Monte Carlo simulations actually provide. Once a best fit is found, the distributions of the ten thousand evaluations of various parameters give us the parameter uncertainties and their correlations, based on the best fit, the phases of observation, and the observational scatter of the real light curves. This is a valid method of analysis if the best fits are close to the true characteristics of the dEB. The reality of this assumption can be assessed using independent solutions of different light curves, for example the U , B and V observations here. The Monte Carlo analysis then serves as an indication of the validity of uncertainties estimated from the interagreement of different light curves.

Sample plots of the distributions of different parameter values are shown in Figure 4.4. It is notable that the ratio of the radii and the ratio of the surface brightnesses are not correlated as this system exhibits total eclipses. However the ratio of the radii and third light show a very strong correlation and illustrate why third light has not been included in previous light curve analyses. This effect is because, for a given

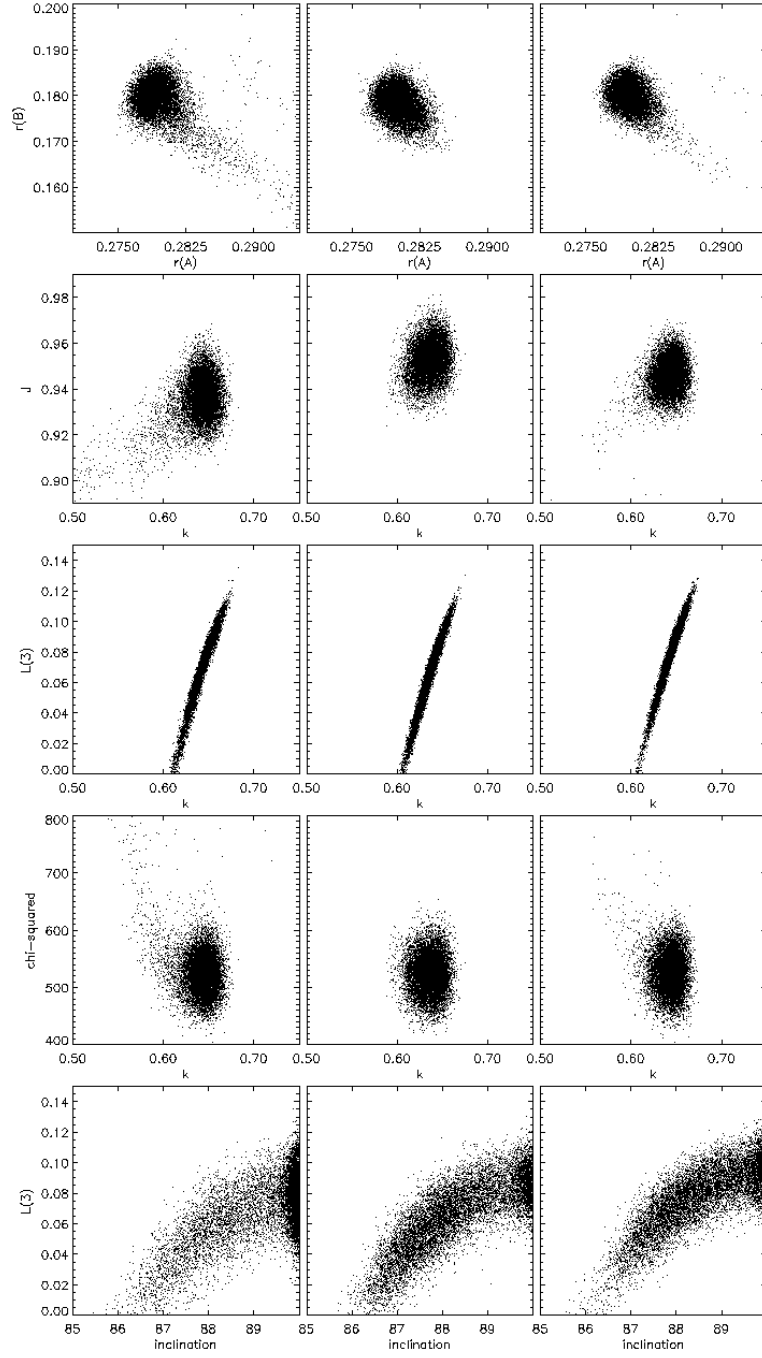


Figure 4.4: Sample distributions of the best-fitting parameters evaluated during the Monte Carlo analysis. The units and parameter symbols are as in Table 4.11. Each distribution between two parameters is shown for the U (left), B (middle) and V (right) light curves.

value of the ratio of the radii, a well-defined value of third light is required to fit the well-determined eclipse depths.

The best-fitting photometric parameters of V453 Cyg, their 1σ uncertainties, and the final adopted parameters are given in Table 4.11. The adopted parameters were determined using weighted means and standard errors of the values determined from the individual light curves; the standard errors are similar to but slightly larger than the standard deviations of the individual values.

4.6.2 Comparison with previous photometric studies

Table 4.12 compares the photometric parameters found in previous studies of V453 Cyg to the results found in this work. The main difference has been caused by our inclusion of third light, which has had a large effect on the derived orbital inclination as well as a significant effect on the radius of the primary star. These two effects are precisely those expected by a change in inclination. Some variations in parameter values will also have been caused by the use of the outdated Russell-Merrill analysis method in three of the literature studies.

4.7 Absolute dimensions and comparison with stellar models

The derived physical parameters for the component stars of V453 Cyg have been collected in Table 4.13; the masses and radii of the two stars have been measured to accuracies of better than 1.4%. The radius of the primary star is extremely well determined because it depends mainly on the duration of totality during secondary eclipse. The rotational velocities of the stars are slightly uncertain: using our spectroscopic data we have been unable to derive values more accurate than those of Olson (1984). The primary star rotates synchronously with the orbital velocity but the secondary rotates somewhat faster (although with a large uncertainty).

Table 4.12: Comparison between some photometric parameters of V453 Cyg from this work and from previous studies.

Photometric parameter	Wachmann (1974)	Cohen (1974)	Cester et al. (1978)	Zakirov (1992)	This work
r_A	0.2923	0.290	0.294	0.302 ± 0.012	0.2795 ± 0.0009
r_B	0.1804	0.178	0.178	0.184 ± 0.009	0.1795 ± 0.0018
k	0.6175	0.61	0.606	0.607 ± 0.009	0.644 ± 0.008
i (degrees)	85.82	86.4	86.1	85.9 ± 0.36	89.0 ± 0.7

Table 4.13: Absolute dimensions of the dEB V453 Cygni in the open cluster NGC 6871.

* Calculated using the combined magnitude and V flux ratio, the assumed cluster distance modulus and reddening, and the reddening law $A_V = 3.1E_{B-V}$.

† Calculated using the T_{eff} –BC calibration of Bessell, Castelli & Plez (1998).

‡ Taken from Olson (1984).

V_{eq} and V_{synch} are the equatorial and synchronous rotational velocities, respectively.

	V453 Cyg A	V453 Cyg B
Cluster age $\log \tau$ (years)	6.3 to 6.7	
Cluster distance modulus	11.65 ± 0.07	
Orbital period (days)	3.889825 ± 0.000017	
Mass ratio q	0.773 ± 0.008	
Mass ($M_{\odot}$)	14.36 ± 0.20	11.11 ± 0.13
Radius ($R_{\odot}$)	8.551 ± 0.055	5.489 ± 0.063
$\log g$ (cm s^{-2})	3.731 ± 0.012	4.005 ± 0.015
Effective temperature (K)	$26\,600 \pm 500$	$25\,500 \pm 800$
M_V^* (mag)	-4.44 ± 0.38	-3.39 ± 0.39
Luminosity ($\log L/L_{\odot}$)	4.69 ± 0.21	4.24 ± 0.28
Distance [†] (pc)	1667 ± 80	
$V_{\text{eq}}^{\ddagger}$ (km s^{-1})	107 ± 9	97 ± 20
V_{synch} (km s^{-1})	111.3 ± 0.7	71.4 ± 0.8
Systemic velocity (km s^{-1})	-13.1 ± 0.3	-16.2 ± 1.8
Apsidal motion period (yr)	66.4 ± 1.8	
$\log k_2$	-2.226 ± 0.024	

The timescales for orbital circularisation and rotational synchronisation (section 1.7.1.1) are all significantly greater than the derived age of V453 Cyg (see below) which is consistent with the presence of orbital eccentricity.

The distance to V453 Cyg has been found to be 1667 ± 80 pc (Table 4.13) using a method involving the bolometric corrections of Bessell, Castelli & Plez (1998) (section 1.6.3.1). This corresponds to a distance modulus of 11.11 ± 0.10 mag.

4.7.1 Stellar model fits

The absolute parameters of the components of V453 Cyg have been compared to the predictions of stellar models from four different groups:– (1) the Granada95 models (section 1.3.2.1), (2) the Padova93 models (section 1.3.2.3; the more recent models of Girardi et al. 2000 only extend to stellar masses of $7 M_{\odot}$), (3) the Geneva92 models (section 1.3.2.2), and (4) the Cambridge2000 models (section 1.3.2.4). For each set of models we have interpolated over age using cubic spline functions and plotted the resulting predictions in the mass–radius and logarithmic T_{eff} –surface gravity diagrams. Comparisons with the properties of V453 Cyg were performed simultaneously in both diagrams and the two stars were assumed to have the same age and chemical composition (as expected for close binary stars). The Granada95, Padova93 and Geneva92 models all include a moderate amount of convective core overshooting (although with different formalisations). Happily, the Cambridge2000 models are available both with and without a moderate amount of overshooting, allowing us to test whether the inclusion of this effect provides a better fit to the observational data.

Panels (a) and (b) of Figure 4.5 show the parameters of V453 Cyg compared to the predictions of the Granada95 models. A good fit is obtained for an age of 9.9 Myr and a chemical composition of $(Z, Y) = (0.02, 0.28)$ (i.e., normal helium abundance). Attempts to fit the stars with a higher or lower helium abundance (Claret 1995) or metal abundances of $Z = 0.01$ (Claret & Giménez 1995) or $Z = 0.03$ (Claret 1997) were unsuccessful.

The predictions of the Padova93 and the Geneva92 models are compared to the

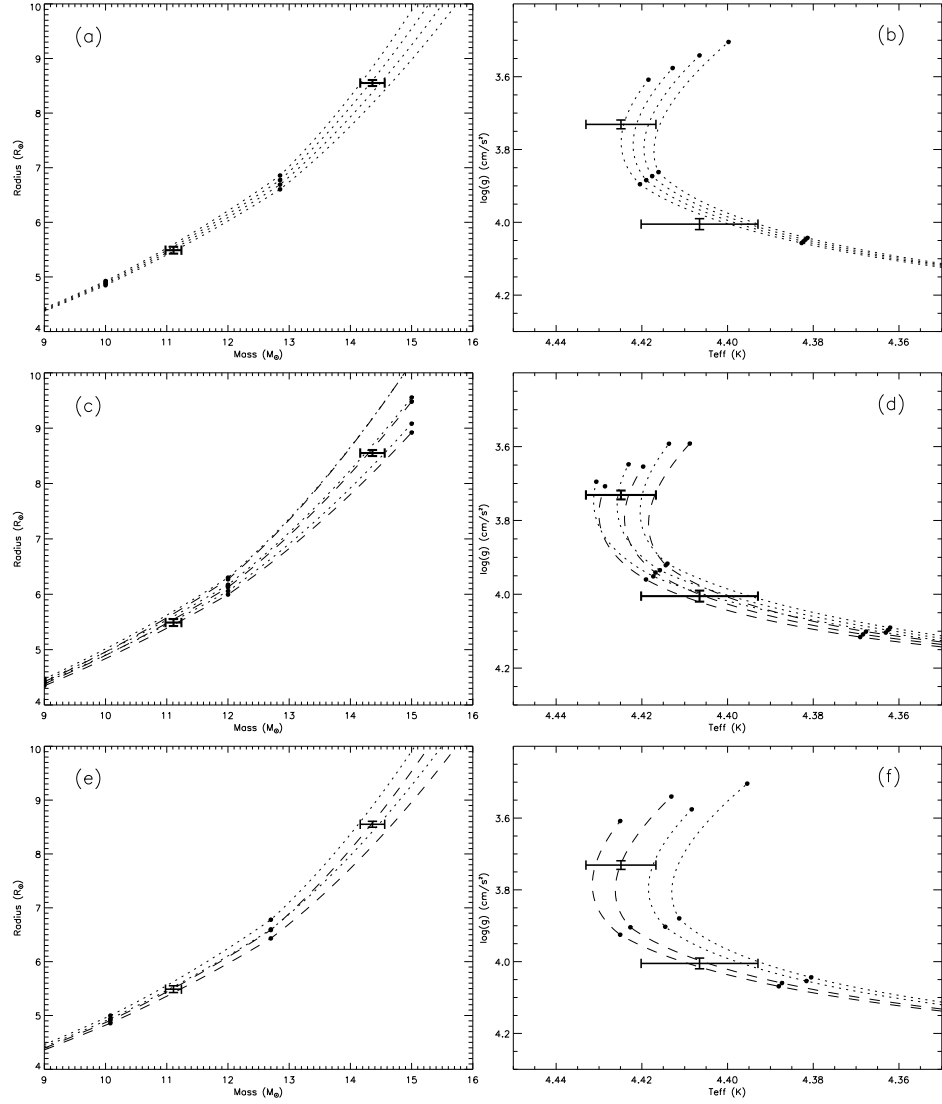


Figure 4.5: Comparison between stellar models and the absolute dimensions of V453 Cyg in the mass–radius and the $T_{\text{eff}}\text{--}\log g$ diagrams. Isochrones have been plotted to represent the stellar models, with circles showing their points of evaluation. Broken lines have been plotted by interpolating over mass using cubic splines. Panels (a) and (b) show the Granada stellar models for $(X,Y) = (0.02,0.28)$ and ages of 9.7, 9.9, 10.1 and 10.3 Myr (radii increase and T_{eff} s decrease as age increases). Panels (c) and (d) show the Padova models for $(X,Y) = (0.02,0.28)$ (dotted lines) and Geneva models for $(X,Y) = (0.02,0.30)$ (dashed lines) for ages of 9.4, 9.8 and 10.2 Myr. Panels (e) and (f) show the Cambridge models for $(X,Y) = (0.02,0.28)$ with overshooting (dashed lines) and without overshooting (dotted lines, for ages of 9.8 and 10.2 Myr (with overshooting) or 9.4 and 9.8 Myr (with no overshooting)).

parameters of V453 Cyg in panels (c) and (d) of Figure 4.5. The two sets of model predictions are plotted for $Z = 0.02$ and the same three ages, so are directly comparable apart from a slight difference in the assumed helium abundance ($Y = 0.28$ for the Padova93 models and $Y = 0.30$ for the Geneva92 models). It is notable that the two sets of models agree very well not only with each other but with the comparable Granada95 models discussed above. Whilst both the Padova93 and Geneva92 models fit the components of V453 Cyg best for an age of 9.8 Myr, a marginally better fit is provided by the slightly higher predicted T_{eff} values of the Padova93 models. Attempts were also made to fit the components of V453 Cyg using the Padova93 and Geneva92 models with larger or smaller metal abundances but no good fit was found.

The Cambridge2000 model set differs from the other model sets considered here in that it is available with and without a moderate amount of convective core overshooting, but does not include any mass loss; this should be unimportant for these stars. Panels (e) and (f) of Figure 4.5 show the parameters of V453 Cyg compared to the predictions of the Cambridge2000 models. Both overshooting and standard-mixing isochrones are plotted for an age of 9.8 Myr for comparison. The overshooting models are also plotted for the best-fitting age of 10.2 Myr and the standard-mixing models are plotted for their best-fitting age of 9.4 Myr. The overshooting models are notably more successful than the standard-mixing models, which predict T_{eff} values which are slightly too low for the stars of V453 Cyg. As with the Padova93 and Geneva92 models, we were unable to perform fits for different helium abundances as such models have not been published.

The above comparisons demonstrate that a good agreement has been reached between different sets of theoretical evolutionary models for stars similar to the components of V453 Cyg. All sets of models were successful in fitting the observations for an age of 10.0 ± 0.2 Myr and solar metal and helium abundances. We also attempted to fit models to the absolute dimensions of V453 Cyg derived with zero third light. This changes the radii to 8.649 and 5.250 $R_{\odot}$ and the surface gravities to $\log g = 3.723$ and 4.045 (cm s^{-2}), with other quantities, and the uncertainties, unaffected. Using the Granada95 models we were able to achieve a fit in the mass-radius plane for low metal abundance ($Z = 0.01$), high helium abundance ($Y = 0.36$) and an age of 8.2 Myr.

However, the T_{eff} values were predicted to be 2000 K greater than observed, and a combination of low metal abundance and high helium abundance does not agree with the predictions of Galactic chemical evolution theory (see e.g., Binney & Merrifield 1998). We were unable to find a simultaneous fit in the mass–radius and T_{eff} – $\log g$ diagrams for the Geneva92, Padova93 or Cambridge2000 models.

BMM97 successfully fitted the Geneva92 theoretical models to the observed masses and luminosities of the components of V453 Cyg. As they did not compare stellar radii, they were not subject to errors from the assumption of no third light. The luminosities of V453 Cyg are also much more uncertain than the radii, so fitting in the $\log T_{\text{eff}} - M_{\text{bol}}$ plane allows a wider range of predictions to fit the observed data.

4.7.2 Comparison between the observed apsidal motion constant and theoretical predictions

The observed value of the internal structure constant $\log k_2$ was calculated as described in section 1.7.2.2 from the apsidal period and the properties of V453 Cyg. Theoretical values for each star were interpolated from the tabulated predictions of the Granada95 models (section 1.3.2.1) and the general relativistic contribution was removed from the observed value (see section 1.7.2.1). The observed and theoretical values are

$$\log \overline{k_2^{\text{obs}}} = -2.254 \pm 0.024$$

$$\log \overline{k_2^{\text{theo}}} = -2.255$$

The agreement with observations is excellent. This agreement is particularly important for assessing the assumed amount of convective core overshooting, on which theoretical values of $\log k_2$ have a significant dependence (Claret & Giménez 1991).

4.8 Membership of the open cluster NGC 6871

V453 Cyg is traditionally considered to be a member of the NGC 6871 open cluster, and appears on the cluster MS in all photometric diagrams. Further proof of membership

comes from its systemic velocity, $-13.2 \pm 0.3 \text{ km s}^{-1}$ using a weighted mean of systemic velocities calculated for each star. This agrees well with the value of $-15 \pm 6 \text{ km s}^{-1}$ quoted by Hron (1987) and the RV of the cluster member NGC 6871 11 which was measured to be $-14.6 \pm 2.7 \text{ km s}^{-1}$ using the same instrumental setup as we used for V453 Cyg. However, the RV of NGC 6871 is given as $-7.7 \pm 3.2 \text{ km s}^{-1}$ by Rastorguev et al. (1999), which differs from our value for V453 Cyg by 1.6σ . The proper motion of V453 Cyg is consistent with cluster membership (Perryman et al. 1997).

Massey et al. (1995) give an age of 2 to 5 Myr for the members of NGC 6871 with the earliest spectral types, but their photometric diagrams contain somewhat evolved $15 M_{\odot}$ stars which are also claimed to be cluster members. This suggests that the stars in NGC 6871 have either a spread in ages or were created by two distinct bursts of star formation. We cannot currently distinguish between the two possibilities; the age of $10.0 \pm 0.2 \text{ Myr}$ derived for V453 Cyg using theoretical models is consistent with both evolutionary scenarios.

The distance modulus found for V453 Cyg, $11.11 \pm 0.10 \text{ mag}$ (section 4.7) is in poor agreement with the distance modulus of $11.65 \pm 0.07 \text{ mag}$ found for NGC 6871 from CCD *UBV* photometry by Massey, Johnson & DeGioia-Eastwood (1995). However, if the bolometric corrections of Code et al. (1976) are used we find a distance modulus of $11.49 \pm 0.14 \text{ mag}$, which is in much better agreement. This confirms that the theoretical bolometric corrections of Bessell, Castelli & Plez (1998) (and those of Girardi et al. 2002) are quite different from the empirically-determined bolometric corrections of Code et al. (1976).

4.9 Summary

We have derived the absolute dimensions of the components of the high-mass dEB V453 Cygni, a member of the open cluster NGC 6871. T_{effs} were found using the helium ionisation balance derived from high-resolution spectra, which also suggest an enhanced photospheric helium abundance relative to solar. RVs were derived from the

spectra using only the weak spectral lines and the TODCOR cross-correlation algorithm.

The apsidal motion rate of the system has been determined using an extended version of the photometric method of Lacy (1992), which includes times of minimum light and spectroscopic determinations of eccentricity and ω . The apsidal motion period is well constrained, and allows the derivation of eccentricity and ω to a greater accuracy than possible with the light curves and RV curves.

We have reanalysed the *UBV* light curves of Cohen (1974) in order to determine the radii of the components of the dEB. The best-fitting parameters include a small amount of third light, which was previously undetected. Robust parameter uncertainties were derived using a Monte Carlo analysis, allowing us to quantify and illustrate the effect of correlations between different photometric parameters. The ratio of the radii and the amount of third light are strongly correlated, due to their dependence on the depths of the eclipses; previous photometric studies which did not include third light are systematically biased towards values of the stellar radii which are 1% higher and 5% lower for primary and secondary respectively.

The accurate absolute dimensions presented here allow V453 Cyg to be added to the list of dEBs with the best-determined values of mass, radius and T_{eff} (Andersen 1991). However, our analysis would clearly be much improved with better observational data. The inclusion of only a few new times of minima would greatly increase the accuracy of the results of the apsidal motion analysis, and more accurate rotational velocities would allow a more accurate derivation of the internal structure constant, $\log k_2$. A definitive spectroscopic orbit will require observations with a higher signal to noise ratio than those presented here, and should give masses determined to accuracies of better than 1%. Definitive light curves of the system would allow determination of the limb darkening coefficients for both stars, providing an important test of model atmosphere codes.

The absolute masses, radii and T_{eff} s of the components of V453 Cyg have been compared to several stellar models in the mass–radius and $\log T_{\text{eff}}$ – $\log g$ planes, assuming the same age for both stars. Not only is there impressive agreement between different theoretical models, but all model sets are able to fit the observational data

for a solar helium and metal abundance. Moreover, the Granada models provide a perfect match to the observed apsidal motion rate once the relativistic contribution has been subtracted from the overall effect. Stellar models have for a long time appeared to predict that the central condensations of stars are lower than those found using observations of apsidal motion (section 1.7.2.3). This apparent discrepancy has been reduced by the discovery that the internal structure constants change significantly through a star's evolution. The current generation of theoretical models, incorporating OPAL opacity data (section 1.3.1.2), are in good agreement with observations. It is noticeable that some observers have not removed the general relativistic effect from their observed $\log k_2$ values before comparison with theory; in many cases this will have a negligible effect but for the stars of V453 Cyg it causes about 6% of the observed apsidal motion, changing $\log k_2$ by an amount similar to its uncertainty.

The normal helium abundance implied by stellar model fits also conflicts with the slight overabundance noted in our spectral synthesis analysis. We note that the photospheric helium abundance is not directly comparable to the initial internal helium abundance used in model calculations.

Fits to the Cambridge stellar models support the inclusion of a moderate amount of overshooting in most stellar evolutionary models. Whilst models without overshooting were able to fit the masses and radii of the stars, the predicted T_{eff} s are slightly lower than that determined from the helium ionisation balance.

The stellar models were extremely successful in fitting the absolute dimensions and T_{eff} s of a high-mass slightly-evolved dEB, with component masses and radii differing by ten and twenty-five times their combined uncertainties, respectively. For observational stellar astrophysicists, this fact implies that we must either observe systems so thoroughly that their masses and radii are known to accuracies of 0.5% and the T_{eff} s to 2%, or target particular types of stars to critique the success of one set of stellar models compared to another. Such targets include low-mass, high-mass, pulsating, and Population II stars, as well as dEBs found in Local Group galaxies. dEBs in open clusters can satisfy this requirement if the cluster they belong to is well-studied or otherwise interesting.

5 V621 Per in the open cluster χ Persei

V621 Per is a very interesting dEB which is a member of the young open cluster χ Persei. This cluster is often thought to be physically related to h Persei (chapter 3.1) so the following work ties in well with the studies of V615 Per and V618 Per presented above. V621 Per itself is interesting more for its potential usefulness, because the evolved nature of the primary component would make its absolute dimensions particularly useful for studying convective core overshooting (section 1.3.1.4), rather than for what we can currently discover. This study is therefore only the first step in a full understanding of V621 Per, but is of fundamental importance to further investigation of this difficult object; this idea will be revisited in the summary at the end of this chapter.

Dr. S. Zucker was involved in the analysis presented in this chapter. Whilst he contributed no text, his input was important in the execution of several analyses and in a consultative capacity.

5.1 V621 Per

V621 Per (Table 5.1) was discovered to be a dEB by Krzesiński & Pigulski (1997, hereafter KP97) from approximately 1200 images, through the broad-band B and V passbands, of the nucleus of χ Persei. The eclipses are total, last for approximately 1.3 days, and are about 0.12 mag deep in both B and V . The ascending and descending branches of one eclipse were observed in BV on two successive nights but the only other observations during eclipse were 102.1 days earlier, and during totality, so the period could not be determined.

The B2 giant component of V621 Per is one of the brightest members of χ Persei and has been studied several times using high-resolution optical spectroscopy to determine accurate chemical abundances. Lennon, Brown & Dufton (1988) derived a normal helium abundance but state that different lines gave different results, which they claim could be due to the high surface gravity used ($\log g = 3.6$). Dufton et al.

Table 5.1: Identifications and photometric indices for V621 Per from various studies. All photometric parameters refer to the combined system light (although the secondary star is much fainter than the primary). Most photometric quantities have been determined many times and the quoted values have been selected as the most representative of all determinations.

* Calculated from the system magnitude in the V passband, the adopted cluster distance modulus and reddening (see section 5.1.1) and the canonical reddening law $A_V = 3.1E_{B-V}$.

References: (1) Argelander (1903); (2) Oosterhoff (1937); (3) Keller et al. (2001); (4) Slesnick et al. (2002); (5) Capilla & Fabregat (2002); (6) Two Micron All Sky Survey (section 1.6.3); (7) Crawford, Glaspey & Perry (1970); (8) Uribe et al. (2002) based on proper motion and position.

	V621 Per	Reference
Bonner Durchmusterung	BD +56°576	1
Oosterhoff number	Oo 2311	2
Keller number	KGM 43	3
Slesnick number	SHM 47	4
α_{2000}	02 22 09.7	5
δ_{2000}	+57 07 02	5
V	9.400	4
$B - V$	0.294	4
$U - B$	-0.505	4
J	8.753 ± 0.021	6
H	8.755 ± 0.026	6
K_s	8.712 ± 0.020	6
$b - y$	0.282	7
m_1	-0.064	7
c_1	0.160	7
β	2.621	7
Spectral type	B2 III	7
M_V	-4.04 ± 0.19	*
Membership probability	0.94	8

(1990) found a normal abundance of helium and various metals, but a deficiency of 0.4 dex in nitrogen and aluminium. These authors may also have been the first to note that V621 Per is a spectroscopic binary.

Vrancken et al. (2000) derived a precise T_{eff} and surface gravity of $T_{\text{eff}} = 22\,500 \pm 500$ K and $\log g = 3.40 \pm 0.05$, based on the silicon ionization balance and direct fitting of the $\text{H}\beta$ and $\text{H}\gamma$ absorption lines using non-LTE model atmosphere calculations. They also derived a high microturbulent velocity of 9 km s^{-1} (or 13 km s^{-1} from the O II lines) consistent with evolution away from the MS. It is notable that abundance analyses generally find larger microturbulence velocities than those usually assumed (see e.g., Dufton, Durrant & Durrant 1981). Vrancken et al. derived abundances of C, N, O, Mg, Al and Si comparable to other bright B stars in χ Persei, but abundances of the overall sample seem to be lower than the Sun by 0.5 ± 0.2 dex. Venn et al. (2002) derived a boron abundance using ultraviolet spectra taken with the STIS spectrograph on board the Hubble Space Telescope. They found a lower microturbulent velocity of 4 km s^{-1} , as usual in the ultraviolet wavelength region, but a macroturbulent velocity of 20 km s^{-1} . They also report a value of $[\frac{\text{M}}{\text{H}}] = -0.16 \pm 0.17$ dex from abundance analyses of the spectral lines of light metals.

V621 Per is a member of the young open cluster χ Persei and is more evolved, and composed of more dissimilar stars, than V453 Cyg. χ Persei is also regarded as a physical relation of h Persei (section 3.1.1) so a full analysis of this system would allow the simultaneous comparison of the observed masses and radii of six stars (the components of V615 Per, V618 Per and V621 Per) with theoretical predictions.

5.1.1 χ Persei

The open clusters χ Persei (NGC 884) and h Persei (NGC 869) together form the Perseus Double Cluster. The co-evolutionary nature of h and χ Persei has been studied many times since the seminal work of Oosterhoff (1937). The results of recent photometric studies (Marco & Bernabeu 2001; Keller et al. 2001; Slesnick, Hillenbrand & Massey 2002, Capilla & Fabregat 2002) seem to be converging to identical values of

distance modulus (11.70 ± 0.05 mag) and age ($\log \tau = 7.10 \pm 0.01$ years), which implies that h and χ Persei are physically related although there do appear to be some differences in stellar content, for example the large number of Be stars in χ Persei. The reddening of χ Persei is $E_{B-V} = 0.56 \pm 0.05$, but h Persei displays differential reddening. These issues were discussed in detail in Chapter 3.

5.2 Observations

Spectroscopic observations were carried out during the same observing run and using the same observational and data reduction techniques as for V615 Per and V618 Per (section 3.2.1). The spectral windows chosen for observation were again 4450–4715 Å (6 spectra) and 4230–4500 Å (24 spectra). An additional spectrum was observed around H β (4861 Å) to provide an additional T_{eff} indicator for spectral analysis. The signal to noise ratios per pixel of the observed spectra are approximately 60. An observing log is given in Table 5.2.

5.3 Spectroscopic orbit

The INT spectra contain identifiable spectral lines from only the primary star. RVs were derived from the spectra by cross-correlation with a synthetic template spectrum, using the XCOR routine in MOLLY (section 3.2.1). Several template spectra were investigated and the resulting RVs were found to be insensitive to the choice of template.

Our spectroscopic observations cover less than the full orbital period of V621 Per and the orbit is eccentric. Our observations cannot provide a unique value of the period, so literature RVs were taken from Liu, Janes & Bania (1989, 1991) and Venn et al. (2002). Additional high-resolution spectra were generously made available by Dr. P. Dufton and Dr. D. Lennon. These were originally observed for an abundance analysis (Dufton et al. 1990; Vrancken et al. 2000) so the wavelength calibrations may

Table 5.2: Observing log for the spectroscopic observations of V621 Per.

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
V621 Per	323111	4450–4710	2452559.42678	300	11/10/02	22:09:26
V621 Per	323112	4450–4710	2452559.43050	300	11/10/02	22:14:48
V621 Per	323113	4450–4710	2452559.43423	300	11/10/02	22:20:10
V621 Per	323327	4450–4710	2452560.45020	300	12/10/02	22:43:06
V621 Per	323328	4450–4710	2452560.45390	300	12/10/02	22:48:26
V621 Per	323329	4450–4710	2452560.45761	300	12/10/02	22:53:46
V621 Per	323565	4450–4710	2452561.54122	300	14/10/02	00:54:06
V621 Per	323741	4450–4710	2452562.49053	300	14/10/02	23:41:03
V621 Per	323774	4450–4710	2452562.61487	300	15/10/02	02:40:06
V621 Per	323885	4450–4710	2452563.43990	300	15/10/02	22:28:05
V621 Per	323942	4450–4710	2452563.71957	300	16/10/02	05:10:48
V621 Per	324048	4450–4710	2452564.39418	300	16/10/02	21:22:12
V621 Per	324085	4450–4710	2452564.59844	300	17/10/02	02:16:20
V621 Per	324271	4450–4710	2452565.38518	300	17/10/02	21:09:11
V621 Per	324306	4450–4710	2452565.53126	300	18/10/02	00:39:32
V621 Per	324493	4450–4710	2452566.37575	300	18/10/02	20:55:33
V621 Per	324513	4450–4710	2452566.59947	280	19/10/02	02:17:42
V621 Per	324518	4450–4710	2452566.68738	300	19/10/02	04:24:17
V621 Per	324628	4450–4710	2452568.66096	300	21/10/02	03:46:08
V621 Per	324795	4450–4710	2452569.41556	300	21/10/02	21:52:44
V621 Per	324840	4450–4710	2452569.55706	300	22/10/02	01:16:29
V621 Per	325172	4450–4710	2452570.48284	300	22/10/02	23:29:34
V621 Per	325218	4450–4710	2452570.57305	300	23/10/02	01:39:28
V621 Per	325267	4450–4710	2452570.68548	300	23/10/02	04:21:22
V621 Per	325394	4710–4970	2452571.46962	300	23/10/02	23:10:29
V621 Per	325397	4450–4710	2452571.47473	300	23/10/02	23:17:51
V621 Per	325472	4450–4710	2452571.68893	300	24/10/02	04:26:17
V621 Per	325666	4450–4710	2452572.54620	500	25/10/02	01:00:43
V621 Per	325686	4450–4710	2452572.59940	500	25/10/02	02:17:19
V621 Per	325735	4450–4710	2452572.73174	500	25/10/02	05:27:53

Table 5.3: RV observations of V621 Per and the $O-C$ values with respect to the final spectroscopic orbit.

References: (1) Liu, Janes & Bania (1989); (2) Liu, Janes & Bania (1991); (3) measured from spectra generously provided by Dr. P. Dufton; (4) Venn et al. (2002); (5) This work.

HJD – 2 400 000	Radial velocity (km s^{-1})	$O-C$ (km s^{-1})	Reference
47439.8623	3.1	–5.1	1
47824.8013	5.4	–0.9	2
49678.742	–124.0	–5.5	3
49682.772	–29.5	–2.0	3
49683.643	–12.5	–0.5	3
49684.665	0.9	1.0	3
51221.71538	1.0	–0.1	4
52559.42678	–89.0	–0.1	5
52559.43050	–88.8	–1.8	5
52559.43423	–89.2	–1.6	5
52560.45020	–97.8	2.0	5
52560.45390	–97.4	2.3	5
52560.45761	–96.9	2.8	5
52561.54122	–112.4	–0.7	5
52562.49053	–118.4	0.6	5
52562.61487	–119.9	–0.3	5
52563.43990	–120.5	–0.8	5
52563.71957	–120.0	–1.9	5
52564.39418	–110.5	–0.4	5
52564.59844	–106.1	0.4	5
52565.38518	–87.5	1.1	5
52565.53126	–83.0	1.7	5
52566.37575	–62.0	–1.2	5
52566.59947	–54.3	0.1	5
52566.68738	–52.0	0.1	5
52568.66096	–11.0	–0.5	5
52569.41556	–3.3	–1.5	5
52569.55706	–1.9	–1.4	5
52570.48284	6.2	0.7	5
52570.57305	6.5	0.6	5
52570.68548	8.3	1.9	5
52571.47473	9.4	1.1	5
52571.68893	8.9	0.5	5
52572.54620	7.1	–1.0	5
52572.59940	7.8	–0.2	5
52572.67052	8.0	0.1	5
52572.73174	7.4	–0.4	5

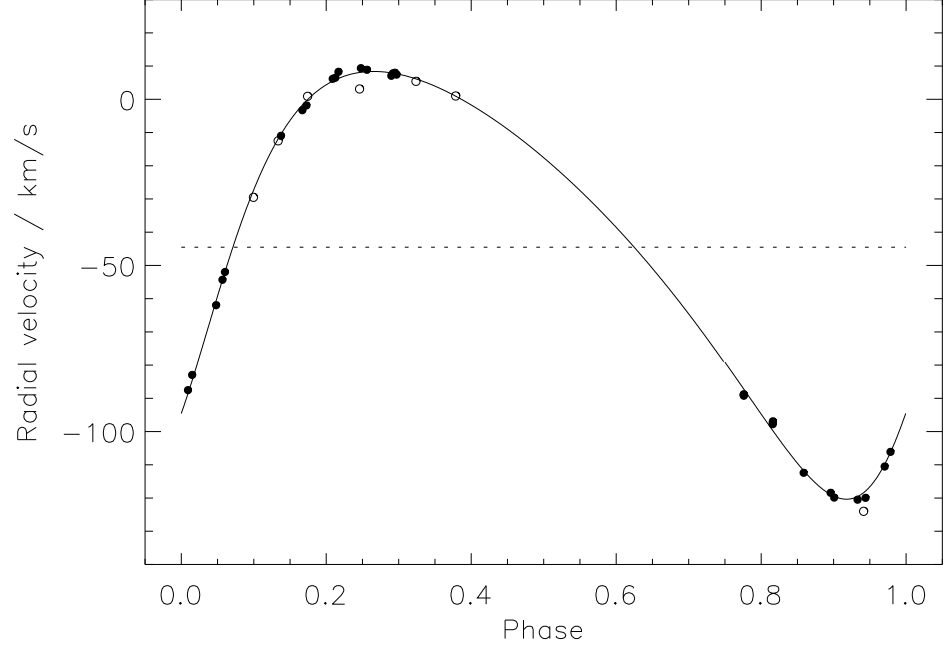


Figure 5.1: Spectroscopic orbit for V621 Per. Filled circles denote RVs derived from the INT spectra and open circles show RVs obtained from other sources. The systemic velocity is indicated by a dotted line.

Table 5.4: Parameters of the spectroscopic orbit derived for V621 Per.

Orbital period (days)	P	25.53018 ± 0.00020
Reference time (HJD)	T_{peri}	$2\,452\,565.150 \pm 0.097$
Eccentricity	e	0.2964 ± 0.0057
Periastron longitude ($^{\circ}$)	ω	233.2 ± 2.0
Semi-amplitude (km s^{-1})	K	64.46 ± 0.40
Systemic velocity (km s^{-1})	V_{γ}	-44.53 ± 0.46
Mass function ($M_{\odot}$)	$f(M)$	0.617 ± 0.012

only be accurate to a few km s^{-1} (P. Dufton, 2004, private communication). RVs were derived by fitting Gaussian functions to strong lines, predominantly from He I and O II, and excellent agreement was found between different lines in the same spectra.

Using the photometric constraint that the orbital period of V621 Per must be a submultiple of 102.1 days (KP97), the possible periods were investigated by analysing the RVs with SBOP (section 2.2.4.1) and the KP97 light curves with JKTEBOP (section 3.7.1). Only a period around 25.5 days can provide a good fit to all the data. The photometric and spectroscopic data were fitted simultaneously by requiring the spectroscopically-derived orbital period to correctly predict the phase at which the primary eclipse occurs. The midpoints of the primary and secondary eclipses occur at phases 0.67 and 0.06, respectively.

The final spectroscopic orbit was calculated using the RVs derived from the INT spectra and fixing the orbital period at the value given above. The orbit is plotted in Figure 5.1, the RVs and $O-C$ values are given in Table 5.3 and the parameters of the orbit are given in Table 5.4.

The projected rotational velocity of the primary component of V621 Per was found by fitting Gaussian functions to the Si III 4575 Å spectral line singlet. Using the orbital inclination found in Section 5.5, the total line broadening corresponds to an equatorial rotational velocity of $V_{\text{eq}} = 32.2 \pm 1.2 \text{ km s}^{-1}$ where the quoted error is the 1σ error of the individual values.

The INT spectra are single-lined in character, and in experiments with the two-dimensional cross-correlation algorithm TODCOR (section 2.2.3.3) and with spectral disentangling (section 2.2.3.4) we were unable to detect any signal from the secondary star. By simulating the spectrum of the secondary star with a rotationally broadened primary spectrum we have constructed several trial composite spectra. From analysis of these using cross-correlation, we estimate that we would have detected secondary spectral lines if it contributed more than 5% of the total light, for a rotational velocity of 50 km s^{-1} . If it rotates faster than this, or has a spectrum very different to that of the primary star, then the detection threshold will increase.

5.4 Determination of effective temperature and surface gravity

5.4.1 Temperatures and surface gravities in the literature

Lennon, Brown & Dufton (1988) found the atmospheric parameters of V621 Per to be $T_{\text{eff}} = 21\,500\text{ K}$ and $\log g = 3.6$, from several different Strömgren photometric calibrations and fitting Balmer lines with synthetic profiles. Dufton et al. (1990) found $T_{\text{eff}} = 21\,700\text{ K}$ and $\log g = 3.6$ using a similar method, but found $T_{\text{eff}} = 23\,000\text{ K}$ from the silicon ionization equilibrium, for which the corresponding $\log g$ is 3.7.

Vrancken et al. (2000) derived $T_{\text{eff}} = 22\,500 \pm 500\text{ K}$ and $\log g = 3.40 \pm 0.05$ from the silicon ionization equilibrium and fitting Balmer lines with synthetic profiles. These atmospheric parameters were adopted by Venn et al. (2002).

5.4.2 Effective temperature and surface gravity for V621 Per

The work in this section was undertaken by Dr. B. Smalley and is included here for completeness.

Using Strömgren $uvby\beta$ data taken from Crawford, Glaspey & Perry (1970), and the calibration of Moon & Dworetzky (1985), we derive $T_{\text{eff}} = 21\,700 \pm 800\text{ K}$ and $\log g = 3.69 \pm 0.07$ (where the uncertainty is a formal error of the fit). The Strömgren photometry of Marco & Bernabeu (2001) gives $T_{\text{eff}} = 19\,300 \pm 800\text{ K}$ and $\log g = 3.56 \pm 0.07$, and of Capilla & Fabregat gives $T_{\text{eff}} = 20\,900 \pm 800\text{ K}$ and $\log g = 3.36 \pm 0.07$. The Crawford et al. (1970) photometry should be preferred as the filters are closest to the filters used to define the Strömgren $uvby$ and Crawford β systems, and because photoelectric $uvby\beta$ photometry has been shown to be superior to CCD $uvby\beta$ photometry (see e.g., Mermilliod & Paunzen 2003).

Using Geneva photometry from Rufener (1976) and the calibration of Kunzli et al. (1997) we find $T_{\text{eff}} = 22\,230 \pm 250\text{ K}$ and a high $\log g$ value of 3.97 ± 0.18 . Use of the Geneva photometry of Waelkens et al. (1990) gives $T_{\text{eff}} = 23\,200 \pm 420\text{ K}$ and a low

$\log g$ value of 3.31 ± 0.26 .

We have assumed a surface gravity value of $\log g = 3.6$, which agrees well with Dufton et al. (1990), the calibration results using the Crawford et al. (1970) data, and with our spectroscopic and photometric analyses (see section 5.6), and fitted the $H\gamma$ and $H\beta$ spectra with synthetic profiles calculated using UCLSYN (section 1.4.3.2). The spectra were rotationally broadened as necessary and instrumental broadening was applied to match the resolution of the observations. For $\log g = 3.60$ we find $T_{\text{eff}} = 22\,500 \pm 500$ K, in agreement with Vrancken et al. (2000).

5.5 Light curve analysis

We have analysed the BV light curves of KP97 using JKTEBOP (section 3.7.1). The orbital eccentricity and longitude of periastron were fixed at the spectroscopic values, initial passband-specific linear limb darkening coefficients of 0.30 (primary star) and 0.25 (secondary star) were taken from van Hamme (1993) and the gravity darkening exponents β_1 were fixed at 1.0 (Claret 1998). Changes in the limb darkening and gravity brightening values for the secondary star have a negligible effect on the photometric solutions because this star contributes very little of the light of the system. Third light was fixed at zero, as solutions in which it was a free parameter were not significantly different from solutions with third light fixed at zero.

Initial light curve solutions converged to an orbital inclination, i , of 90° , but values of i from about 88 to 90° fit the observations almost equally well. Solutions in which the surface brightness ratio, J , was freely adjusted towards the best fit generally converged to a value of J below zero, which is unphysical. We therefore present separate solutions (Table 5.5) for the B and V light curves in which orbital inclination is fixed at $i = 88, 89$ and 90° and the surface brightness ratio is fixed at $J = 0.0, 0.5$ and 1.0 . The light curves are of insufficient quality to solve for the limb darkening coefficients so these have been fixed during solution.

Robust errors were estimated using Monto Carlo simulations (section 4.6.1). This

Table 5.5: Results of the light curve analysis of V621 Per for several different (fixed) values of the surface brightness ratio and orbital inclination. The final entry gives the adopted values and uncertainties of the parameters (see text for discussion); the uncertainties are confidence intervals, not 1σ errors.

Light curve	Inclination i ($^\circ$)	Surface brightness ratio J	Primary radius (a)	Secondary radius (a)	Light ratio	σ (mmag) (one observation)
B	90.0	0.0	0.09754 ± 0.00038	0.02944 ± 0.00008	0.0	4.82
B	90.0	0.5	0.09753 ± 0.00039	0.03019 ± 0.00009	0.0477 ± 0.0006	4.76
B	90.0	1.0	0.09751 ± 0.00039	0.03098 ± 0.00009	0.1005 ± 0.0014	4.71
B	89.0	0.0	0.09951 ± 0.00037	0.03020 ± 0.00008	0.0	4.79
B	89.0	0.5	0.09950 ± 0.00038	0.03096 ± 0.00009	0.0482 ± 0.0006	4.75
B	89.0	1.0	0.09948 ± 0.00038	0.03178 ± 0.00009	$0.1016 \pm 0.$	4.71
B	88.0	0.0	0.10517 ± 0.00036	0.03236 ± 0.00008	0.0	4.91
B	88.0	0.5	0.10515 ± 0.00037	0.03320 ± 0.00009	0.0496 ± 0.0007	4.90
B	88.0	1.0	0.10514 ± 0.00037	0.03411 ± 0.00010	0.1046 ± 0.0016	4.91
V	90.0	0.0	0.09792 ± 0.00032	0.02984 ± 0.00007	0.0	4.83
V	90.0	0.5	0.09791 ± 0.00032	0.03062 ± 0.00007	0.0487 ± 0.0007	4.78
V	90.0	1.0	0.09790 ± 0.00216	0.03147 ± 0.00205	0.1028 ± 0.2972	4.74
V	89.0	0.0	0.09995 ± 0.00033	0.03066 ± 0.00006	0.0	4.73
V	89.0	0.5	0.09991 ± 0.00112	0.03147 ± 0.00283	0.0494 ± 0.0994	4.71
V	89.0	1.0	0.09992 ± 0.00158	0.03236 ± 0.00147	0.1044 ± 0.2070	4.71
V	88.0	0.0	0.10580 ± 0.00031	0.03307 ± 0.00007	0.0	4.83
V	88.0	0.5	0.10579 ± 0.00122	0.03398 ± 0.00281	0.0513 ± 0.0960	4.91
V	88.0	1.0	0.10579 ± 0.00032	0.03498 ± 0.00009	0.1087 ± 0.0017	5.02
Adopted	89.0 ± 1.0	0.25 ± 0.25	0.1016 ± 0.00039	0.0316 ± 0.0020		

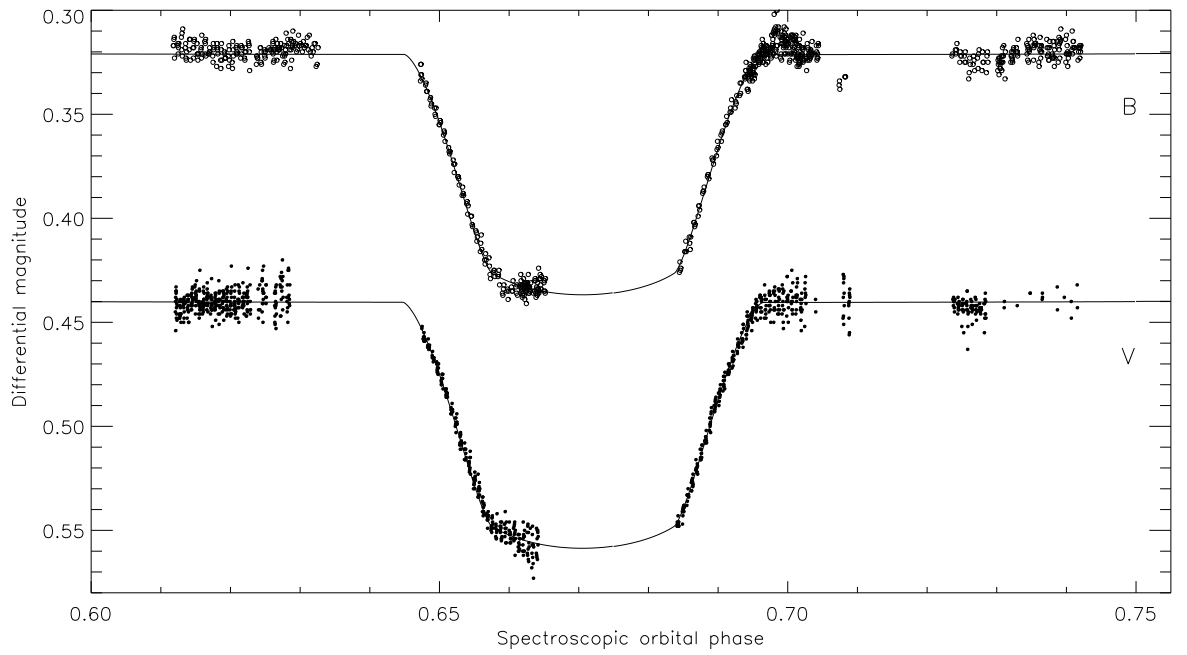


Figure 5.2: The KP97 B and V light curves of V621 Per around the primary eclipse, phased using the spectroscopic ephemeris, with the best-fitting JKTEBOP model light curves. The B light curve, offset by -0.1 mag for clarity, is shown using open circles and the V light curve is shown using filled circles. The best-fitting curves were generated with $J = 0.25$ and $i = 89.0^\circ$.

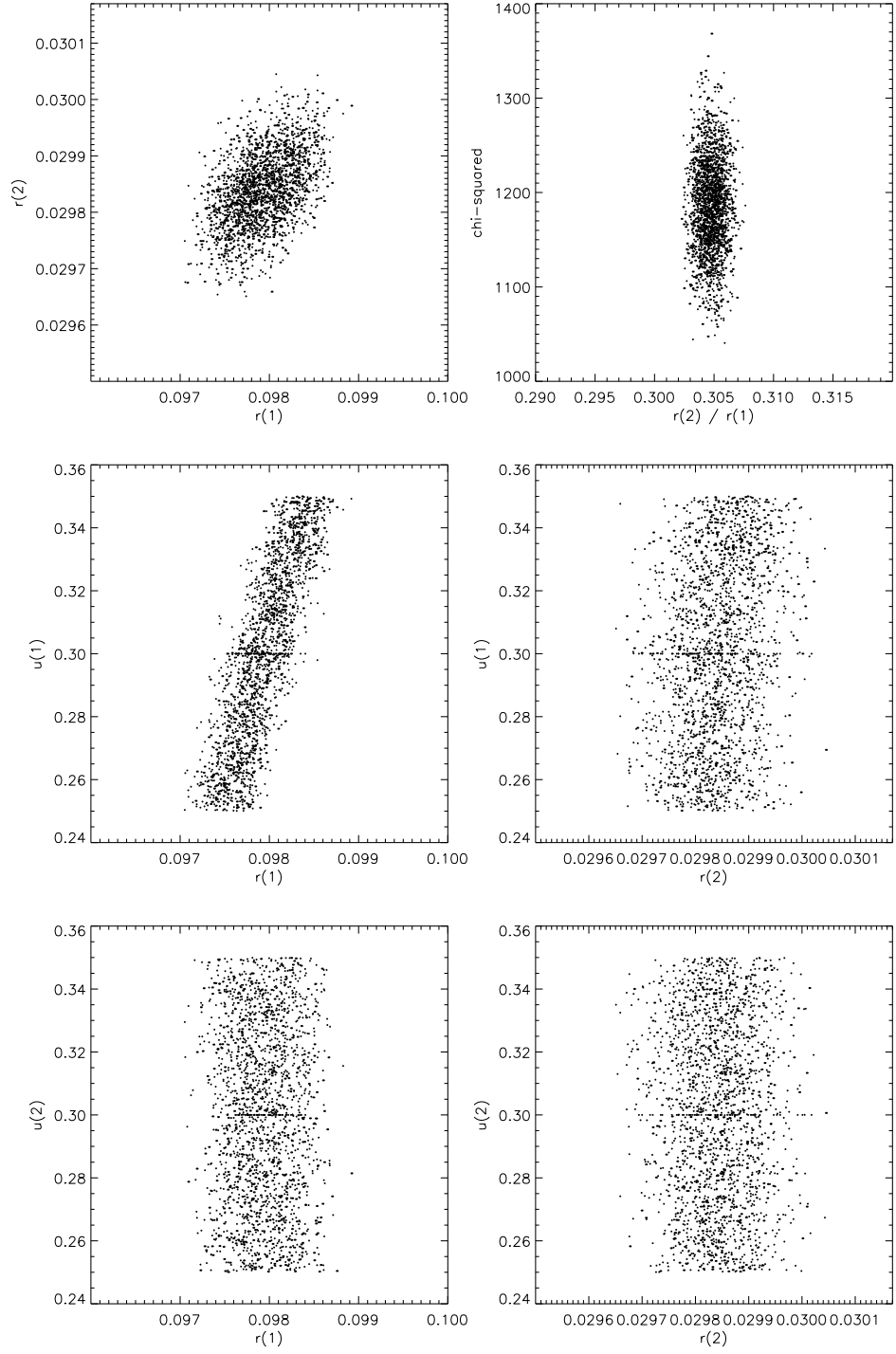


Figure 5.3: Results of the Monte Carlo analysis for $J = 0.0$ and $i = 90.0^\circ$. The limb darkening coefficients, u_1 and u_2 , were chosen randomly on a flat distribution between $u - 0.05$ and $u + 0.05$ for each synthetic light curve, and fixed during solution of the light curve.

method was modified to explicitly include uncertainties due to the use of assumed limb darkening coefficients by fixing them at random values on a flat distribution within ± 0.05 of the original value.

As no trace of the secondary star was found in the observed spectra, the B passband light ratio must be 0.05 or less. The maximum light ratio in the V passband will be slightly greater than this as the secondary star is expected to have a lower T_{eff} than the primary star. For simplicity, we have adopted a maximum light ratio of 0.05 for both the B and V light curves. We have therefore calculated best-estimate parameters by evaluating the ranges of possible parameter values in the two light curves and then averaging the midpoints of the ranges for the two light curves (Table 5.5). The quoted uncertainties are confidence intervals which encompass the range of possible values for each parameter, so are not 1σ errors. This procedure is simple but is quite adequate considering the nature of the observations analysed here.

Figure 5.2 shows the observed light curves and the best-fitting models with $J = 0.25$ and $i = 89^\circ$. Figure 5.3 represents the relation between different parameters of the fit to the V light curve. As V621 Per exhibits total eclipses, the radii of the two stars are only weakly correlated. Changes in the limb darkening coefficients used do affect the derived radii of both stars, but this effect is quite small and easily quantified.

5.6 Absolute dimensions and comparison with stellar models

Although the absolute masses and radii of the component stars of V621 Per cannot be found directly, the mass function and fractional radii (the stellar radii expressed as a fraction of the semi-major axis of the orbit) are accurately known. This allows us to empirically determine the surface gravity of the secondary star despite not knowing its actual mass or radius.

The mass function of a spectroscopic binary is given by

$$f(M) = \frac{K_1^3 P}{2\pi G} = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} \quad (5.1)$$

where K_1 is the velocity semiamplitude of the primary star, P is the orbital period, G is the gravitational constant, i is the orbital inclination and M_1 and M_2 are the masses of the primary and secondary stars. Kepler's third law is

$$P^2 = \frac{4\pi^2 a^3}{G(M_1 + M_2)} \quad (5.2)$$

where a is the semimajor axis. Rearranging and combining these two equations gives

$$M_1 + M_2 = \frac{4\pi^2 a^3}{GP^2} = \frac{(M_2 \sin i)^{3/2}}{f(M)^{1/2}} \quad (5.3)$$

The definitions of surface gravity, g , and fractional radius, r , can be combined to give

$$a^2 = \frac{GM}{gr^2} \quad (5.4)$$

Rearrangement of the last two equations gives

$$a^3 = \left(\frac{P}{2\pi}\right)^2 G \frac{(M_2 \sin i)^{3/2}}{f(M)^{1/2}} = \left(\frac{GM}{gr^2}\right)^{3/2} \quad (5.5)$$

so

$$g = \left(\frac{2\pi}{P}\right)^{4/3} \frac{[Gf(M)]^{1/3}}{r^2 \sin i} \left(\frac{M}{M_2}\right) \quad (5.6)$$

Therefore the surface gravities of the primary and secondary stars are given by

$$g_1 = \left(\frac{P}{2\pi}\right)^{4/3} \frac{[Gf(M)]^{1/3}}{r_1^2 \sin i} \left(\frac{M}{M_2}\right) \quad (5.7)$$

$$g_2 = \left(\frac{P}{2\pi}\right)^{4/3} \frac{[Gf(M)]^{1/3}}{r_2^2 \sin i} \quad (5.8)$$

where q is the mass ratio of the binary. Taking the logarithm of both sides and expressing all quantities in the usual astrophysical units (g in cm s^{-2} ; $f(M)$, M_1 and M_2 in $M_\odot$; P in days) gives

$$\log g_2 = 3.18987 + \frac{\log f(M)}{3} - \frac{4 \log P}{3} - \log(r_2^2 \sin i) \quad (5.9)$$

$$\log g_1 = 3.18987 + \frac{\log f(M)}{3} - \frac{4 \log P}{3} - \log(r_1^2 \sin i) - \log q \quad (5.10)$$

where $q = \frac{M_2}{M_1}$ is the mass ratio of the binary system.

Equation 5.9 contains only known quantities so, despite not knowing the mass *or* radius of the secondary component of V621 Per we can empirically calculate its surface gravity to be $\log g_2 = 4.244 \pm 0.054$. We cannot calculate the surface gravity of the primary star because we do not know the mass ratio accurately. Although we have found the surface gravity from spectral analysis the result is too uncertain to be useful in calculating the mass ratio.

Alternatively, it is possible to use V621 Per's membership of the open cluster χ Persei to infer the properties of the primary star. The absolute magnitudes of the V621 Per system, found from the apparent magnitudes, the distance modulus and reddening of the cluster (Table 5.1 and section 5.1.1) and the reddening laws $A_V = 3.1E_{B-V}$ and $A_K = 0.38E_{B-V}$ (Moro & Munari 2000), are $M_V = -4.04 \pm 0.16$ and $M_K = -3.20 \pm 0.06$. Adopting bolometric corrections of -2.20 ± 0.05 and -2.93 ± 0.08 (Bessell, Castelli & Plez 1998) gives absolute bolometric magnitudes of -6.24 ± 0.17 and -6.13 ± 0.10 for the V and K passband data respectively. The two values are in good agreement but the K passband value is more accurate because it is less affected by the uncertainty in E_{B-V} . We will adopt the K passband value as it is more accurate, and the 2MASS apparent magnitudes are known to be very reliable. Adopting a solar absolute bolometric magnitude of 4.74 (Bessell, Castelli & Plez 1998), which is consistent with the adopted bolometric corrections, gives a luminosity of $\log \frac{L}{L_\odot} = 4.348 \pm 0.039$. This gives a radius of $9.9 \pm 0.7 R_\odot$ for the primary star. If we assume that the secondary star's contribution to the total light of the system is 5%, this will cause the primary radius to be overestimated by about $0.25 R_\odot$, which is negligible at this level of accuracy.

Table 5.6: Absolute masses and radii of the components of V621 Per calculated using different values of $\log g_1$. The luminosity is that of the primary component only as we do not know the T_{eff} of the secondary star.

$\log g_1$ (cm s^{-2})	Mass ratio	Primary mass ($M_\odot$)	Secondary mass ($M_\odot$)	Primary radius ($R_\odot$)	Secondary radius ($R_\odot$)	Luminosity ($\log L/L_\odot$)
3.20	1.071 ± 0.078	2.2 ± 0.3	2.3 ± 0.2	6.1 ± 0.5	1.9 ± 0.1	3.936 ± 0.051
3.25	0.955 ± 0.069	2.7 ± 0.4	2.6 ± 0.2	6.5 ± 0.5	2.0 ± 0.1	3.986 ± 0.053
3.30	0.851 ± 0.062	3.4 ± 0.6	2.9 ± 0.3	6.9 ± 0.6	2.1 ± 0.2	4.038 ± 0.054
3.35	0.758 ± 0.055	4.4 ± 0.8	3.3 ± 0.3	7.3 ± 0.6	2.3 ± 0.2	4.094 ± 0.055
3.40	0.676 ± 0.049	5.6 ± 1.0	3.8 ± 0.4	7.8 ± 0.7	2.4 ± 0.2	4.152 ± 0.057
3.45	0.602 ± 0.044	7.3 ± 1.3	4.4 ± 0.4	8.4 ± 0.7	2.6 ± 0.2	4.213 ± 0.058
3.50	0.537 ± 0.039	9.4 ± 1.8	5.1 ± 0.5	9.0 ± 0.8	2.8 ± 0.2	4.277 ± 0.059
3.55	0.478 ± 0.035	12.3 ± 2.4	5.9 ± 0.6	9.8 ± 0.9	3.0 ± 0.2	4.343 ± 0.061
3.60	0.426 ± 0.031	16.2 ± 3.2	6.9 ± 0.8	10.6 ± 1.0	3.3 ± 0.3	4.412 ± 0.062
3.65	0.380 ± 0.028	21.4 ± 4.3	8.1 ± 0.9	11.5 ± 1.1	3.6 ± 0.3	4.483 ± 0.063
3.70	0.339 ± 0.025	28.5 ± 5.9	9.6 ± 1.2	12.5 ± 1.2	3.9 ± 0.3	4.557 ± 0.064
3.75	0.302 ± 0.022	38.0 ± 8.0	11.5 ± 1.4	13.6 ± 1.4	4.2 ± 0.4	4.633 ± 0.065
3.80	0.269 ± 0.019	51.1 ± 10.9	13.7 ± 1.7	14.9 ± 1.5	4.6 ± 0.4	4.711 ± 0.066
3.85	0.240 ± 0.017	68.8 ± 15.0	16.5 ± 2.1	16.3 ± 1.7	5.1 ± 0.5	4.790 ± 0.067
3.90	0.214 ± 0.015	93.2 ± 20.6	19.9 ± 2.6	17.9 ± 1.9	5.6 ± 0.5	4.872 ± 0.068

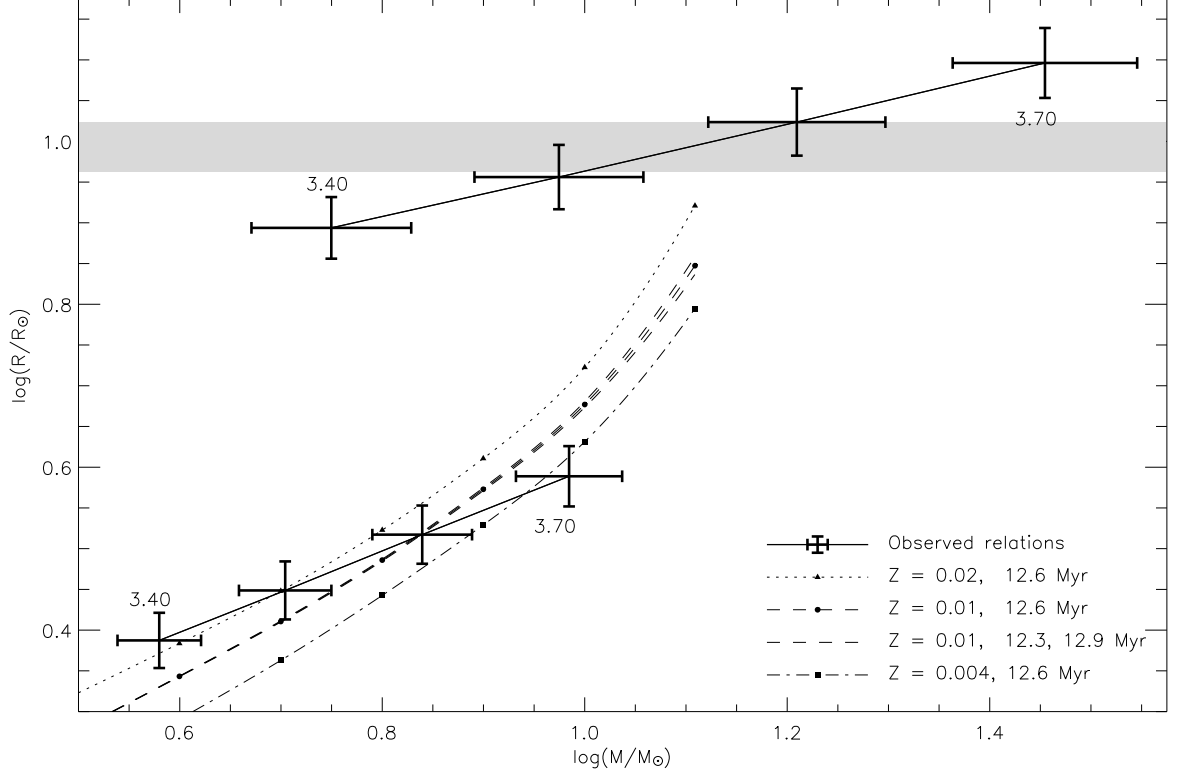


Figure 5.4: The logarithmic mass–radius plot for the two components of V621 Per. The possible combinations of mass and radius for each star, for different values of the primary surface gravity, $\log g_1$, are plotted with errorbars connected by solid lines. Numbers on the diagram indicate the value of $\log g_1$ used to calculate the adjacent datapoint. The Granada stellar model predictions are plotted for an age of 12.6 Myr for $Z = 0.004$ (dash-dotted line), $Z = 0.01$ (dashed line) and $Z = 0.02$ (dotted line). The 12.3 and 12.9 Myr $Z = 0.01$ predictions are plotted using a dashed line with omission of the symbols representing the points of model evaluation. Curves have been calculated using a cubic spline interpolation. The radius of the primary star, calculated from its known distance and apparent magnitude, has been shown using light shading to indicate the range of possible values.

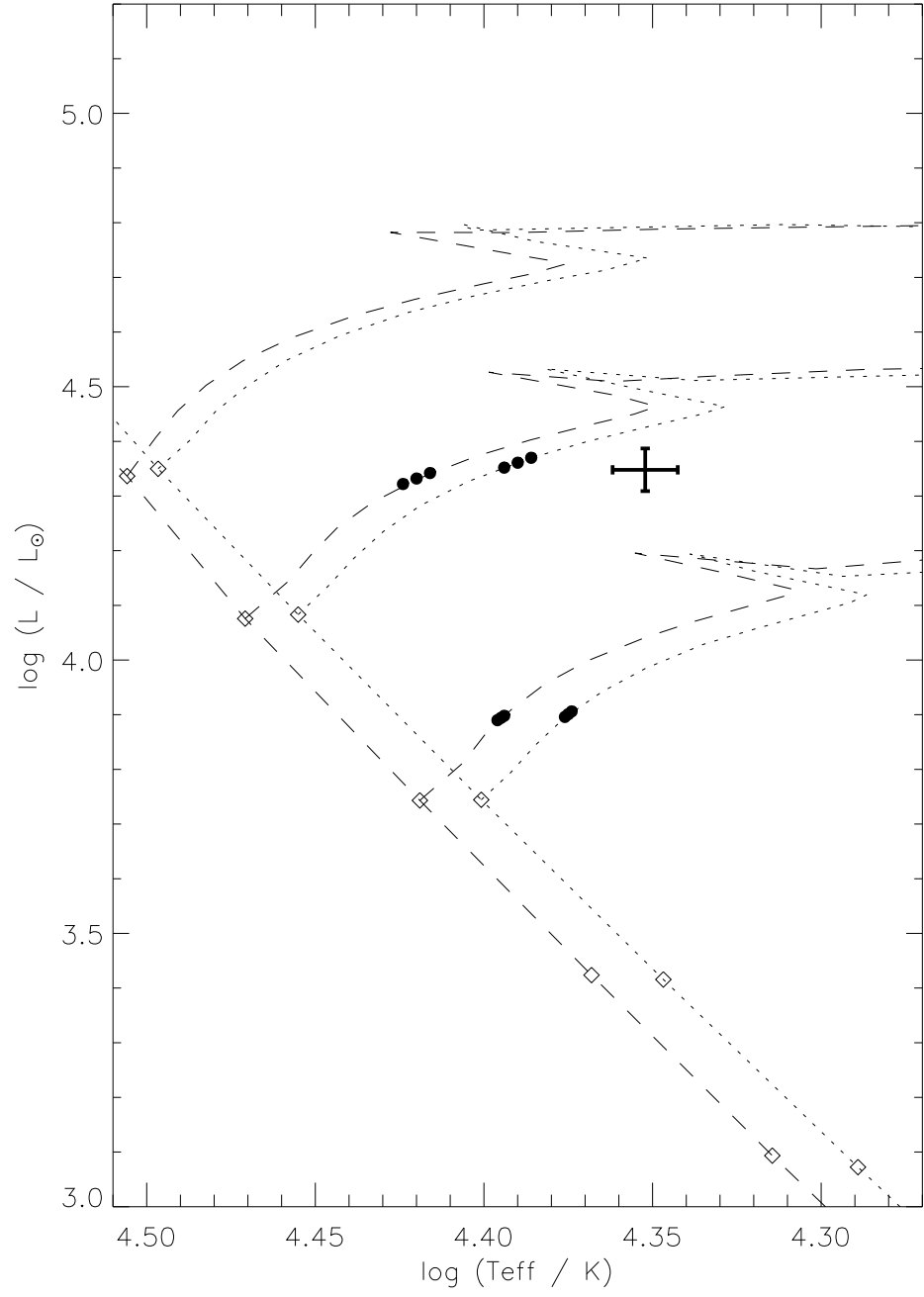


Figure 5.5: HR diagram showing the luminosity and T_{eff} derived for V621 Per. The Granada evolutionary model predictions are plotted for masses of 10.0, 12.8 and 15.8 $M_{\odot}$, and for metal abundances of $Z = 0.01$ (dashed lines) and 0.02 (dotted lines). The ZAMSs are plotted using the same line styles and filled circles denote predictions for ages of 12.3, 12.6 and 12.9 Myr.

5.6.1 Comparison with stellar models

The mass ratio of V621 Per can in principle be found from the equations in section 5.6, but it has a very sensitive dependence on the value of $\log g_1$. An alternative approach is to evaluate the mass ratio, and hence the absolute masses and radii of the two stars, for several different values of $\log g_1$ and compare the possibilities with the predictions of stellar models. Substituting the mass ratio into the definition of the mass function, we can derive the absolute masses of the two stars using

$$M_1 = \frac{f(M)}{\sin^3 i} \frac{(1+q)^2}{q^3} \quad (5.11)$$

$$M_2 = qM_1 \quad (5.12)$$

The absolute stellar radii can then be found from the masses and surface gravities.

We have used the equations above to determine the absolute masses and radii of both components of V621 Per for several assumed values of $\log g_1$ between 3.40 and 3.70. These have been compared to the predictions of the Granada stellar evolutionary models (section 1.3.2.1) for ages close to the age of the χ Persei open cluster, 12.6 ± 0.3 Myr (section 3.1.1).

Figure 5.4 shows possible values of the absolute masses and radii of the components of V621 Per compared to predictions of the Granada evolutionary models for ages around 12.6 Myr. Also shown (by a shaded area) is the range of possible values of the primary radius, derived from the known distance and apparent magnitude of the dEB. Whilst all three possible metal abundances can fit the two stars for $\log g_1 \sim 3.55$, the $Z = 0.01$ predictions provide the best fit to the predicted properties of the secondary component. This diagram suggests that $\log g_1$ is probably between 3.55 and 3.60. If the best fit is sought for metal abundances of $Z = 0.02$ and 0.004, ages of roughly 5 and 40 Myr, respectively, are found.

The primary component of V621 Per has been plotted in the HR diagram (Figure 5.5) and compared with Granada theoretical model predictions for masses of 10.0, 12.8 and 15.8 $M_\odot$ and for metal abundances of $Z = 0.01$ and 0.02. The χ Persei open cluster has been found to have an age of 12.6 ± 0.3 Myr (section 3.1.1) and this age

has been indicated on the evolutionary track for each model mass. The position of the primary component of V621 Per in these diagrams suggests that its mass is a little below $12.8 M_{\odot}$, also consistent with the form of the mass–radius diagram (Figure 5.4). However, its age derived by comparison with evolutionary models is somewhat greater than the 12.6 Myr expected due to its membership of χ Persei, and the discrepancy is larger for the $Z = 0.01$ model predictions than for the $Z = 0.02$ predictions.

The age of 12.6 ± 0.3 Myr for χ Persei was derived, from comparison between photometric observations of the cluster and the predictions of theoretical models, by researchers who assumed that $Z = 0.02$ (section 3.9). As the metal abundance of χ Persei has been found to be $Z = 0.01$ (section 3.8), this age may have a systematic error. Therefore the age discrepancy found here is of only minor significance, but deserves investigating when more accurate parameters are found for V621 Per. Also, the amount of overshooting present in stellar models is known to significantly change the predicted ages of giant stars (Schröder & Eggleton 1996) and the inclusion of rotation also affects the MS lifetime of high-mass stars (Maeder & Meynet 2000).

From comparison with the Granada evolutionary models, the surface gravity of the primary component of V621 Per is approximately 3.55. This conclusion is valid for metal abundances of $Z = 0.01$ and 0.02. From Table 5.6 the masses and radii of V621 Per corresponding to $\log g_1 = 3.55$ are about 12 and $6 M_{\odot}$, and 10 and $3 R_{\odot}$, for primary and secondary star respectively. This means that the primary star is near the age at which it passes through the ‘blue loop’ evolutionary stage (the point at which core hydrogen exhaustion causes the T_{eff} and surface gravity to rise temporarily), so accurate masses and radii for it would provide extremely good tests of the predictions of theoretical models. The Granada stellar models predict a luminosity ratio of about 0.05 for the inferred properties of V621 Per. The light ratio in the blue will be smaller than the overall luminosity ratio because the secondary star is expected to have a lower T_{eff} than the primary star. This suggests that the quality of our spectroscopic observations was almost sufficient to detect the secondary star. Accurate velocities for both stars should be measurable on spectra of a high signal to noise ratio, depending on the rotational velocity of the secondary star.

5.6.2 Membership of the open cluster χ Persei

V621 Per is situated, on the sky, in the centre of the χ Persei open cluster. It also appears in the correct place on the colour-magnitude diagrams of the cluster in the literature (see references in Section 5.1.1) and has the correct proper motion (Uribe et al. 2002) for cluster membership. The systemic velocity of the dEB, $-44.5 \pm 0.4 \text{ km s}^{-1}$, is consistent with the measured cluster systemic velocities of Oosterhoff (1937), Bidelman (1943), Hron (1987), Liu et al. (1989, 1991) and Chen, Hou & Wang (2003). In section 3.6.3 we measured the systemic velocity of the h Persei cluster to be $-44.2 \pm 0.3 \text{ km s}^{-1}$, indicating that h and χ Persei have the same systemic velocities, which is consistent with them having a common origin.

5.7 Summary

V621 Persei is a dEB in the young open cluster χ Persei, composed of a bright B2 giant star and an unseen MS secondary star. From blue-band spectroscopic data and RVs taken from the literature, we have derived an orbital period of 25.5302 days and a mass function of $f(M) = 0.617 \pm 0.012 M_{\odot}$. The discovery light curve of KP97 shows that the system exhibits a total primary eclipse lasting around 1.3 days and about 0.12 mag deep in B and V . No data exist around phase 0.06, where the secondary eclipse is expected to occur. The secondary eclipse may be up to about 0.06 mag deep. The light curves have been solved using JKTEBOP and Monte Carlo simulations to find robust uncertainties, and accurate fractional radii have been determined. The surface gravity of the secondary component has been found to be $\log g_2 = 4.244 \pm 0.054$.

Using the data above, possible values of the absolute masses and radii of the two stars were calculated by assuming different values of the primary surface gravity. A comparison in the mass-radius diagram of these possible values with theoretical predictions from the Granada stellar evolutionary models suggests that $\log g_1 \approx 3.55$. This surface gravity value agrees well with the values determined by Lennon et al.

(1988) and Dufton et al. (1990) by fitting observed Balmer line profiles with synthetic spectra. The luminosity of V621 Per has been derived from the known distance of the χ Persei cluster and the apparent magnitude of the dEB. This has been used to place the primary star in the HR diagram, and a comparison with the Granada evolutionary models confirms that its mass is roughly $12 M_{\odot}$, although a small discrepancy exists between the inferred age of V621 Per and the age of χ Persei.

The value of $\log g_1$ leads to masses of approximately 12 and $6 M_{\odot}$ and radii of 10 and $3 R_{\odot}$ for the components of the dEB. This conclusion is not strongly dependent on use of the Granada stellar models; predictions of the Geneva, Padova and Cambridge models are close to those of the Granada models (section 4.9). V621 Persei is a potentially important object for the information it holds about the evolution of high-mass stars. The expected luminosity ratio of the system, about 0.05, suggests that spectral lines of the secondary component should be detectable in spectra of a high signal to noise ratio. Better light curves will be needed for detailed studies of the properties of V621 Per, but the long period and lengthy eclipses mean that a large amount of telescope time will be required. In particular, the secondary eclipse is expected to occur around phase 0.06 and may be up to 0.06 mag deep. Observations of the light variation through secondary eclipse will be needed to provide a definitive study of the system. The absolute dimensions of the primary star, a B2 giant which is close to the blue loop evolutionary stage, could provide a good test of the success of theoretical stellar models and of the amount of convective core overshooting which occurs in stars.

6 HD 23642 in the Pleiades open cluster

The work in this section was undertaken in light of a recent paper (Munari et al. 2004) on HD 23642, in which the distance to the Pleiades was measured very precisely using the dEB. Dr. Maxted and I both suspected that there were two areas in which our understanding of HD 23642 could be improved, given the same observational data. Firstly, the photometric parameter uncertainties were formal errors, which are known to often be somewhat optimistic. Secondly, there are alternative ways of finding the distance to a dEB which may be better than the usual method involving bolometric corrections, as used by Munari et al. This research is of importance to our understanding of the stellar distance scale, because there is currently a disagreement in the astronomical community about the distance to the Pleiades (see below).

The real uncertainties which we derived were found to be somewhat larger than the formal errors quoted by Munari et al. (2004), substantiating comments by Zwahlen et al. (2004). We also introduced a new surface-brightness-based method to find the distance to a dEB, which is in some aspects superior to the bolometric correction method. We also found that Munari et al. had made a subtle calculation error which affected their final distance estimate by an amount the same size as its quoted error. In light of these findings, we submitted our study of HD 23642 to the *Astronomy and Astrophysics* journal, leading to its publication in early 2005.

6.1 The eclipsing binary HD 23642

HD 23642 (Table 6.1) was discovered to be a double-lined spectroscopic binary by Pearce (1957) and Abt (1958), and both components have been found to display slight spectral peculiarities (Abt & Levato 1978). Torres (2003) discovered shallow secondary eclipses in the Hipparcos photometric data of HD 23642 and also presented an accurate spectroscopic orbit. M04 derived precise absolute masses and radii of both components from high-resolution spectra and complete *BV* light curves. M04 found a distance of

Table 6.1: Identifications and astrophysical data for HD 23642.

References: (1) Perryman et al. (1997); (2) Abt & Levato (1978); (3) Two Micron All Sky Survey (section 1.6.3); (4) M04.

	HD 23642	References
Hipparcos number	HIP 17704	1
Hipparcos distance (pc)	111 ± 12	1
Spectral type	A0 Vp (Si) + Am	2
B_T	6.923 ± 0.011	1
V_T	6.839 ± 0.011	1
$J_{2\text{MASS}}$	6.635 ± 0.023	3
$H_{2\text{MASS}}$	6.641 ± 0.026	3
$K_{2\text{MASS}}$	6.607 ± 0.024	3
Orbital period (days)	2.46113400(34)	4
Reference time (HJD)	2 452 903.5981(13)	4

131.9 ± 2.1 pc, in disagreement with the Hipparcos parallax distance of 111 ± 12 pc for HD 23642.

6.2 The Pleiades open cluster

The Pleiades is a nearby, young open star cluster which is of fundamental importance to our understanding of stellar evolution and the cosmic distance scale. It has been exhaustively studied by many researchers and its distance and chemical composition were, until recent observations, considered to be well established. The distance derived from data obtained by the Hipparcos satellite, however, is in disagreement with traditional values, leading to claims that stellar evolutionary theory is much less reliable than previously thought.

The ‘long’ distance scale of 132 ± 3 pc was established by MS fitting analyses (e.g., Johnson 1957; Meynet, Mermilliod & Maeder 1993). Recent parallax observations from terrestrial telescopes (Gatewood, de Jonge & Han 2000), and from the Hubble Space

Telescope (Soderblom et al. 2005) are in good agreement with this distance.

The astrometric binary HD 23850 (Atlas) was recently studied by Pan, Shao & Kulkarni (2004) using the Palomar Testbed Interferometer (Colavita et al. 2003). These authors did not have a spectroscopic orbit for HD 23850, but were able to show that the distance to Atlas was greater than 127 pc, and probably between 133 and 137 pc. Zwahlen et al. (2004) have subsequently published a spectroscopic orbit and new interferometric measurements which, combined with the observations of Pan, Shao & Kulkarni (2004), give an entirely geometrical distance of 132 ± 4 pc to HD 23850.

A ‘short’ distance scale of 120 ± 3 pc (van Leeuwen 2004) has been found using trigonometrical parallaxes observed by the Hipparcos space satellite (Perryman et al. 1997). This is 2.8σ different to the traditional ‘long’ distance scale for the Pleiades, which is an important discrepancy. In an attempt to explain this, van Leeuwen (1999) placed the MSs of other nearby open clusters in the HR diagram using Hipparcos parallaxes, and found that five of the eight clusters have MSs as faint as the Pleiades.

Castellani et al. (2002) have shown that current theoretical stellar evolutionary models can fit the Pleiades MS if a low metal abundance of $Z = 0.012$ is adopted. However, Stello & Nissen (2001) used a metallicity-insensitive photometric technique to demonstrate that, if the Hipparcos parallaxes were correct, the MS Pleiades stars were implausibly fainter than their counterparts in the field. Also, Boesgaard & Friel (1990) have measured the iron abundance of the Pleiades to be approximately solar ($[\frac{\text{Fe}}{\text{H}}] = -0.034 \pm 0.024$) from high-resolution spectra of twelve F-type dwarfs in the cluster (further references can be found in Stauffer et al. 2003).

Narayanan & Gould (1999) have presented evidence that the Hipparcos parallaxes are correlated on angular scales of two to three degrees. They used a variant of the moving cluster method to find a distance of 130 ± 11 pc, in agreement with both the ‘long’ distance scale and the ‘short’ Hipparcos distance (van Leeuwen 2004). Makarov (2002) has reanalysed the Hipparcos data, allowing for this suggested correlation, and found the Pleiades distance to be 129 ± 3 pc. Until this result is confirmed, however, the ‘long’ and ‘short’ distance scales cannot yet be considered to be reconciled.

Munari et al. (2004; hereafter M04) studied the dEB HD 23642 and found a

Table 6.2: Spectroscopic orbital parameters for HD 23642.

	Primary	Secondary
Semi-amplitude K (km s^{-1})	99.10 ± 0.58	140.20 ± 0.57
Systemic velocity (km s^{-1})	6.07 ± 0.39	
Mass ratio q	0.7068 ± 0.0050	
$a \sin i$ ($R_{\odot}$)	11.636 ± 0.040	
$M \sin^3 i$ ($M_{\odot}$)	2.047 ± 0.021	1.447 ± 0.017

distance of 132 ± 2 pc, in good agreement with the ‘long’ distance scale. The method used by M04 is commonly used to find the distances to EBs but depends on theoretical calculations to provide bolometric corrections (BCs). We have reanalysed the data of M04 (which U. Munari has made available over the internet) to investigate alternative, empirical, methods of finding the distance to HD 23642 and similar EBs by the use of surface brightness relations.

6.3 Spectroscopic analysis

M04 observed HD 23642 five times with the Élodie échelle spectrograph on the 1.93 m telescope of the Observatoire de Haute-Provence. The RVs derived were combined by M04 with the spectroscopic observations of Pearce (1957) and Abt (1958), using lower weights for the older data, to calculate a circular spectroscopic orbit.

The low weight – and low precision – of the data of Pearce (1957) and Abt (1958) mean that they contribute little to the accuracy of the spectroscopic orbit. For comparison with the results of M04 we have chosen to derive the orbit using only the five échelle velocities for each star. The orbit was computed using SBOP (section 2.2.4.1), with the orbital ephemeris from M04, eccentricity fixed at zero, and equal systemic velocities for both stars. The root-mean-squares of the residuals of the resulting spectroscopic orbit are 0.4 and 1.2 km s^{-1} for the primary and secondary stars,

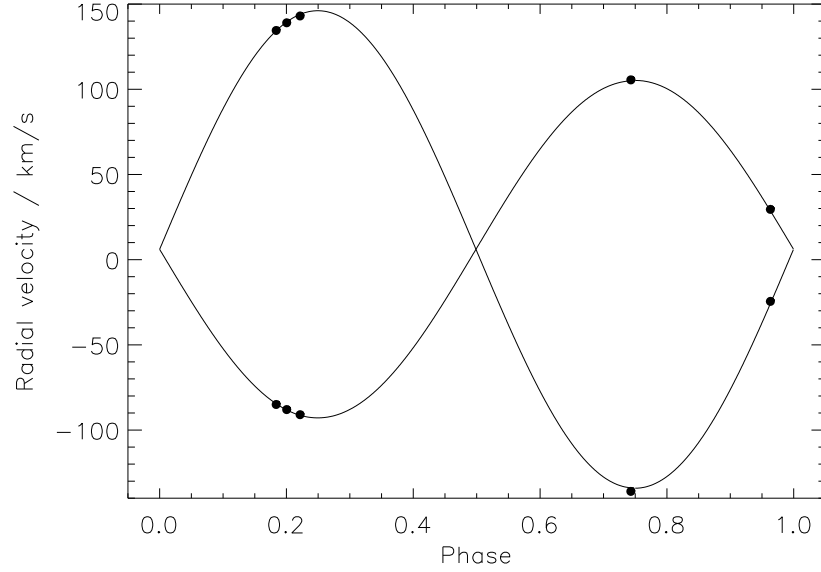


Figure 6.1: Spectroscopic orbit for HD 23642 from the RVs given by M04.

respectively. The spectroscopic orbit is plotted in Figure 6.1 and its parameters are given in Table 6.2. The orbital parameters are in acceptable agreement with those of M04 and Torres (2003) (see Table 6.3).

6.3.1 Determination of effective temperatures

The work in this section was undertaken by Dr. B. Smalley and is included here for completeness.

Atmospheric parameters were derived for the components of HD 23642 by comparing the observed spectra (from M04) with synthetic spectra calculated using UCLSYN (section 1.4.3.2). The spectra were rotationally broadened as necessary and instrumental broadening was applied to match the resolution of the observations. Surface gravities of 4.25 were assumed for both stars.

For the primary star, spectroscopic fitting gives a T_{eff} of 9750 ± 250 K with

Table 6.3: Comparison between spectroscopic orbits from the literature and from this study for HD 23642. Some results from Munari et al. (2004) are not included because these authors did not quote velocity semiamplitudes. The probable errors quoted by Pearce (1957) have been converted into standard errors.

Parameter	Pearce (1957)	Abt (1958)	Torres (2003)	Munari et al. (2004)	This study
Period (d)	2.46399	2.4611	2.46113329	2.46113400	2.46113400
$\pm$	0.00001		0.00000066	0.00000034	
Eccentricity	0.0	0.018	0.0	0.0	0.0
$\pm$	fixed		fixed	fixed	fixed
K_A (km s $^{-1}$)	100.6	98.1	97.40		99.10
$\pm$	3.8		0.84		0.58
K_B (km s $^{-1}$)	148.9	140.6	140.47		140.20
$\pm$	5.0		0.85		0.57
V_γ (km s $^{-1}$)	+6.8	+4.99	+6.1	+5.17	+6.07
$\pm$	2.1		1.7	.024	0.39

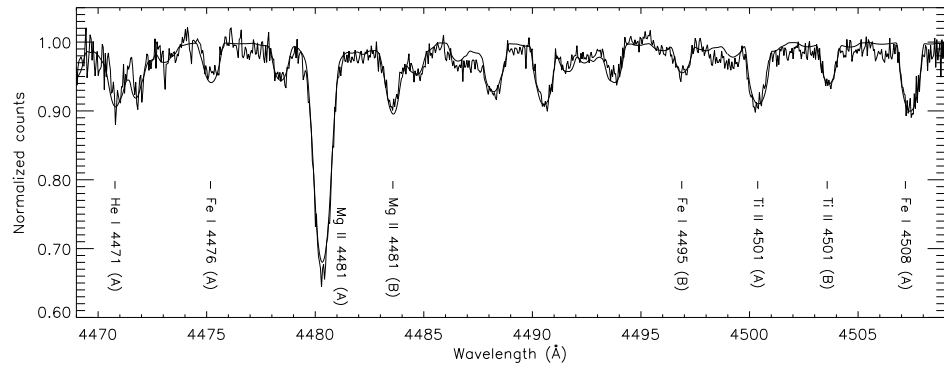


Figure 6.2: Comparison between a spectrum of HD 23642 and the best-fitting synthetic spectrum used to determine the atmospheric parameters of the stars. The spectrum of HD 23642 plotted here is a recombination of the individual spectra of the two stars, which were obtained by spectral disentangling (section 2.2.3.4). The source ion of some lines of interest have been indicated, with the rest wavelength of the line (Ångströms) and which star is producing it (A for the primary or B for the secondary). The effective RVs of the primary and secondary stars in this diagram are -58 and $+160$ km s $^{-1}$, respectively.

a microturbulent velocity of $\zeta_{T,A} = 2 \text{ km s}^{-1}$ and a projected rotational velocity of $V_A \sin i = 37 \pm 2 \text{ km s}^{-1}$. For the secondary star we find $T_{\text{eff}} = 7600 \pm 400 \text{ K}$, $\zeta_{T,B} = 4 \text{ km s}^{-1}$ and $V_B \sin i = 32 \pm 3 \text{ km s}^{-1}$. The microturbulent velocities are consistent with those typically found for stars of these T_{eff} s (section 1.4.3.1). The quoted uncertainties are limits of high confidence (roughly 2σ) and are larger than the formal fitting errors. A monochromatic light ratio of 0.25 ± 0.05 was obtained at $4480 \text{ \AA}$. The observations and best-fitting synthetic spectrum are shown in Figure 6.2.

Atmospheric parameters have also been estimated from *uvby* β photometry obtained from Hauck & Mermilliod (1998) and dereddened using $E_{b-y} = 0.008$, calculated from $E_{B-V} = 0.012$ (M04) and $E_{b-y} \approx 0.73E_{B-V}$ (section 2.3.1.3). Using the semi-empirical grid calibrations of Moon & Dworetsky (1985) and the TEFFLOGG program (Moon 1985), we obtained $T_{\text{eff}} = 9200 \text{ K}$ and $\log g = 4.30$, for the combined light of the system. To evaluate the effects of the secondary we have subtracted the photometry of the classical Am star 63 Tauri using a V -passband magnitude difference of 1.44. This gave the parameters $T_{\text{effA}} = 9870 \text{ K}$ and $\log g_A = 4.37$ for the primary component, in good agreement with our observationally determined parameters for this star.

A near-fundamental determination of T_{eff} can be obtained using the infra-red flux method (Smalley 1993). Ultraviolet fluxes were obtained from the IUE archive, optical fluxes from Kharitonov et al. (1988) and infrared fluxes from the 2MASS catalogue. From this the total integrated flux at the Earth was found to be $(5.44 \pm 0.44) \times 10^{-8} \text{ erg s}^{-1} \text{ cm}^{-2}$, for a reddening of $E_{B-V} = 0.012$. The IRFM then yielded $T_{\text{eff}} = 8900 \pm 350 \text{ K}$, which is rather low compared to the above values but is affected by the flux contribution of the cooler secondary star, which is proportionately brighter in the infrared. Allowing for the presence of the secondary star using the method of Smalley (1993) we find that the primary would have $T_{\text{effA}} = 9250 \pm 400 \text{ K}$ for a secondary star with $T_{\text{effB}} = 7500 \pm 500 \text{ K}$, which is consistent with the values determined above.

Fundamental T_{eff} s can be obtained for binary systems using total integrated fluxes and angular diameters obtained from system parameters, and known distances (Smalley & Dworetsky 1995; Smalley et al. 2002). In the case of HD 23642 the properties of the system have been found using a model-dependent method, so application of this

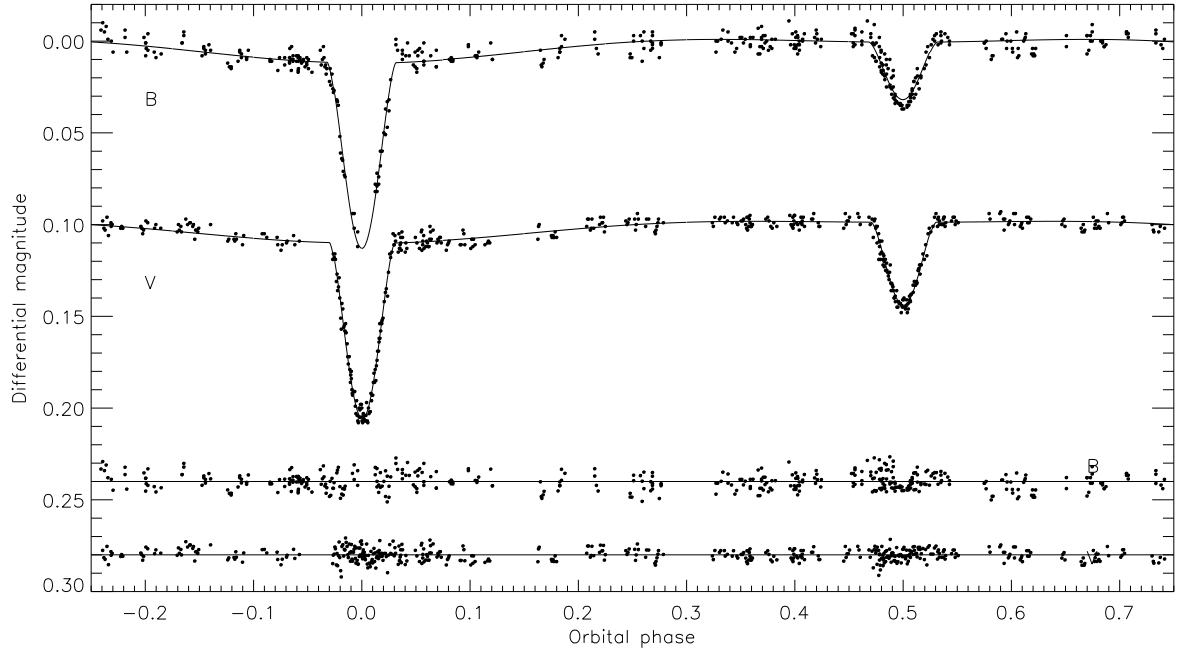


Figure 6.3: The M04 B and V light curves with our best fit overplotted. The V light curve is shifted by $+0.1$ mag for clarity. The residuals of the fit are offset by $+0.24$ mag and $+0.28$ mag for the B and V light curves respectively. Note that the poor fit around the secondary eclipse in the B light curve is due to scattered data, as suggested by the distribution of the residuals for this light curve. Rejection of the offending data makes the fit look better but is otherwise unjustified (see text for discussion).

procedure would lead to a circular argument. However, the method does allow for a consistency check on the two T_{eff} s and, importantly, their error estimates. Using the parameters obtained in the present work, we find $T_{\text{effA}} = 9620 \pm 280$ K and $T_{\text{effB}} = 7510 \pm 430$ K for the primary and secondary, respectively. Similar results are obtained for the parameters given by M04. However, use of the Hipparcos parallax of HD 23642 (which gives a distance of 111 ± 12 pc) would give $T_{\text{effA}} = 8640 \pm 540$ K and $T_{\text{effB}} = 6690 \pm 570$ K, which are clearly inconsistent with the values obtained above. The ‘short’ Pleiades distance (120 ± 3 pc) would give $T_{\text{effA}} = 9000 \pm 310$ K and $T_{\text{effB}} = 6970 \pm 450$ K, which is closer but still somewhat discrepant.

Using several techniques we have found the T_{eff} s of the two components stars of HD 23642 to be $T_{\text{effA}} = 9750 \pm 250$ K for the primary and $T_{\text{effA}} = 7600 \pm 400$ K for the secondary. Our error estimates are higher than those reported in M04, primarily because we have assessed the influence of external uncertainties, in addition to the internal precision of fits to spectra.

6.4 Photometric analysis

The B and V light curves contain 432 and 492 individual measurements, respectively, obtained with a 28 cm Schmidt-Cassegrain telescope and photometer by M04. The two light curves were solved separately using JKTEBOP (section 3.7.1). Linear limb darkening coefficient values of 0.496 and 0.596 (B) and 0.421 and 0.548 (V), for the primary and secondary stars respectively, were adopted from van Hamme (1993) as the light curves are not of sufficient quality to include them as free parameters. Gravity darkening exponents β_1 were fixed at 1.0 (Claret 1998) and the mass ratio was fixed at the spectroscopic value. The ephemeris given in M04 was used and the orbit was assumed to be circular. Although the M04 light curves have very low observational scatter, deriving accurate parameters from them is problematic due to the shallow eclipses. This is exacerbated by some scattered data in the B light curve, which makes it less reliable than the V light curve. As third light, L_3 , is poorly constrained by the

Table 6.4: Photometric solution A, found by fitting the light curves without using a spectroscopic light ratio to constrain the ratio of the radii. The individual errors are from Monte Carlo simulations. The combined parameters (bottom line) are the weighted means of the two different values for each parameter. The quality of the fit is given by σ_{rms} (see text for definition).

Light curve	Surface brightness ratio, J	Light ratio	Primary radius, r_A (a)	Ratio of the radii, k	Secondary radius, r_B (a)	Inclination of orbit, i ($^\circ$)	σ_{rms} (mmag)
<i>B</i>	0.316 ± 0.018	0.264 ± 0.083	0.1536 ± 0.0069	0.929 ± 0.116	0.1427 ± 0.0108	76.91 ± 0.41	4.639
<i>V</i>	0.493 ± 0.018	0.366 ± 0.094	0.1487 ± 0.0057	0.882 ± 0.093	0.1312 ± 0.0083	77.80 ± 0.24	3.254
<i>BV</i>			0.1507 ± 0.0044	0.900 ± 0.073	0.1355 ± 0.0066	77.57 ± 0.21	

Table 6.5: Photometric solution B, found using JKTEBOP and the light ratio of Torres (2003). The individual uncertainties are from Monte Carlo simulations with a fixed ratio of the radii, k . Symbols have the same meaning as in Table 6.4. The adopted parameter values (bottom line) are the weighted means of the different parameter values for each light curve. The adopted uncertainties depend mainly on the possible values of k .

Light curve	Light ratio	k	J	r_A (a)	r_B (a)	i ($^\circ$)	σ_{rms} (mmag)
<i>V</i>	0.300	0.8075	0.483 ± 0.014	0.1531 ± 0.0009	0.1237 ± 0.0007	78.04 ± 0.08	3.254
<i>V</i>	0.335	0.8477	0.489 ± 0.015	0.1508 ± 0.0009	0.1278 ± 0.0008	77.90 ± 0.08	3.254
<i>V</i>	0.370	0.8858	0.493 ± 0.015	0.1485 ± 0.0009	0.1315 ± 0.0008	77.79 ± 0.08	3.254
<i>B</i>	0.190 ± 0.008	0.8075	0.304 ± 0.014	0.1599 ± 0.0019	0.1291 ± 0.0015	77.47 ± 0.19	4.654
<i>B</i>	0.214 ± 0.008	0.8477	0.309 ± 0.015	0.1580 ± 0.0019	0.1339 ± 0.0016	77.26 ± 0.19	4.647
<i>B</i>	0.237 ± 0.009	0.8858	0.313 ± 0.015	0.1560 ± 0.0019	0.1382 ± 0.0016	77.08 ± 0.19	4.642
Adopted		0.848 ± 0.039		0.1538 ± 0.0024	0.1300 ± 0.0037	77.78 ± 0.17	

observations, we have made separate solutions for $L_3 = 0$ and 0.05 (in units of the total light of the eclipsing stars) and included differences in the parameter values derived in the uncertainties quoted below. As there are no features in the spectra of HD 23642 known to come from a third star, it is unlikely that third light is greater than 5%.

Initial solutions provided an inadequate fit to the light variation outside eclipse so the reflection effect for the secondary star was separately adjusted towards best fit rather than being calculated from the system geometry. We also solved the light curves using the WD98 code (section 2.4.1.2), using a detailed treatment of reflection. As the differences between the JKTEBOP and WD98 solutions were negligible, further analysis was undertaken using JKTEBOP. This code has two important advantages; a detailed error analysis is not prohibitively expensive in terms of computer time, and the philosophy of the JKTEBOP code is to solve for the set of parameters most directly related to the light curve shape.

As the photometry of M04 is not supplied with observational errors, we have weighted all observations equally. We will judge the quality of the fit of a model light curve to observations using the root mean square of the residuals of the fit, σ_{rms} .

6.4.1 Light curve solution

The ratio of the radii, k , is poorly constrained by the light curves because the eclipses are very shallow. Whilst a reasonable photometric solution can be obtained from the light curves alone, an alternative is to use a spectroscopic light ratio to constrain k . A light ratio of $\frac{l_B}{l_A} = 0.31 \pm 0.03$ was given by Torres (2003), based on a cross-correlation analysis of a 45 Å wide spectral window centred on 5187 Å. This spectroscopic light ratio is noted to be preliminary so we provide separate solutions without (‘solution A’) and with (‘solution B’) its inclusion in the light curve fitting procedure, but preference is given to solutions including the spectroscopic constraint. We have used the light ratio of Torres (2003) as it is based on a larger amount of observational data than the light ratio found in section 6.3.1, and because the wavelength it was obtained at is closer to the central wavelength of the V passband.

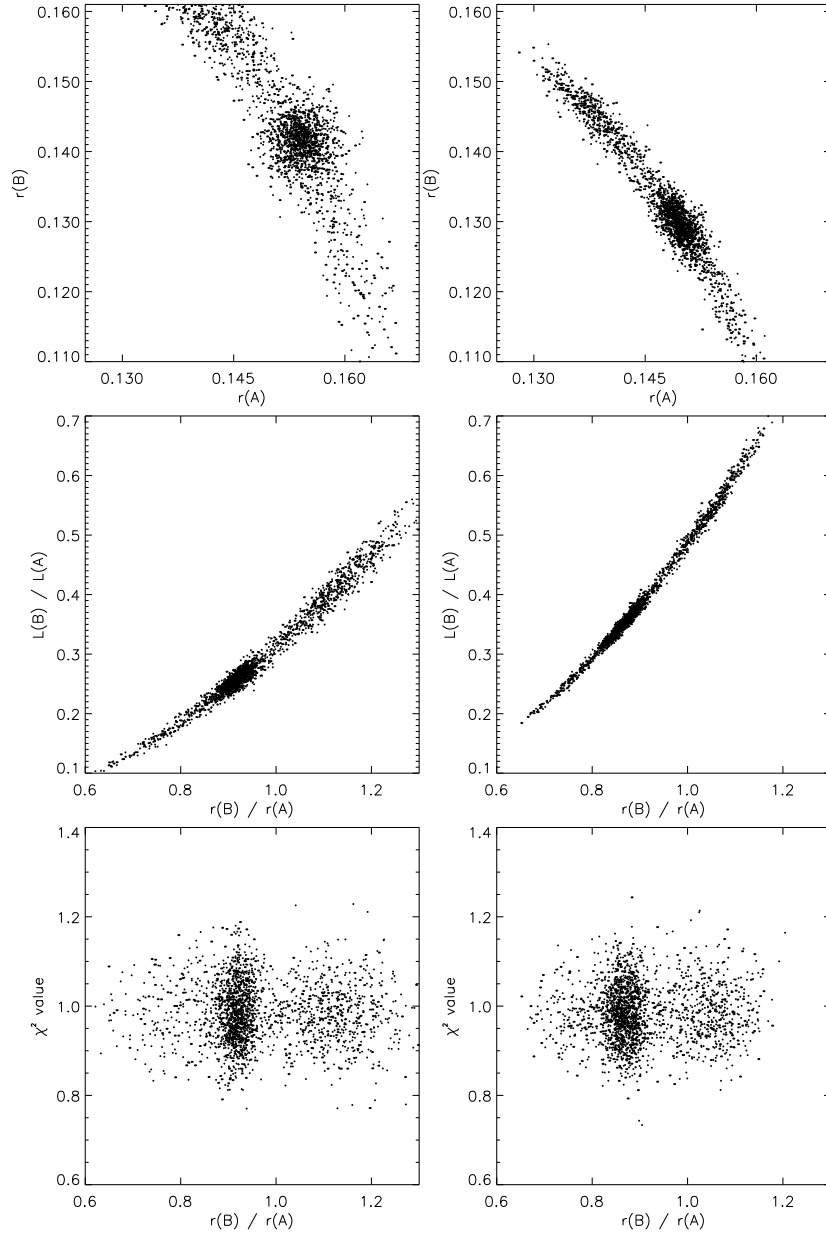


Figure 6.4: Results of the Monte Carlo analysis for the B (left panels) and V (right panels) light curves. The top two panels show r_A plotted against r_B . The middle two panels show k versus the light ratios. The lower two panels show the reduced χ^2 versus k . These values were calculated using the residuals of the best fits to the observed light curves. Note the greater scatter of the Monte Carlo solutions for the B light curve, which is due to the larger observational errors. Only 2000 of the 10 000 points have been plotted in each panel.

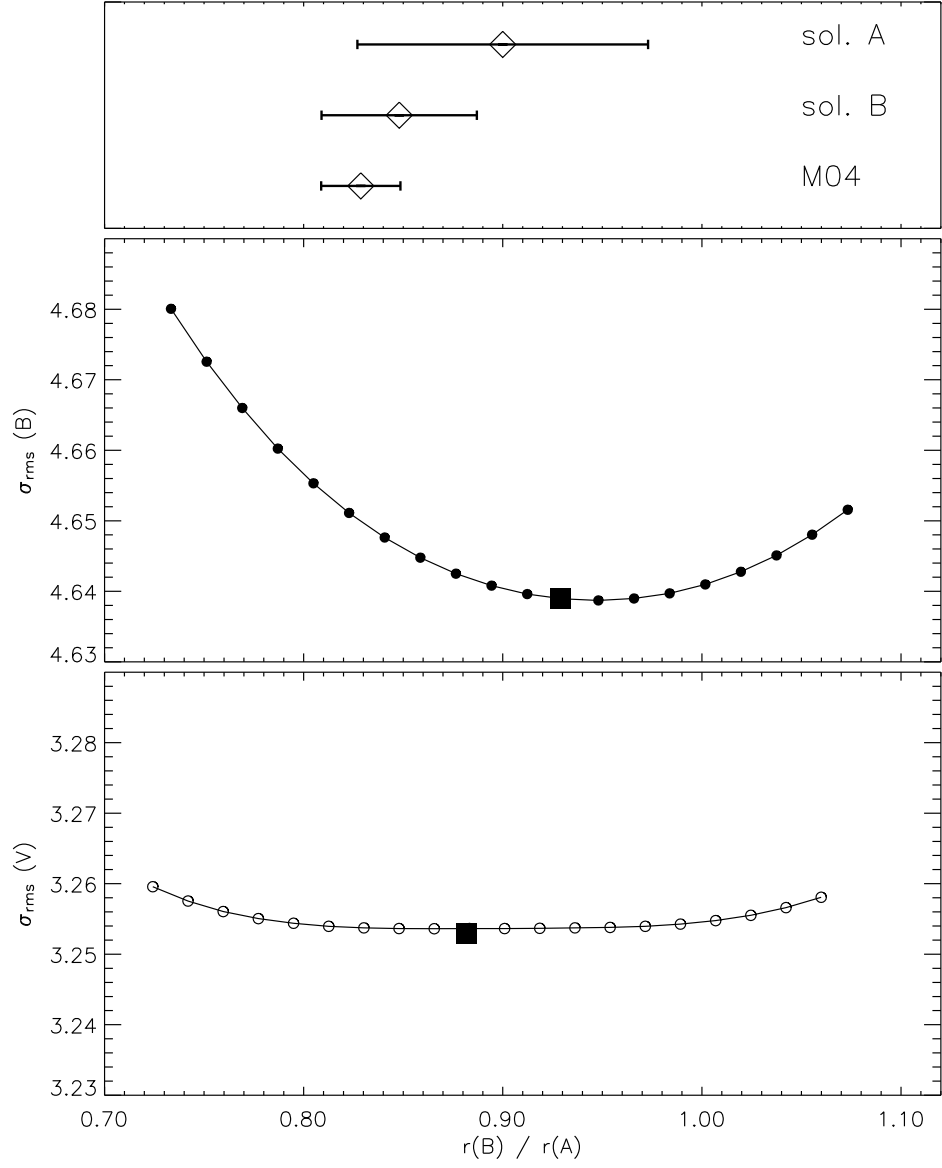


Figure 6.5: Comparison between the quality of the fit for different values of k and the solutions derived in this work and by M04. σ_{rms} has been plotted against k for the B light curve (middle panel) and V light curve (lower panel) using the same scales. The optimum values of k found in the individual solutions of the two light curves are indicated by filled squares. The upper panel shows the values and uncertainties of k found in solution A, solution B, and by M04. Note that the quality of the photometry is excellent, particularly for the V light curve where the photometric error of an individual datapoint is 3.25 millimagnitudes.

The light ratio of Torres (2003) was converted to a V passband light ratio using a V passband response function and synthetic spectra, calculated from ATLAS9 model atmospheres, for the T_{eff} s and surface gravities found in our preliminary analyses. The resulting V passband light ratio of 0.335 ± 0.035 (where the uncertainties include a small contribution due to possible systematic errors from the use of ATLAS9 model atmospheres) has been used to constrain k using the V light curve. The resulting values of k were then adopted for solution of the B light curve.

Table 6.4 gives solution A, and Table 6.5 gives solution B. The best fit for the former solution is compared to the observational data in Figure 6.3; the light variation for the latter solution is almost identical so has not been plotted. Note that the B light curve appears to be badly fitted at the centre of the secondary eclipse. Investigation has revealed that this is not a problem with the EBOP or JKTEBOP model, but is caused by scatter present in the observational data. A better fit can be obtained by rejecting one night's data, around phase 0.48, which is brighter than the model light curve. After rejection of these data, the fit is significantly improved but the derived parameters are quite similar. We have therefore included all observational data in the fitting procedure, and suggest that further photometric data should be obtained.

The uncertainties in the fitted parameters were estimated using the Monte Carlo analysis implemented in JKTEBOP (sections 4.6.1 and 5.5) and are included in Table 6.4 and Table 6.5. Some results of the Monte Carlo simulations are shown in Figure 6.4. Uncertainties in the theoretically-derived limb darkening coefficients have been incorporated by perturbing the values of the coefficients by ± 0.05 , on a flat distribution, for each Monte Carlo simulation.

The fractional stellar radii given by solution A are $r_A = 0.1507 \pm 0.0044$ and $r_B = 0.1355 \pm 0.0066$, whereas inclusion of the spectroscopic light ratio (solution B) gives $r_A = 0.1538 \pm 0.0024$ and $r_B = 0.1300 \pm 0.0037$. The fractional radii found by M04 were $r_A = 0.1514 \pm 0.0025$ and $r_B = 0.1254 \pm 0.0022$. Our spectroscopic-constraint result is in agreement with the M04 result, but we are unable to reproduce the small uncertainties claimed by M04. Figure 6.5 compares the values of k found for solution A, solution B and by M04, to the residuals of the fit for a range of different values

Table 6.6: Absolute dimensions of the components of HD 23642, calculated using photometric solution B. Symbols have their usual meanings and the equatorial rotational velocities are denoted by V_{eq} . The absolute bolometric magnitudes have been calculated using a solar luminosity of 3.826×10^{26} W and absolute bolometric magnitude of 4.75.

	Primary star	Secondary star
Mass ($M_{\odot}$)	2.193 ± 0.022	1.550 ± 0.018
Radius ($R_{\odot}$)	1.831 ± 0.029	1.548 ± 0.044
$\log g$ (cm s^{-1})	4.254 ± 0.014	4.249 ± 0.025
T_{eff} (K)	9750 ± 250	7600 ± 400
$\log(L/L_{\odot})$	1.437 ± 0.047	0.858 ± 0.095
M_{bol}	1.16 ± 0.12	2.60 ± 0.24
V_{eq} (km s^{-1})	38 ± 2	33 ± 3
V_{synch} (km s^{-1})	37.6 ± 0.6	31.8 ± 0.9

of k for each light curve. Solution A has the largest uncertainty but is close to the minima of the residuals in the B and V light curves. Solution B, for which k was found using a spectroscopic light ratio, has a smaller uncertainty in k but has a slight dependence on theoretical model atmospheres. The results of M04 were obtained from the same data as our solution A, but the two values of k are quite different. The low uncertainties quoted by M04 mean that their value of k (calculated by us from the stellar radii given by M04) is inconsistent with the minima in the residuals curves. M04 adopted formal errors from their photometric analysis, which are known to be optimistic (section 2.4.3).

6.5 Absolute dimensions and comparison with stellar models

The absolute dimensions of the two stars have been calculated from the spectroscopic results and photometric solution B, and are given in Table 6.6. For comparison, solution

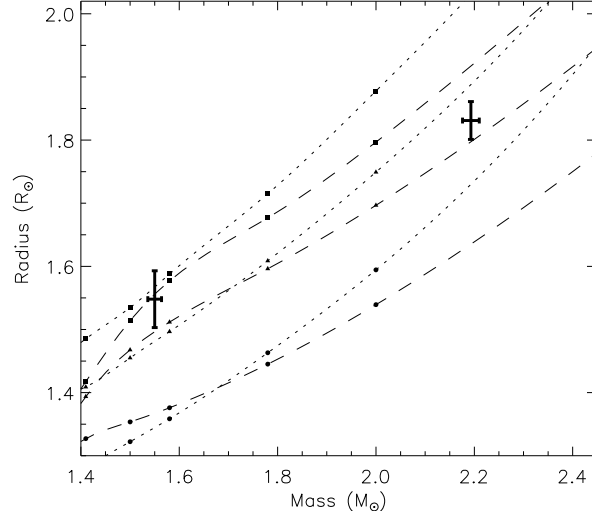


Figure 6.6: Comparison between the observed properties of HD 23642 and the Granada stellar evolutionary models for metal abundances of $Z = 0.01$ (circles), $Z = 0.02$ (triangles) and $Z = 0.03$ (squares). Predictions for normal helium abundances are plotted with dashed lines and helium-rich model predictions are plotted using dotted lines. An age of 125 Myr was assumed.

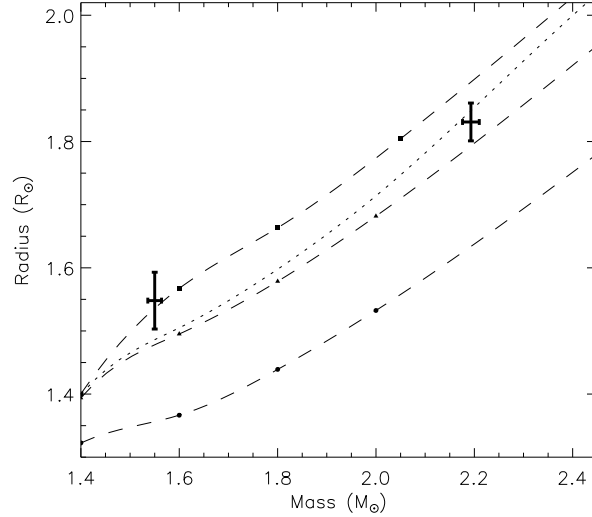


Figure 6.7: Comparison between the observed properties of HD 23642 and the Cambridge stellar evolutionary models for metal abundances of $Z = 0.01$ (circles), $Z = 0.02$ (triangles) and $Z = 0.03$ (squares). An age of 125 Myr was assumed (dashed lines), but predictions for a solar chemical composition and an age of 175 Myr are also shown (dotted line).

A gives $R_A = 1.796 \pm 0.053 R_\odot$ and $R_B = 1.615 \pm 0.079 R_\odot$, and the masses and radii determined by M04 are $M_{rmA} = 2.24 \pm 0.017 M_\odot$, $M_B = 1.56 \pm 0.014 M_\odot$, $R_A = 1.81 \pm 0.030 R_\odot$ and $R = 1.50 \pm 0.026 R_\odot$.

The masses and radii of the stars in HD 23642 can be compared to evolutionary models to estimate the metal abundance of the dEB. This is important because unusual chemical compositions have been suggested as possible reasons why the ‘long’ distance scale of the Pleiades disagrees with the Hipparcos parallax distances.

In Figure 6.6 the masses and radii of the components of HD 23642 (from solution B) have been compared to predictions of the Granada stellar models (section 1.3.2.1). An age of 125 Myr has been adopted (Stauffer, Schultz & Kirkpatrick 1998); a change in this age by -25 or $+75$ Myr does not affect our conclusions. Figure 6.6 shows predictions for metal abundances of $Z = 0.01$, 0.02 and 0.03 . For each metal abundance we have plotted predictions for normal helium abundance (dashed lines) and for significantly enhanced helium abundances (dotted lines). Figure 6.7 compares the masses and radii of HD 23642 to the predictions of the Cambridge models (section 1.3.2.4) for metal abundances of $Z = 0.01$, 0.02 and 0.03 . The solar chemical composition isochrone is also shown for an age of 175 Myr.

The masses and radii of the components of HD 23642 suggest that the metal abundance of the dEB is slightly greater than solar ($Z \approx 0.02$). Predictions for enhanced helium abundances are ruled out as they predict a mass-radius relation for HD 23642 much steeper than observed. The approximately solar Pleiades iron abundance found by Boesgaard & Friel (1990), from high-resolution spectroscopy of F dwarfs, is confirmed. The ‘short’ and ‘long’ Pleiades distances therefore cannot be reconciled by adopting an unusual chemical composition for the cluster.

6.6 The distance to HD 23642 and the Pleiades

We will now derive the distance to HD 23642 using three methods, one of which is introduced here. We investigate the normal technique using BCs to find absolute

Table 6.7: The distances derived for HD 23642 by using different sources of BCs.

Bolometric corrections	passband	$\left[\frac{M}{H}\right]$	Distance (pc)
Code et al. (1976)	<i>V</i>		138.1 ± 6.2
Bessell et al. (1998)	<i>V</i>		139.5 ± 5.3
Bessell et al. (1998)	<i>K</i>		138.6 ± 3.3
Girardi et al. (2000)	<i>B</i>	0.0	140.2 ± 6.1
Girardi et al. (2000)	<i>V</i>	0.0	139.8 ± 5.3
Girardi et al. (2000)	<i>J</i>	0.0	138.7 ± 3.8
Girardi et al. (2000)	<i>H</i>	0.0	138.0 ± 3.3
Girardi et al. (2000)	<i>K</i>	0.0	138.8 ± 3.3
Girardi et al. (2000)	<i>V</i>	-0.5	137.9 ± 5.3
Girardi et al. (2000)	<i>V</i>	+0.5	142.4 ± 5.3
Girardi et al. (2000)	<i>K</i>	-0.5	137.9 ± 3.3
Girardi et al. (2000)	<i>K</i>	+0.5	139.8 ± 3.3

visual magnitudes, the use of surface brightness calibrations in terms of observed colour indices, and a new method based on infrared surface brightness calibrations in terms of T_{eff} . All the techniques require reliable apparent magnitudes, which for HD 23642 are available from the Tycho experiment on board the Hipparcos satellite and from 2MASS (section 1.6.3). The apparent *B* and *V* magnitudes observed by Tycho have been converted to the Johnson photometric system using the calibration of Bessell (2000). The *JHK* photometry from 2MASS has been converted to the SAAO near-infrared photometric system using the calibration of Carpenter (2001).

6.6.1 Distance from the use of bolometric corrections

The traditional method of determining the distance to an EB is to calculate the luminosity of each star from its radius and T_{eff} . The resulting absolute bolometric magnitudes are then converted to absolute visual magnitudes using BCs. The combined absolute visual magnitude of the two stars is then compared to the apparent visual

magnitude to find the distance modulus (section 1.6.3.1).

To find the distance to HD 23642, we have adopted the astrophysical parameters of the system given in Table 6.6. An interstellar reddening of $E_{B-V} = 0.012 \pm 0.004$ mag has been adopted from M04. We have calculated the distance to HD 23642 using the empirical BCs given by Code et al. (1976), with a calibration uncertainty of 0.05 mag (Clausen 2004) added in quadrature. We have also derived the distance using the theoretical BCs of Bessell, Castelli & Plez (1998) and of Girardi et al. (2000). For the primary component of HD 23642, the uncertainty resulting from its T_{eff} measurement is reduced due to the form of the BC function around 10 000 K. The main contribution to the overall uncertainty comes from the uncertainties in the T_{eff} s of the stars.

The distances found by using BCs are given in Table 6.7 and show that a value around 139 pc is obtained consistently. The distance found using the empirical BCs of Code et al. (1976) is very similar to the distances found by using the theoretically-derived BCs, suggesting that systematic errors due to the use of model atmospheres are small. Distances derived using BCs for the *JHK* passbands are more precise because the uncertainties in T_{eff} and interstellar reddening are less important. If the results of photometric solution A are used to find the distance to HD 23642 using this method, distances of around 139 pc are also found, but with larger uncertainties.

The BCs given by Girardi et al. (2000) are available for several different metallicities, $[\frac{M}{H}]$. We have investigated the effect of non-solar metallicities on the distance derived using BCs for $[\frac{M}{H}] = -0.5$ and $+0.5$ (Table 6.7). Whilst a significant change in distance is found for distances derived using BCs for the *V* passband, the effect is much smaller for the *K* passband, underlining the usefulness of infrared photometry in determining distances to dEBs.

M04 found the distance to HD 23642, using the Bessell et al. (1998) BCs, to be 131.9 ± 2.1 pc. However, we have been unable to reproduce their result using the properties of HD 23642 found by these authors. The absolute bolometric magnitudes given by M04 appear to have been taken from output files produced by the WD98 code (section 2.4.1.2), which uses a solar luminosity of $L_{\odot} = 3.906 \times 10^{26}$ W and absolute bolometric magnitude of $M_{\text{bol}\odot} = 4.77$. The BCs used by M04, however, were calcu-

lated using different values: $L_{\odot} = 3.855 \times 10^{26} \text{ W}$ and $M_{\text{bol}\odot} = 4.74$ (Bessell, Castelli & Plez 1998). This inconsistency appears to be sufficient to explain the discrepancy between their result and our results. If we adopt the masses, radii and T_{eff} s of M04, we find a distance of $135.5 \pm 2.3 \text{ pc}$. This is about 3 pc smaller than most of our results, mainly due to the smaller stellar radii found by M04 for the components of HD 23642.

The discussion above shows that it is possible to calculate consistent distances to EBs, using different sets of BCs. One weakness of this method is that it is difficult to evaluate the systematic error introduced by the uncertainty in the zeropoint of the T_{eff} scale. The use of BCs derived from theoretical models also introduces a systematic error which is likely to be small in this case but, in general, is difficult to quantify. This systematic error is due to deficiencies in the model, e.g., the approximate treatment of convection and the lack of complete spectral line lists. It may also be the case that the star has properties which are not accounted for by the model, e.g., spectral peculiarity due to magnetic fields or slow rotation. For these reasons it is desirable to develop an empirical distance determination method which is less sensitive to systematic errors in T_{eff} and in which the sources of uncertainty are more explicit.

6.6.2 Distance from relations between surface brightness and colour

In this method the angular diameter of the star is calculated and compared to its linear diameter, determined from photometric analysis, to find the distance (section 1.6.3.2). The advantage of this procedure is that the calculations can be entirely empirical, depending on the surface brightness calibration. One disadvantage is that individual apparent magnitudes and colour indices of the components of the dEB, calculated from the light ratios found in light curve analyses, must be used. The uncertainties in these quantities can cause the derived distance to have a low precision.

We have determined a distance to HD 23642 of $138 \pm 19 \text{ pc}$ using the S_V versus $B - V$ calibration given by Di Benedetto (1998). The main uncertainty in this result comes from the uncertainties in the light ratios in the B and V passbands. The more

recent calibration, given by Kervella et al. (2004, hereafter KTDS04), has not been used because it does not allow for the nonlinear dependence of S_V on $B - V$. The $B - V$ colour index is also not a good indicator of surface brightness because the B passband is known to be sensitive to metallicity through the effect of line blanketing. The $V - K$ and $B - L$ indices are good surface brightness indicators (KTDS04; Di Benedetto 1998) because they are more sensitive to surface brightness and because the intrinsic scatter in the calibrations falls below 1%.

6.6.3 Distance from relations between surface brightness and T_{eff}

The concept behind this section, and the derivation of equation 6.3, is due to Dr. P. F. L. Maxted.

Empirical relations between T_{eff} and the wavelength-dependent surface brightness of a star, S_{m_λ} where m_λ is an apparent magnitude in passband λ , have been derived by KTDS04 from interferometric observations. This allows the surface brightnesses of the components of a dEB to be found without using the light ratios of the system found during the light curve analysis.

Using the definition of the zeroth magnitude angular diameter (section 1.1.1.5) and the small-angle approximation for the angular diameter of a star, we can derive

$$m_\lambda = 5 \log_{10} \left(\frac{\phi^{(m_\lambda=0)} d}{2R} \right) \quad (6.1)$$

where m_λ is a apparent magnitude in passband λ , d and R are the stellar distance and linear radius and $\phi^{(m_\lambda=0)}$ is the zeroth magnitude angular diameter in passband λ . The equation for summing two apparent magnitudes, $m_{\lambda,1}$ and $m_{\lambda,2}$ into a combined apparent magnitude, $m_{\lambda,1+2}$ is

$$m_{\lambda,1+2} = -2.5 \log_{10}(10^{-0.4m_{\lambda,1}} + 10^{-0.4m_{\lambda,2}}) \quad (6.2)$$

Substituting equation 6.1 for each star into equation 6.2 gives

$$d = 10^{0.2m_{\lambda,1}+2} \sqrt{\left(\frac{2R_1}{\phi_1^{(m_{\lambda}=0)}}\right)^2 + \left(\frac{2R_2}{\phi_2^{(m_{\lambda}=0)}}\right)^2} \quad (6.3)$$

where the stellar radii R_1 and R_2 are in AU, $\phi_1^{(m_{\lambda}=0)}$ and $\phi_2^{(m_{\lambda}=0)}$ are in arcseconds and distance d is in parsecs.

Calibrations for $\phi^{(m_{\lambda}=0)}$ are given in terms of T_{eff} by KTDS04 (where they are denoted using $ZMLD_{\lambda}$). These calibrations are for the broad-band *UBVRIJHKL* passbands and are valid for T_{eff} s between 10 000 K and 3600 K for MS stars. The *JHKL* passband calibrations have the least scatter and are also the least affected by interstellar reddening. For the *K* passband the scatter around the calibration is undetectable at a level of 1% so we conservatively adopt a scatter of 1%. We have applied the calibrations for *B* and *V* (derived from Tycho data) and for *J*, *H* and *K* (derived from 2MASS data). As there is no “standard” infrared photometric system, the calibrations of KTDS04 use data from several different *JHKL* systems, so the systematic uncertainty of not having a standard system is already included in the quoted scatter in the calibrations. For the A stars only, the scatter around the *B* and *V* calibrations is much smaller than the overall scatter quoted, so for HD 23642 the *B* and *V* distance uncertainties are overestimated by a factor of about two.

The distances found using equation 6.3 and the KTDS04 calibrations are given in Table 6.8. We have calculated the distances from the results of the light curve solution and the spectroscopic velocity semiamplitudes (K_A and K_B), in order to carefully assess how important the uncertainties in each basic quantity are to the final distance uncertainty. The intrinsic scatter in the calibrations is marked as “cosmic” scatter in Table 6.8 and is the main contributor to the uncertainty in the *B* and *V* passband distances. Note that the uncertainties in the *JHK* calibration distances are much smaller than in the *BV* calibration distances, because the calibrations have much smaller scatter and reddening is less important. We will adopt the *K* passband calibration distance of 139.1 ± 3.5 pc as our final distance to HD 23642. Note that we cannot treat any of the distance estimates investigated above as being independent of each other as they are all

calculated using the same values for reddening, stellar radii and T_{eff} s. If we adopt the results of photometric solution A, the K passband distance we find is 139.7 pc with an error of 4.7 pc, which is entirely consistent with our adopted distance of 139.1 ± 3.5 pc.

One shortcoming of finding distances using equation 6.3 is that the T_{eff} scales used in analysis of the dEB and for the calibration must be the same to avoid systematic errors. This is, however, a more relaxed constraint on the T_{eff} scale than that involved in finding distance using BCs. We note that our T_{eff} uncertainties include contributions due to possible spectral peculiarity and systematic offset relative to the (inhomogeneous) T_{eff} s used in the KTDS04 calibration. The uncertainty in distance could be reduced by further observations and estimations of the T_{eff} s of the two stars, using the same technique as for the stars used to calibrate the surface brightness relations.

6.7 Conclusion

The ‘long’ distance scale of the Pleiades is 132 ± 3 pc and is supported by MS fitting, the distance of the astrometric binary Atlas, and by ground-based and Hubble Space Telescope parallax measurements. The ‘short’ distance is 120 ± 3 pc and is derived from parallaxes observed by the Hipparcos satellite. These results have been summarised in Table 6.9. It has been suggested that the two distances could be reconciled if the Pleiades cluster is metal-poor, but determinations of the atmospheric metal abundances of Pleiades F dwarfs suggest that the cluster has a solar iron abundance.

We have studied the dEB HD 23642, a member of the Pleiades with an Hipparcos parallax, to calculate reliable absolute dimensions and uncertainties of the component stars. By comparing the radii of the components of HD 23642 to theoretical models we find that the metal and helium abundances are approximately solar, which removes the possibility that the ‘long’ and ‘short’ distance scales could be reconciled by adopting a low metal abundance or high helium abundance for the Pleiades.

We have investigated the use of BCs for determining the distances to dEBs, using the empirical BC calibration of Code et al. (1976) and two sources of theoretically-

Table 6.8: The results and individual error budgets for distance estimates using the T_{eff} s and overall apparent magnitudes of the HD 23642 system in the $BVJHK$ passbands. All distances are given in parsecs and the total uncertainties are the sums of the individual uncertainties added in quadrature.

Uncertainty source	B	V	J	H	K
Spectroscopic K_A	0.3	0.3	0.3	0.3	0.3
Spectroscopic K_B	0.3	0.3	0.3	0.3	0.3
Orbital inclination, i	0.1	0.1	0.1	0.1	0.1
Fractional radius, r_A	1.8	1.7	1.5	1.4	1.4
Fractional radius, r_B	0.8	1.0	1.3	1.4	1.5
Primary T_{eff}	4.8	3.3	1.7	0.7	0.7
Secondary T_{eff}	3.8	3.5	2.1	1.5	1.4
Reddening E_{B-V}	1.1	0.8	0.3	0.2	0.1
Apparent magnitude	1.0	1.0	1.9	1.9	1.9
“Cosmic” scatter	9.0	8.3	1.7	1.9	1.4
Total uncertainty	11.1	9.9	4.2	3.8	3.5
Distance	142.8	141.4	139.6	138.4	139.1

Table 6.9: Summary of the different distances found for the Pleiades or for Pleiades members, both from the literature (upper part of the table) and for HD 23642 in this work (lower part of the table). References are also given in the text and abbreviations and symbols have their usual meanings.

Source of distance measurement	Distance (pc)
Hipparcos parallaxes (van Leeuwen 2004)	120 $\pm$ 3
Hipparcos parallax of HD 23642	111 $\pm$ 12
Hipparcos parallax of HD 23850	117 $\pm$ 14
MS fitting (Stello & Nissen 2001)	132.4 $\pm$ 1.8
Ground-based parallaxes	130.9 $\pm$ 7.4
HST parallaxes of three Pleiades stars	134.6 $\pm$ 3.1
Narayanan & Gould (1999)	131 $\pm$ 11
Makarov (2002)	129 $\pm$ 3
Eclipsing binary HD 23642 (Munari et al. 2004)	131.9 $\pm$ 2.1
Astrometric binary HD 23850	132 $\pm$ 4
Code et al. (1976) empirical BCs (V passband)	138.1 $\pm$ 6.2
Girardi et al. (2000) theoretical BCs (V passband)	139.8 $\pm$ 5.3
Girardi et al. (2000) theoretical BCs (K passband)	138.8 $\pm$ 3.3
Surface brightness–($B - V$) relation	138 $\pm$ 19
Surface brightness– T_{eff} relation (V passband)	141.4 $\pm$ 9.9
Surface brightness– T_{eff} relation (K passband)	139.1 $\pm$ 3.5

calculated BCs. We find that the empirical and theoretical BCs give distances to HD 23642 in good agreement with each other. Distances determined using BCs for near-infrared passbands are more precise and reliable due to a smaller dependence on interstellar reddening and metal abundance.

We have presented a new, almost entirely empirical, technique for determining the distance to dEBs composed of two components with T_{eff} s between 10 000 K and 3600 K, based on interferometrically-derived calibrations between T_{eff} and surface brightness (Kervella et al. 2004). This method does not explicitly require a light ratio for calculation of distance. Distances determined using the near-infrared *JHKL* calibrations are more precise as uncertainties in interstellar reddening and T_{eff} are less important. Using this technique and *K*-passband photometry from 2MASS, we find that HD 23642 is at a distance of 139.1 ± 3.5 pc (Table 6.9). This distance is consistent with the ‘long’ distance scale of the Pleiades (1.5σ) but in disagreement with the distances to HD 23642 (2.2σ) and to the Pleiades derived from Hipparcos parallax observations (4.1σ).

Further observations of HD 23642, to find more accurate dimensions, would provide very precise metal abundance and T_{eff} measurements for both stars. This would reduce the uncertainty in its distance and allow further investigation of the system, which is itself an interesting object due to the metallic-lined nature of the secondary star. Further infrared observations would also allow the use of entirely empirical surface brightness relations in finding an accurate distance to HD 23642.

7 The metallic-lined eclipsing binary WW Aurigae

Towards the end of our spectroscopic observing run using the Isaac Newton Telescope (see section 3.2.1) it became clear that we had significant spare time in the second half of each night during which very few primary targets (dEBs in open clusters) were visible. Five additional targets were selected by myself at the telescope and added to our observing list. One of these, WW Aurigae, was observed many times and it soon became clear that we had acquired an excellent spectroscopic dataset for this system.

Upon further investigation we found that good *UBV* light curves existed for WW Aur in a journal which is not accessible from the NASA ADS internet tool. I followed up a mention of WW Aur in the IAU *Archive of unpublished variable star observations* (Breger 1988) and found that extensive *uvby* observations of WW Aur had been made by Dr. P. B. Etzel for his Master's thesis but not subsequently published.

I decided that a full analysis of WW Aur would be very useful because the available data was of sufficient quality to give mass and radius measurements of excellent accuracy. When studying dEBs it is not easy to know which are most worth further analysis without doing that further analysis. However, the usefulness of measurements of mass and radius generally increases as the accuracy increases, so a study of WW Aur was likely to be interesting and useful.

An additional advantage of WW Aur is that both components exhibit a pronounced metallic-lined character, so accurate masses and radii for these stars may help to increase our understanding of this phenomenon, in particular whether its existence has any effect on the bulk properties of stars (e.g., radius). Metallic-lined A stars are well represented in the compilation of accurate dEB data by Andersen (1991), but the details of the physical processes and particular conditions of occurrence are still not fully understood (section 1.4.4.1).

Table 7.1: Identifications, location, and combined photometric indices for WW Aurigae. **References:** (1) Perryman et al. (1997); (2) Cannon & Pickering (1918); (3) Peters & Hoffleit (1992); (4) Argelander (1903); (5) KK75; (6) Two Micron All Sky Survey (section 1.6.3); (7) Crawford et al. (1972).

	WW Aurigae	Reference
Hipparcos number	HIP 31173	1
Henry Draper number	HD 46052	2
Bright Star Catalogue	HR 2372	3
Bonner Durchmusterung	BD +32°1324	4
α_{2000}	06 32 27.19	1
δ_{2000}	+32 27 17.6	1
Hipparcos parallax (<i>mas</i>)	11.86 ± 1.06	1
Spectral type	A4 m + A5 m	5
B_T	6.036 ± 0.005	1
V_T	5.839 ± 0.005	1
$J_{2\text{MASS}}$	5.498 ± 0.021	6
$H_{2\text{MASS}}$	5.499 ± 0.026	6
$K_{2\text{MASS}}$	5.481 ± 0.021	6
$b - y$	$+0.081 \pm 0.008$	7
m_1	$+0.231 \pm 0.011$	7
c_1	$+0.944 \pm 0.011$	7
β	2.862 ± 0.013	7

7.1 WW Aurigae

WW Aurigae ($P = 2.52$ days, $m_V = 5.8$) is a bright Northern metallic-lined dEB. It has an accurate Hipparcos parallax, so the geometric distance to the system is known to an accuracy of 10% (Table 7.1). Its eclipsing nature was discovered independently by Solviev (1918) and Schwab (1918). Joy (1918) presented spectra which showed lines of both components moving with an orbital period of 2.525 days. Wylie (1923) verified this period photometrically. Dugan (1930) made extensive photometric observations but his investigation was complicated by the slight variability of his comparison star.

Huffer & Kopal (1951) and Piotrowski & Serkowski (1956) undertook photoelectric observations of WW Aur but were both hampered by bright observing conditions.

Etzel (1975, hereafter E75) observed excellent photoelectric light curves in the Strömgren *uvby* passbands, consisting of about one thousand observations in each passband. The light curve analysis code EBOP (section 2.4.1.1) was introduced in E75 and used to derive the photometric elements of WW Aur from the *uvby* light curves.

Kiyokawa & Kitamura (1975, hereafter KK75) published excellent *UBV* light curves of WW Aur and analysed them using a procedure based on rectification (section 2.4.1). Kitamura, Kim & Kiyokawa (1976, hereafter KKK76) published good photographic spectra and derived accurate absolute dimensions of WW Aur by combining their results with those of KK75. The light curves of KK75 were also analysed by Cester et al. (1978) in their program to determine accurate and homogeneous photometric elements of many dEBs with the light curve code WINK (Wood 1971a).

The rotational velocities of the components were found to be 35 and 55 km s⁻¹ from CCD spectra by Abt & Morrell (1995), who also classified the stars as Am (A2,A5,A7) where the bracketed spectral types have been obtained by studying the Ca II K line, Balmer lines, and metallic lines, respectively.

7.2 Observations and data acquisition

7.2.1 Spectroscopic observations

Spectroscopic observations were carried out during the same observing run and using the same observational and data reduction techniques as for V615 Per and V618 Per (section 3.2.1). The spectral window chosen for observation was 4230–4500 Å and 59 spectra were obtained. One additional spectrum was observed around H β (4861 Å) to provide an additional T_{eff} indicator for spectral analysis. The signal to noise ratios per pixel of the observed spectra are about 450. A log of the spectroscopic observations is given in Table 7.2.

Table 7.2: Observing log for the spectroscopic observations of V615 Per and V618 Per.

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
WW Aur	324643	4230–4500	2452568.71912	120	21/10/02	05:12:38
WW Aur	324846	4230–4500	2452569.56375	120	22/10/02	01:28:47
WW Aur	324847	4230–4500	2452569.56533	120	22/10/02	01:31:04
WW Aur	324848	4230–4500	2452569.56691	120	22/10/02	01:33:21
WW Aur	324892	4230–4500	2452569.63472	120	22/10/02	03:10:59
WW Aur	324922	4230–4500	2452569.69819	120	22/10/02	04:42:22
WW Aur	324947	4230–4500	2452569.73041	120	22/10/02	05:28:46
WW Aur	324948	4230–4500	2452569.73200	120	22/10/02	05:31:03
WW Aur	325224	4230–4500	2452570.59939	120	23/10/02	02:19:59
WW Aur	325225	4230–4500	2452570.60098	120	23/10/02	02:22:16
WW Aur	325226	4230–4500	2452570.60256	120	23/10/02	02:24:32
WW Aur	325227	4230–4500	2452570.60414	120	23/10/02	02:26:49
WW Aur	325228	4230–4500	2452570.60572	120	23/10/02	02:29:06
WW Aur	325252	4230–4500	2452570.65503	120	23/10/02	03:40:05
WW Aur	325253	4230–4500	2452570.65661	120	23/10/02	03:42:22
WW Aur	325254	4230–4500	2452570.65819	120	23/10/02	03:44:39
WW Aur	325291	4230–4500	2452570.74037	120	23/10/02	05:42:58
WW Aur	325292	4230–4500	2452570.74195	120	23/10/02	05:45:15
WW Aur	325293	4230–4500	2452570.74353	120	23/10/02	05:47:32
WW Aur	325309	4230–4500	2452570.77810	120	23/10/02	06:37:18
WW Aur	325310	4230–4500	2452570.77968	120	23/10/02	06:39:35
WW Aur	325311	4230–4500	2452570.78127	120	23/10/02	06:41:52
WW Aur	325317	4230–4500	2452570.78782	120	23/10/02	06:51:17
WW Aur	325318	4230–4500	2452570.78939	120	23/10/02	06:53:34
WW Aur	325319	4230–4500	2452570.79098	120	23/10/02	06:55:51
WW Aur	325428	4230–4500	2452571.57055	120	24/10/02	01:38:19
WW Aur	325429	4230–4500	2452571.57213	120	24/10/02	01:40:36
WW Aur	325430	4230–4500	2452571.57371	120	24/10/02	01:42:53
WW Aur	325433	4710–4970	2452571.57781	300	24/10/02	01:48:47
WW Aur	325462	4230–4500	2452571.67001	120	24/10/02	04:01:32

continued

Table 7.3: Observing log for the spectroscopic observations of V615 Per and V618 Per (*continued*).

Target	Spectrum number	Wavelength (Å)	HJD of midpoint	Exposure time (s)	Date	Time
WW Aur	325463	4230–4500	2452571.67159	120	24/10/02	04:03:49
WW Aur	325464	4230–4500	2452571.67318	120	24/10/02	04:06:06
WW Aur	325508	4230–4500	2452571.74282	120	24/10/02	05:46:22
WW Aur	325509	4230–4500	2452571.74440	120	24/10/02	05:48:39
WW Aur	325510	4230–4500	2452571.74599	120	24/10/02	05:50:56
WW Aur	325526	4230–4500	2452571.77319	120	24/10/02	06:30:06
WW Aur	325527	4230–4500	2452571.77477	120	24/10/02	06:32:23
WW Aur	325528	4230–4500	2452571.77636	120	24/10/02	06:34:40
WW Aur	325529	4230–4500	2452571.77794	120	24/10/02	06:36:56
WW Aur	325530	4230–4500	2452571.77952	120	24/10/02	06:39:13
WW Aur	325531	4230–4500	2452571.78135	120	24/10/02	06:41:51
WW Aur	325532	4230–4500	2452571.78294	120	24/10/02	06:44:08
WW Aur	325533	4230–4500	2452571.78452	120	24/10/02	06:46:25
WW Aur	325534	4230–4500	2452571.78610	120	24/10/02	06:48:41
WW Aur	325535	4230–4500	2452571.78768	120	24/10/02	06:50:58
WW Aur	325697	4230–4500	2452572.64077	120	25/10/02	03:19:18
WW Aur	325698	4230–4500	2452572.64235	120	25/10/02	03:21:35
WW Aur	325699	4230–4500	2452572.64393	120	25/10/02	03:23:51
WW Aur	325743	4230–4500	2452572.75008	120	25/10/02	05:56:42
WW Aur	325744	4230–4500	2452572.75167	120	25/10/02	05:58:59
WW Aur	325745	4230–4500	2452572.75325	120	25/10/02	06:01:15
WW Aur	325762	4230–4500	2452572.78461	120	25/10/02	06:46:25
WW Aur	325763	4230–4500	2452572.78621	120	25/10/02	06:48:43
WW Aur	325764	4230–4500	2452572.78781	120	25/10/02	06:51:01
WW Aur	325765	4230–4500	2452572.78950	120	25/10/02	06:53:27
WW Aur	325766	4230–4500	2452572.79113	120	25/10/02	06:55:48
WW Aur	325767	4230–4500	2452572.79277	120	25/10/02	06:58:10
WW Aur	325768	4230–4500	2452572.79438	120	25/10/02	07:00:29
WW Aur	325769	4230–4500	2452572.79599	120	25/10/02	07:02:48

7.2.2 Acquisition of light curves

The photoelectric *uvby* light curves of E75 were obtained between 1973 September and 1974 April using a 41 cm Cassegrain telescope at Mt. Laguna Observatory, USA, a single-channel photometer with a refrigerated 1P21 photomultiplier, and a 36 arcsec diaphragm. Observations were taken in the sequence variable–comparison–sky with integration times of 15 s. The comparison star (HD 46251, spectral type A2 V) was compared to a check star (HD 48272) and no variability in brightness was found.

The three photoelectric light curves observed by KK75 in the Johnson *UBV* passbands each contain approximately 1000 observations, which are clearly tabulated. Rather than risk causing several typographical errors by typing these out by hand, the paper copies of the original work were sufficiently clear to enable me to convert them to electronic format using proprietary optical character recognition software. The photocopied sheets were scanned and analysed using Caere Omnipage Pro¹. Optical character recognition software does not make typographical mistakes but is capable of misinterpreting individual characters. In this case several '3's were mistakenly interpreted as '8's by the software so the resulting data was visually inspected and several errors rectified. The light curves were also inspected graphically to identify any remaining mistakes which were larger than the observational errors; smaller mistakes may not have been spotted but will individually have a negligible impact on the results of the light curve analysis below. A full inspection of a small part of the light curve data showed that very few errors remain, probably significantly fewer than ten in the whole 3000 observations, and that these are all smaller than the observational errors.

7.3 Period determination

Available photoelectric times of minima were collected from the literature, and the orbital ephemeris given in the *General Catalogue of Variable Stars* (Khopolov et al.

¹www.caere.com

Table 7.4: Published times of minimum light of WW Aur and $O - C$ values compared to our ephemeris († rejected from solution due to large $O - C$ value).

References: (1) Huffer & Kopal (1951); (2) Piotrowski & Serkowski (1956); (3) Fitch (1964); (4) Broglia & Lenouvel (1960); (5) Chou (1968); (6) D. B. Wood (1973, private communication); (7) Kristenson (1966); (8) Pohl & Kizilirmak (1966); (9) Robinson & Ashbrook (1968); (10) KK75; (11) Baldwin (1973); (12) Popovici (1968); (13) Pohl & Kizilirmak (1970); (14) H. Lanning (E75); (15) Popovici (1971); (16) Pohl & Kizilirmak (1972); (17) Kizilirmak & Pohl (1974); (18) E75; (19) Ebersberger, Pohl & Kizilirmak (1978); (20) Pohl et al. (1982); (21) Caton, Burns & Hawkins (1991).

Time (HJD – 2 400 000)	Cycle number	$O - C$ (HJD)	Ref.	Time (HJD – 2 400 000)	Cycle number	$O - C$ (HJD)	Ref.
32480.9379	–3758.0	0.0025	1	39134.3610	–1123.0	–0.0006	8
32868.5250	–3604.5	–0.0009	2	39184.8586	–1103.0	–0.0034	9
32888.7263	–3596.5	0.0002	1	39527.0020	–967.5	–0.0001	10
32892.5129	–3595.0	–0.0007	2	39537.1022	–963.5	0.0000	10
32936.6998	–3577.5	–0.0016	1	39550.9895	–958.0	–0.0003	10
32945.5389	–3574.0	–0.0001	2	39556.0398	–956.0	–0.0000	10
32945.5403	–3574.0	0.0013	2	39835.0550	–845.5	0.0005	10
32946.8002	–3573.5	–0.0013	1	39836.3167	–845.0	–0.0003	10
33002.3510	–3551.5	–0.0009	2	39852.7308	–838.5	0.0012	11
33031.3878	–3540.0	–0.0019	2	39857.7790	–836.5	–0.0006	11
33190.4670	–3477.0	0.0011	2	39864.0920	–834.0	–0.0002	10
33209.4042	–3469.5	0.0007	2	39869.1424	–832.0	0.0002	11
33215.7165	–3467.0	0.0004	1	39857.7790	–836.5	–0.0006	10
33225.8159	–3463.0	–0.0003	1	39888.0810	–824.5	0.0011	10
33249.8035	–3453.5	–0.0003	1	40154.4692	–719.0	–0.0002	12
33263.6905	–3448.0	–0.0010	1	40288.298	–666.0	0.0026	13
33292.7299	–3436.5	0.0007	1	40684.7235	–509.0	0.0000	14
33297.7776	–3434.5	–0.0016	1	40885.4635	–429.5	0.0010	15
33358.3816	–3410.5	0.0019	2	41024.3382	–374.5	–0.0004	16
33570.4817	–3326.5	0.0004	2	41399.305	–226.0	0.0010	17
33594.4695	–3317.0	0.0005	2	41945.9707	–9.5	0.0000	18
33599.5173	–3315.0	–0.0017	2	41969.9585	0.0	0.0001	18
33646.2338	–3296.5	0.0019	2	41983.8458	5.5	–0.0002	18
33690.4196	–3279.0	–0.0001	2	42022.9841	21.0	0.0003	18
34470.650	–2970.0	–0.0007	3	42026.7718	22.5	0.0005	18
35845.5222	–2425.5	–0.0016	4	42028.0342	23.0	0.0004	18
36586.616	–2132.0	–0.0010	5	42069.6970	39.5	0.0004	18
36591.6714	–2130.0	0.0044	5	42103.7847	53.0	0.0003	18
37654.7002	–1709.0	–0.0000	6	42117.6725	58.5	0.0005	18
38793.4841	–1258.0	0.0001	7	42141.6602	68.0	0.0005	18
38798.5335	–1256.0	–0.0005	7	43477.3949	597.0	–0.0001	19
38802.3215	–1254.5	–0.0000	7	44256.3632	905.5	–0.0002	20
38807.3713	–1252.5	–0.0003	7	44925.4902	1170.5	–0.0034	20
38831.3589	–1243.0	–0.0003	7	46840.7308 †	1929.0	0.0112	21

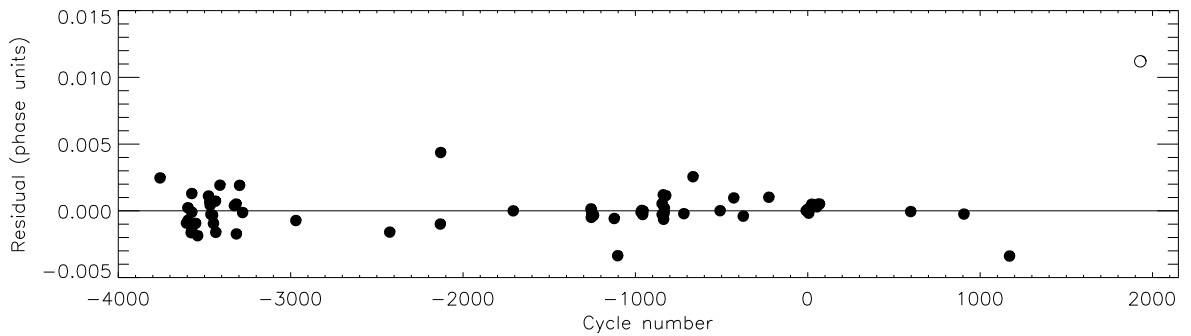


Figure 7.1: Residuals (in units of the orbital period) of the ephemeris which best fits the observed times of minima. The open circle at cycle 1929.0 represents a datapoint which was rejected from the fit.

1999) was used to determine the preliminary cycle number of each minimum (reference time of minimum HJD 2 432 945.5393 and period 2.52501922 days). Equal weights were given to all observations as very few have quoted uncertainties. A straight line was fitted to the resulting cycle numbers and times of minima (Table 7.4) by χ^2 minimisation, using the first time of primary minimum from E75 as the reference time. One time of minimum had a large residual so was rejected. The resulting ephemeris is:

$$\text{Min I} = \text{HJD } 2\,441\,969.95837(23) + 2.52501941(10) \times E \quad (7.1)$$

The residuals of the fit are plotted in Figure 7.1 and give no indication of any form of period change. The root-mean-square of the residuals is 0.0012 days.

7.4 Spectroscopic orbits

Radial velocities were measured from the observed spectra using the two-dimensional cross-correlation algorithm TODCOR (section 2.2.3.3). The metallic-lined nature of both components of WW Aur means that care must be taken to select appropriate template spectra which match the true spectra of the stars as closely as possible. For this reason

Table 7.5: Sample RVs and $O-C$ values (in km s^{-1}) for WW Aur (continued on next page). The results given in this table were calculated using TODCOR and the standard star templates HD 39945 (for the primary star) and HD 32115 (for the secondary star). As discussed in the text, these velocities are for only one of the combinations of templates for which spectroscopic orbits were calculated, so are only a small part of the information used to calculate the final spectroscopic orbit for WW Aur. They are provided for convenience and are plotted in Figure 7.2. Whilst the systemic velocities of the two stars were adjusted in Figure 7.2, in this table the velocities are relative to the velocities of the template spectra. The weights given in column “Wt” were derived from the amount of light collected in that observation and were used in the SBOP analysis.

HJD – 2 400 000	Primary velocity	$O - C$	Secondary velocity	$O - C$	Wt
52568.7191	–22.7	–3.3	–44.2	0.5	0.7
52569.5637	82.7	–2.4	–158.5	–0.8	1.3
52569.5653	83.3	–1.6	–157.6	–0.1	1.6
52569.5669	83.2	–1.5	–156.8	0.5	1.8
52569.6347	70.8	–2.7	–146.4	–1.2	1.1
52569.6982	58.1	–2.6	–131.6	–0.3	1.6
52569.7304	51.4	–2.0	–124.7	–1.2	1.4
52569.7320	51.4	–1.7	–123.8	–0.7	1.2
52570.5994	–136.8	–2.8	77.1	–2.2	1.2
52570.6010	–132.5	1.5	82.1	2.8	0.7
52570.6026	–134.2	–0.1	80.1	0.7	1.1
52570.6041	–133.9	0.1	80.4	1.0	1.1
52570.6057	–134.5	–0.4	79.5	0.1	1.1
52570.6550	–134.8	–1.0	78.9	–0.2	1.4
52570.6566	–133.9	–0.3	80.1	1.1	1.2
52570.6582	–133.4	0.3	80.4	1.4	1.2
52570.7404	–129.3	–0.2	74.9	0.9	1.0
52570.7420	–129.9	–1.0	74.3	0.5	1.3
52570.7435	–129.6	–0.8	74.0	0.3	1.5
52570.7781	–124.4	1.0	71.4	1.4	1.6
52570.7797	–123.5	1.6	71.4	1.6	1.4
52570.7813	–122.9	2.1	71.4	1.8	1.2
52570.7878	–120.9	3.4	71.7	2.9	0.5

HJD - 2 400 000	Primary velocity	<i>O-C</i>	Secondary velocity	<i>O-C</i>	Wt
52570.7894	-120.9	3.2	72.2	3.7	0.4
52570.7910	-123.8	0.1	67.9	-0.4	1.2
52571.5705	63.5	-2.8	-137.4	-0.0	0.8
52571.5721	65.0	-1.6	-136.8	0.9	0.9
52571.5737	66.5	-0.5	-136.2	1.8	0.6
52571.6700	86.1	2.2	-152.4	4.0	0.8
52571.6716	85.8	1.7	-153.6	3.1	0.6
52571.6732	87.0	2.6	-152.2	4.8	0.4
52571.7428	95.1	2.2	-165.2	1.0	1.1
52571.7444	94.8	1.7	-165.8	0.6	1.0
52571.7460	95.1	1.9	-166.1	0.5	1.2
52571.7732	94.8	1.7	-165.8	0.6	1.0
52571.7748	96.3	0.5	-170.2	-0.9	1.2
52571.7764	96.9	1.0	-169.8	-0.4	1.3
52571.7779	97.5	1.4	-170.1	-0.5	1.2
52571.7795	97.8	1.6	-170.2	-0.5	1.1
52571.7813	98.0	1.7	-169.6	0.3	0.8
52571.7829	98.6	2.1	-169.3	0.7	0.8
52571.7845	98.9	2.3	-169.6	0.5	0.6
52571.7861	98.9	2.2	-169.6	0.6	0.5
52571.7877	98.9	2.1	-169.8	0.5	0.4
52572.6408	-52.6	0.6	-10.6	-2.4	0.9
52572.6424	-52.9	0.7	-10.0	-2.3	0.7
52572.6439	-53.7	0.3	-9.4	-2.2	0.9
52572.7501	-81.5	0.2	22.4	-0.3	0.4
52572.7517	-81.8	0.3	22.7	-0.5	0.4
52572.7533	-82.1	0.4	23.3	-0.3	0.3
52572.7846	-89.3	0.5	28.8	-2.8	0.3
52572.7862	-90.2	0.0	28.8	-3.2	0.6
52572.7878	-90.2	0.4	29.4	-3.0	0.5
52572.7895	-91.0	-0.1	28.8	-4.0	0.4
52572.7911	-90.7	0.6	29.7	-3.5	0.6
52572.7928	-90.7	1.0	30.3	-3.3	0.3
52572.7944	-92.2	-0.1	29.7	-4.3	0.6
52572.7960	-92.5	-0.1	30.0	-4.4	0.4

Table 7.6: Parameters of the spectroscopic orbit derived for WW Aur using TODCOR with observed standard star spectra for templates. The systemic velocity was calculated by cross-correlating against a synthetic spectrum to determine an alternative spectroscopic orbit.

	Primary	Secondary
Semi-amplitude K (km s^{-1})	116.81 ± 0.23	126.49 ± 0.28
Systemic velocity (km s^{-1})	-9.10 ± 0.25	-7.84 ± 0.32
Mass ratio q	0.9235 ± 0.0027	
$a \sin i$ ($R_{\odot}$)	12.138 ± 0.018	
$M \sin^3 i$ ($M_{\odot}$)	1.959 ± 0.007	1.809 ± 0.007

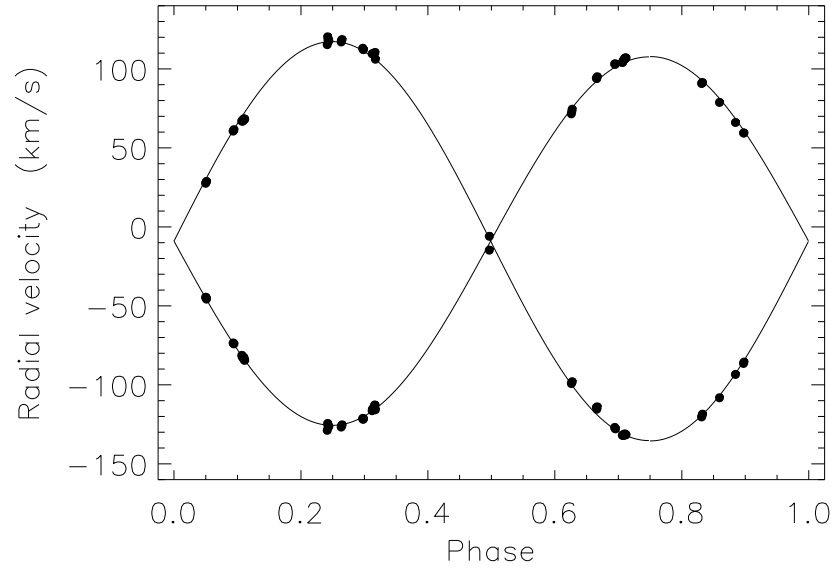


Figure 7.2: Spectroscopic orbit for WW Aur from an SBOP fit to RVs from TODCOR.

synthetic spectra may not be the best choice as undetectable systematic errors could occur from problems such as missing or poorly matching spectral lines.

Spectra from seven standard stars with spectral types between A0 and F5 (luminosity classes V and IV) were selected and used as templates in the TODCOR analysis. The broad $H\gamma$ 4340 Å line was masked in all spectra. Spectroscopic orbits were derived separately for each star and for each combination of two template spectra (for each star) using SBOP (section 2.2.4.1). Circular orbits were assumed as the light curves and initial spectroscopic orbital fits showed no evidence of orbital eccentricity.

Excellent spectroscopic orbits were found for fifteen different combinations of template spectra; the results of other template combinations are in good agreement but with significantly increased standard errors. The final spectroscopic orbital parameters have been derived by calculating the mean and standard deviation of the parameters of the fifteen excellent spectroscopic orbits, and are given in Table 7.6. The final standard deviation is very similar to the individual standard errors calculated by SBOP. As the final parameter values are the means of the fifteen individual orbit solutions, systematic errors due to individual mismatches in T_{eff} , metal abundance and rotational velocity should be negligible. The velocity semiamplitudes are in reasonable agreement with the values found by KKK76: $K_A = 115.62 \pm 0.45 \text{ km s}^{-1}$ and $K_B = 127.73 \pm 0.68 \text{ km s}^{-1}$.

For illustrative reasons the best individual spectroscopic orbit has been selected. The RVs are given in Table 7.5 and the orbit has been plotted in Figure 7.2. The template spectra used to generate this orbit were HD 39945 (spectral type A5 V) for the primary star and HD 32115 (A8 IV) for the secondary star. The velocity semiamplitudes found using these templates are $K_A = 117.03 \pm 0.25 \text{ km s}^{-1}$ and $K_B = 126.66 \pm 0.25 \text{ km s}^{-1}$, where the standard errors from SBOP are given. Please note that these values are not adopted as the final result in this section but are provided for illustrative purposes only.

Systemic velocities have been determined by calculating spectroscopic orbits for each component of WW Aur using TODCOR and synthetic template spectra. The synthetic spectra were constructed using UCLSYN (section 1.4.3.2), so the systemic velocities quoted here are based on terrestrial laboratory spectral line wavelengths.

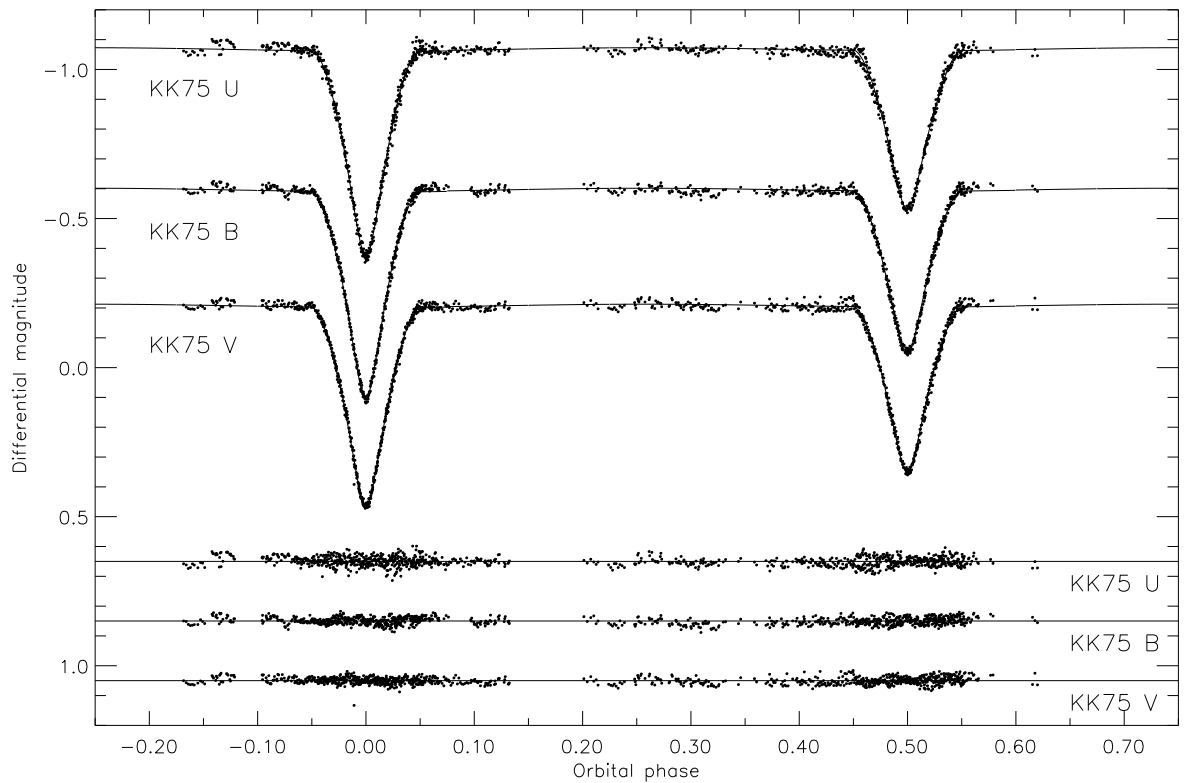


Figure 7.3: The KK75 differential light curves of WW Aur, compared to the best-fitting light curves found using JKTEBOP. The residuals of the fits are plotted with magnitude offsets for clarity.

7.5 Light curve analysis

There are seven good light curves of WW Aur, all obtained using photoelectric photometers. Three light curves, of approximately one thousand observations in each of the Johnson *UBV* passbands, were observed by KK75. Excellent light curves in the Strömgren *uvby* passbands were obtained by E75.

The light curves were analysed using JKTEBOP (section 3.7.1) modified to fit for the sum and the ratio of the stellar radii, rather than the radii directly. Initial values for the passband-specific linear limb darkening coefficients, u_A and u_B , were taken from

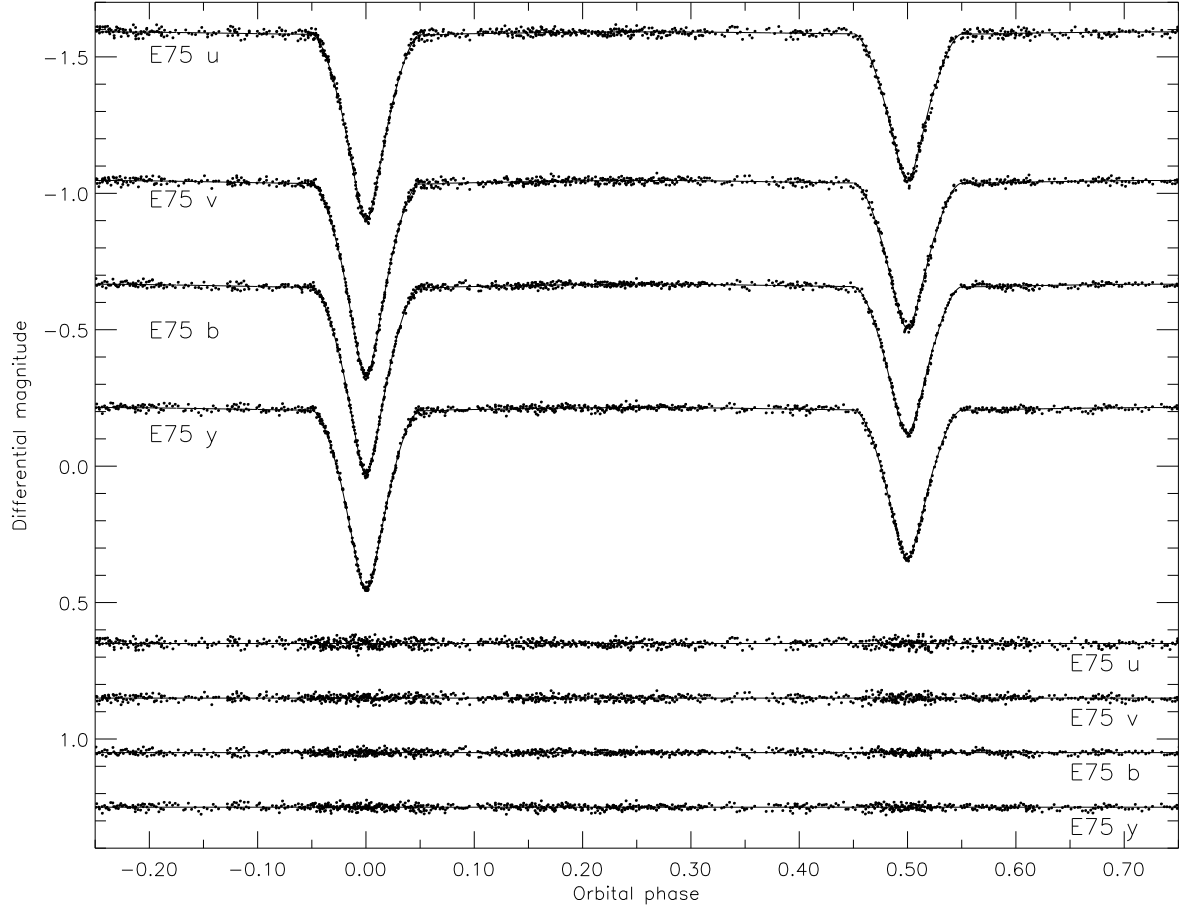


Figure 7.4: The E75 differential light curves of WW Aur, compared to the best-fitting light curves found using JKTEBOP. The residuals of the fits are plotted with magnitude offsets for clarity.

Table 7.7: Results of the light curve analysis for WW Aurigae. The adopted values are the weighted means and 1σ uncertainties of the results for the E75 and KK75 light curves.

	E75				KK75			Adopted values
	u	v	b	y	U	B	V	
Number of datapoints	903	962	902	981	999	980	1058	6785
Observational scatter (mag)	0.012	0.010	0.008	0.009	0.016	0.012	0.012	
Fractional total radii of the stars ($r_A + r_B$)	$\pm$ 0.3084	$\pm$ 0.3101	$\pm$ 0.3100	$\pm$ 0.3099	$\pm$ 0.3084	$\pm$ 0.3092	$\pm$ 0.3118	$\pm$ 0.3099
Ratio of the radii (k)	$\pm$ 0.0012	$\pm$ 0.0008	$\pm$ 0.0009	$\pm$ 0.0009	$\pm$ 0.0011	$\pm$ 0.0009	$\pm$ 0.0009	$\pm$ 0.0004
	$\pm$ 0.978	$\pm$ 0.967	$\pm$ 0.915	$\pm$ 0.968	$\pm$ 0.982	$\pm$ 0.950	$\pm$ 0.996	$\pm$ 0.953
Fractional radius of primary star (r_A)	$\pm$ 0.057	$\pm$ 0.023	$\pm$ 0.020	$\pm$ 0.034	$\pm$ 0.032	$\pm$ 0.027	$\pm$ 0.041	$\pm$ 0.011
	$\pm$ 0.1559	$\pm$ 0.1577	$\pm$ 0.1619	$\pm$ 0.1575	$\pm$ 0.1556	$\pm$ 0.1585	$\pm$ 0.1562	$\pm$ 0.1586
Fractional radius of secondary star (r_B)	$\pm$ 0.0046	$\pm$ 0.0019	$\pm$ 0.0017	$\pm$ 0.0028	$\pm$ 0.0027	$\pm$ 0.0023	$\pm$ 0.0034	$\pm$ 0.0009
	$\pm$ 0.1525	$\pm$ 0.1524	$\pm$ 0.1481	$\pm$ 0.1524	$\pm$ 0.1528	$\pm$ 0.1506	$\pm$ 0.1556	$\pm$ 0.1515
Orbital inclination ($^\circ$) (i)	$\pm$ 0.0041	$\pm$ 0.0018	$\pm$ 0.0018	$\pm$ 0.0026	$\pm$ 0.0023	$\pm$ 0.0022	$\pm$ 0.0029	$\pm$ 0.0009
	$\pm$ 87.66	$\pm$ 87.51	$\pm$ 87.59	$\pm$ 87.37	$\pm$ 87.46	$\pm$ 87.71	$\pm$ 87.64	$\pm$ 87.55
Surface brightness ratio (J)	$\pm$ 0.20	$\pm$ 0.10	$\pm$ 0.14	$\pm$ 0.11	$\pm$ 0.11	$\pm$ 0.11	$\pm$ 0.11	$\pm$ 0.04
	$\pm$ 0.862	$\pm$ 0.828	$\pm$ 0.861	$\pm$ 0.884	$\pm$ 0.761	$\pm$ 0.817	$\pm$ 0.869	$\pm$ 0.869
Primary limb darkening coefficient (u_A)	$\pm$ 0.041	$\pm$ 0.030	$\pm$ 0.029	$\pm$ 0.032	$\pm$ 0.026	$\pm$ 0.027	$\pm$ 0.027	$\pm$ 0.027
	$\pm$ 0.369	$\pm$ 0.675	$\pm$ 0.643	$\pm$ 0.533	$\pm$ 0.709	$\pm$ 0.616	$\pm$ 0.416	$\pm$ 0.416
Secondary limb darkening coefficient (u_B)	$\pm$ 0.088	$\pm$ 0.057	$\pm$ 0.058	$\pm$ 0.065	$\pm$ 0.059	$\pm$ 0.056	$\pm$ 0.060	$\pm$ 0.060
	$\pm$ 0.457	$\pm$ 0.722	$\pm$ 0.658	$\pm$ 0.575	$\pm$ 0.452	$\pm$ 0.512	$\pm$ 0.418	$\pm$ 0.418
Light ratio (assuming $k = 0.953 \pm 0.011$) ($\frac{I_B}{I_A}$)	$\pm$ 0.126	$\pm$ 0.082	$\pm$ 0.074	$\pm$ 0.086	$\pm$ 0.096	$\pm$ 0.078	$\pm$ 0.083	$\pm$ 0.083
	$\pm$ 0.757	$\pm$ 0.737	$\pm$ 0.776	$\pm$ 0.790	$\pm$ 0.768	$\pm$ 0.774	$\pm$ 0.788	$\pm$ 0.788
	$\pm$ 0.019	$\pm$ 0.019	$\pm$ 0.020	$\pm$ 0.020	$\pm$ 0.020	$\pm$ 0.019	$\pm$ 0.019	$\pm$ 0.019

Van Hamme (1993). The gravity darkening exponents, β_1 , were fixed at 1.0 (Claret 1998) and the mass ratio was fixed at the spectroscopic value. Investigations showed that the light curves displayed negligible eccentricity and third light so these quantities were fixed at zero for the final solutions.

The seven light curves are of sufficient quality to directly solve for the limb darkening coefficients, stellar radii, orbital inclination and surface brightness ratio. The results are given in Table 7.7 and the best fits are plotted in Figures 7.3 and Figure 7.4.

A solution was also obtained for the light curve obtained by Huffer & Kopal (1951) using an unfiltered photoelectric photometer equipped with two different amplifiers. This solution is in very poor agreement with the other solutions and the data are not homogeneous. This light curve was therefore not considered further and is not included in the results of our analysis.

7.5.1 Monte Carlo analysis

The primary radius (r_A), secondary radius (r_B), orbital inclination (i), surface brightness ratio (J) and limb darkening coefficients (u_A and u_B) were investigated using Monte Carlo simulations for the seven available light curves and the best-fitting values with 1σ uncertainties are given in Table 7.7. Note that the uncertainties estimated by the use of Monte Carlo simulations are in excellent agreement with the uncertainties found by comparing the values for common parameters found from the solution of different light curves. This demonstrates that the Monte Carlo approach used here provides a very reliable way in which to estimate the uncertainties of parameters derived by analysing light curves.

Figure 7.5 shows some results from the Monte Carlo analysis for the y and V passband light curve solutions, concentrating on the combinations of parameters which show significant correlations or are otherwise interesting.

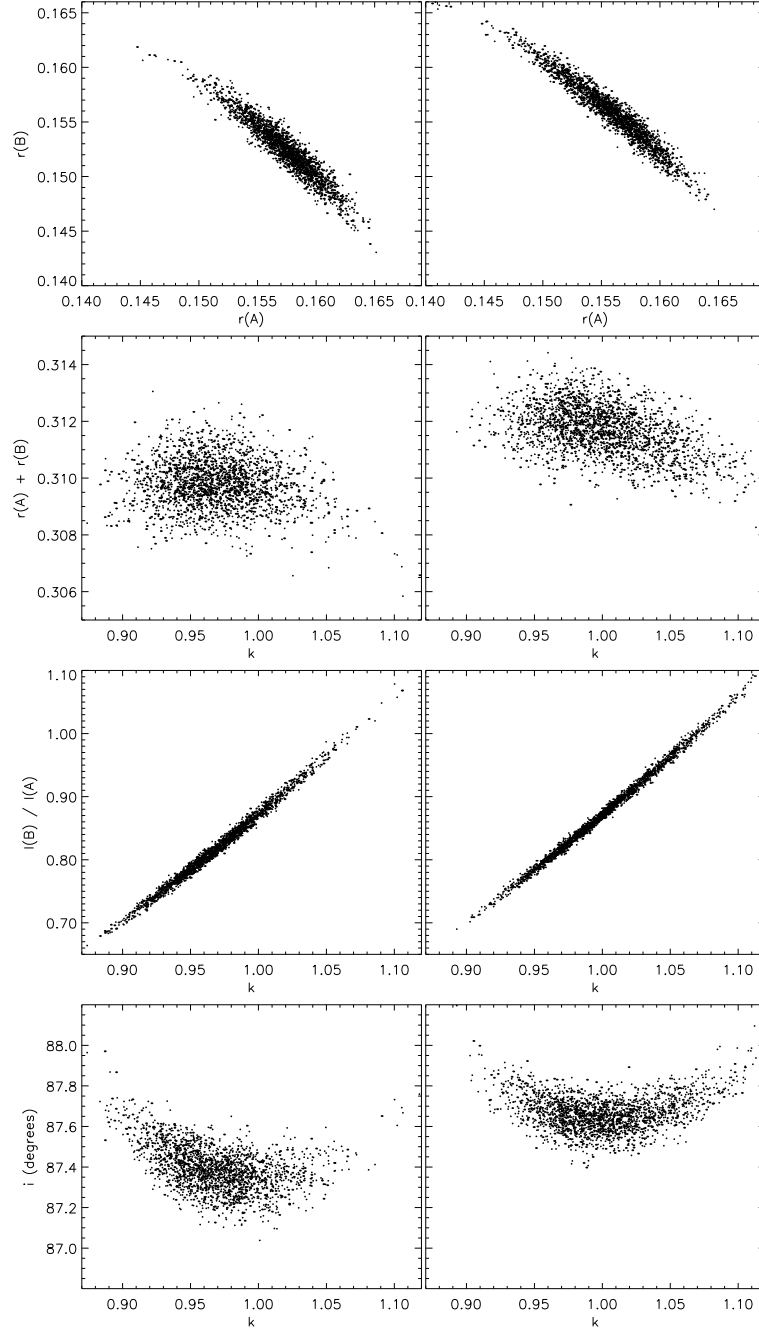


Figure 7.5: Sample distributions of the best-fitting parameters evaluated during the Monte Carlo analysis. The parameter symbols are as in Table 7.7. The y (left-hand column) and V (right-hand column) results are plotted as these passbands have a similar central wavelength but the light curves are from different sources.

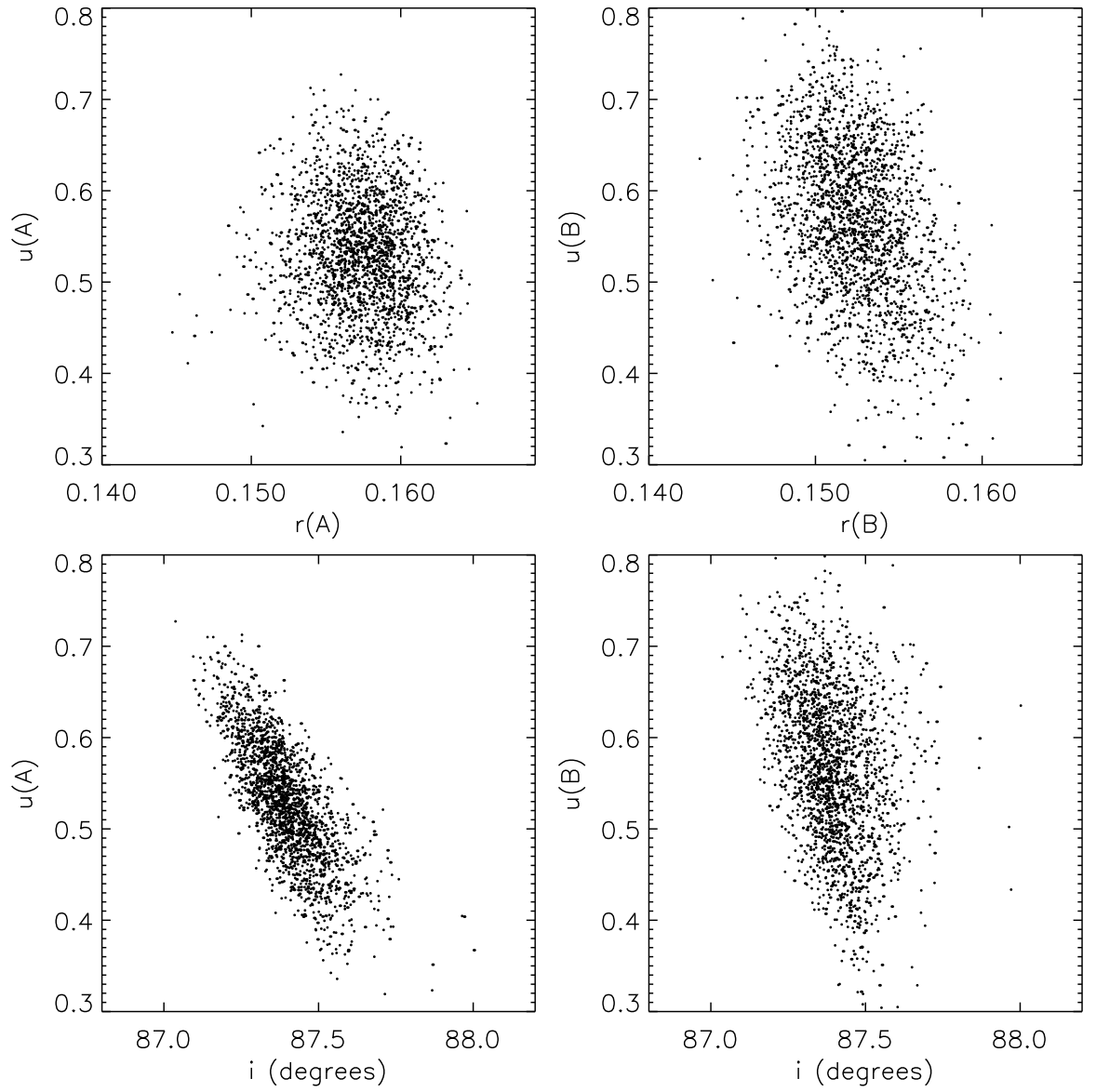


Figure 7.6: As Figure 7.5 but concentrating on the limb darkening in the y passband light curve (all panels).

7.5.2 Limb darkening coefficients

As the limb darkening coefficients were directly fitted in the light curve analysis of WW Aur, it is of interest to compare the best-fitting coefficients with theoretical ones

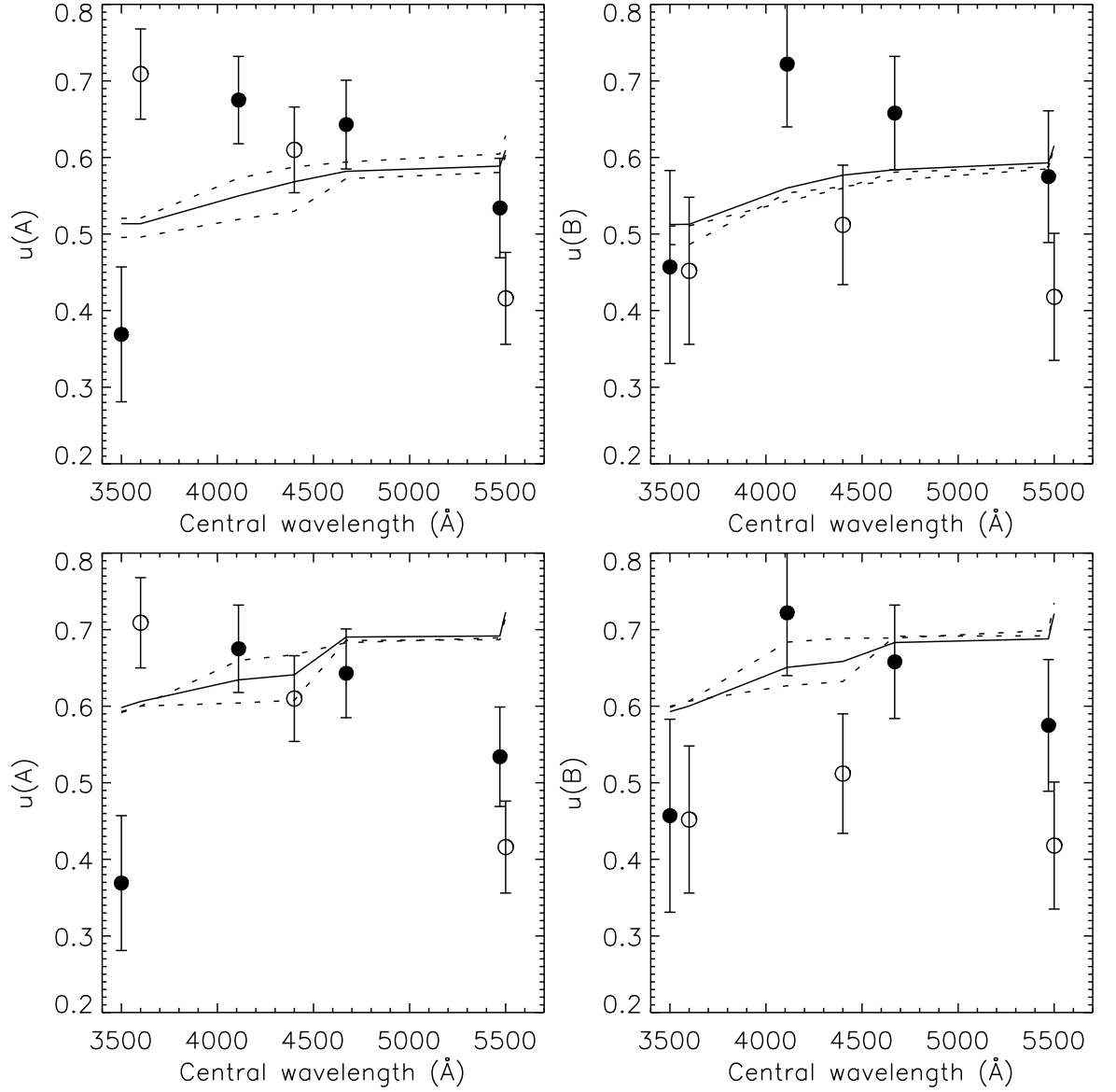


Figure 7.7: The variation of the fitted limb darkening coefficients for the different passbands. The coefficients of the primary star are shown in the left-hand panels whilst those for the secondary are shown in the right-hand panels. Circles represent the coefficients found for WW Aur against the central wavelength of the passband used to observe that light curve. Filled circles represent the *uvby* passbands and open circles the *UBV* passbands. In the upper panels the theoretical coefficients of Van Hamme (1993) have been plotted using a solid line (for the observed T_{eff} s of the stars) and dashed lines (for the T_{eff} s plus or minus their uncertainty). In the lower panels the coefficients of Claret (2000), for a metallicity of $[\frac{M}{H}] = +0.5$, have been plotted.

derived using model atmospheres. Figure 7.6 shows some results of the Monte Carlo analysis, concentrating on the limb darkening coefficient values found for the y light curve. The coefficients appear to be only very weakly correlated with the other fitted light curve parameters, although some correlation is noticeable for the ratio of the radii. Somewhat surprisingly, the coefficients are almost uncorrelated with the radii of the stars.

Figure 7.7 compares the observed linear limb darkening coefficients to coefficients calculated using model atmospheres by Van Hamme (1993) and Claret (2000). Coefficients for a high metallicity of $[\frac{M}{H}] = +0.5$ have been chosen from Claret (2000); the coefficients of Van Hamme (1993) are available only for a solar chemical composition. Comparisons have been made at the central wavelengths of the *uvby* (Strömgren 1963) and *UBV* (Moro & Munari 2000) passbands. Bilinear interpolation has been used to derive theoretical coefficients for the T_{eff} s and surface gravities of the two stars from the tabulated coefficients; these are plotted for the temperatures of the two stars (see next section) and for the temperatures plus or minus their uncertainties.

The agreement between the observed limb darkening coefficients and the values derived using theoretical model atmospheres is generally reasonable, although the Claret (2000) coefficients are larger than those of Van Hamme (1993) and have slightly worse agreement with the observations. It is important to remember, however, that the linear limb darkening law represents the limb darkening of stars less well than other, more complex, limb darkening laws (Van Hamme 1993).

7.5.3 Confidence in the photometric solution

The best-fitting ratio of the radii of a dEB can depend on the program used to analyse the light curve (J. V. Clausen, 2004, private communication), particularly if the light curve is not definitive. This is due to the different representations of the shapes of the stars and the treatment of surface phenomena such as limb darkening. The y light curve of WW Aur has also been analysed by Dr. J. V. Clausen using the WINK code. This photometric solution is in good agreement with the JKTEBOP results, suggesting

Table 7.8: Comparison between published determinations of the photometric parameters of WW Aur and the results obtained in this section.

Parameter	Etzel (1975)	Cester et al. (1978)	KK75	This study
r_A	0.1576 ± 0.0026	0.1623 ± 0.0012	0.1551 ± 0.0012	0.1586 ± 0.0009
r_B	0.1524 ± 0.0026	0.1511 ± 0.0030	0.1546 ± 0.0014	0.1515 ± 0.0009
i ($^\circ$)	87.58 ± 0.11	87.43 ± 0.33	87.556 ± 0.018	87.55 ± 0.04

that any systematic effects present in the JKTEBOP solution are negligible.

It is also useful to compare the photometric solution found here with the results of Etzel (1975). Etzel’s solution of the *uvby* data, using the original version of the EBOP code, is in excellent agreement with that found here. As JKTEBOP is a heavily modified version of EBOP, this provides confirmation that the modifications have not adversely affected our results. The original solution of the *UBV* light curves (KK75) was obtained using the Russell-Merrill method (section 2.4.1; Russell & Merrill 1952) and is in good agreement with the results presented here.

The light ratios found from the light curves are in good agreement with a light ratio obtained from spectral analysis (see below). This is a useful consistency check but is of limited importance here because the light ratio, which was obtained from the $H\beta$ line due to the metallic-lined nature of the spectra, is of low precision.

The results obtained in this section have been compared to previously published determinations of the photometric parameters of WW Aur in Table 7.8.

7.5.4 Photometric indices

The ratio of the radii determined from analysis of the seven light curves is $k = 0.953 \pm 0.011$. This ratio of the radii was used to determine the light ratios of WW Aur in the *uvby* and *UBV* passbands (Table 7.7). Strömgren photometric indices were then found for the two stars from the *uvby* light ratios and the indices of the overall system (Table 7.1). The reason for finding the light ratios using the same ratio of the radii for

Table 7.9: *uvby* photometry and atmospheric parameters for the individual stars (A and B) and for the combined system (AB).

	$b - y$	m_1	c_1	T_{eff}	$\log g$
A	0.081 ± 0.008	0.231 ± 0.011	0.944 ± 0.011	8210 ± 120	4.20 ± 0.05
B	0.073 ± 0.019	0.215 ± 0.032	0.981 ± 0.031	8280 ± 300	4.13 ± 0.12
AB	0.092 ± 0.023	0.252 ± 0.040	0.896 ± 0.041	8120 ± 340	4.29 ± 0.15

each light curve is that the resulting values are more directly comparable to each other as the statistical variation of the individual light curve solutions has been removed.

The resulting Strömgren indices are given in Table 7.9. The uncertainties in these indices have been calculated by adding in quadrature the effects of perturbing each input quantity by its own uncertainty. We have assumed that the *uvby* passbands used by E75 provided a good approximation to the Strömgren photometric system.

7.6 Effective temperature determination

The work in this section was undertaken by Dr. B. Smalley and is included here for completeness.

Fundamental values for the T_{eff} s of the components of WW Aur have previously been determined by Smalley et al. (2002) from the Hipparcos parallax of the system and ultraviolet, optical and infrared fluxes. We have repeated their analysis using our new values for the stellar radii and *V*-passband magnitude difference. We obtain T_{eff} s of 7960 ± 420 K and 7670 ± 410 K for the primary and secondary stars, respectively. These are only slightly different from those obtained previously. The uncertainty in the Hipparcos parallax for WW Aur is the main contributor to the error determination, with the lack of high-quality optical fluxes contributing to a slightly lesser extent.

These fundamental values are in agreement with those obtained from $\text{H}\alpha$ and $\text{H}\beta$ profiles (Smalley et al. 2002). Ribas et al. (1998) determined the T_{eff} s of the

components of twenty dEBs from consideration of their Hipparcos parallaxes, apparent magnitudes and radii. The T_{eff} s found by Ribas et al. for WW Aur are 8180 ± 425 K and 7766 ± 420 K, in good agreement with the values found by us using our updated astrophysical properties of the system.

Strömgren *uvby* photometry for the combined system and inferred values for the individual components (based on the light ratios determined in section 7.5) are given in Table 7.9, along with T_{eff} s and surface gravities obtained from the solar-composition Canuto & Mazzitelli (1991, 1992) grids of *uvby* colours (Smalley & Kupka 1997). These imply slightly hotter T_{eff} s and also a smaller difference between the two components. However, the fundamental values are consistent to within the uncertainties and thus will be preferred.

A light ratio was also obtained using the $H\beta$ line, which is not significantly affected by spectral peculiarities, to provide an external check on the accuracy of the light curve solution. Synthetic spectra were calculated using UCLSYN (section 1.4.3.2). The spectra were rotationally broadened as necessary and instrumental broadening was applied to match the resolution of the observations. Comparison between synthetic spectra and observed spectra of WW Aur gave a light ratio of $\frac{l_2}{l_1} = 0.75 \pm 0.10$, which is in good agreement with the light ratios found by analysing the light curves.

Projected rotational velocities of 35 ± 3 and 37 ± 3 km s⁻¹ were obtained from the sodium D lines of WW Aur in archival MUSICOS spectra (Catala et al. 1993) and converted to equatorial rotational velocities using the orbital inclination found in Section 7.5. These rotational velocities are in excellent agreement with the velocities determined by KKK76.

7.7 Absolute dimensions

The absolute dimensions of WW Aur, found from our spectroscopic and photometric analysis, are given in Table 7.10. The results of our analyses are in good agreement with those previously found by KK75, KKK76 and E75. The light ratios found from

Table 7.10: The absolute dimensions and related quantities determined for the dEB WW Aur. V_{eq} and V_{synch} are the observed equatorial and calculated synchronous rotational velocities, respectively.

	WW Aur A	WW Aur B
Mass ($M_{\odot}$)	1.964 ± 0.007	1.814 ± 0.007
Radius ($R_{\odot}$)	1.927 ± 0.011	1.841 ± 0.011
$\log g$ (cm s^{-2})	4.162 ± 0.007	4.167 ± 0.007
T_{eff} (K)	7960 ± 420	7670 ± 410
$\log(L/L_{\odot})$	1.129 ± 0.092	1.023 ± 0.093
V_{eq} (km s^{-1})	35 ± 3	37 ± 3
V_{synch} (km s^{-1})	38.62 ± 0.23	36.90 ± 0.23

the light curve analysis are also in good agreement with a spectroscopic light ratio obtained from the $H\beta$ line.

Care has been taken to avoid the use of theoretical calculations in any part of our analysis. Our spectroscopic orbit was found by cross-correlating the spectra of WW Aur against observed spectra of a range of standard stars. In our photometric analysis, limb darkening coefficients were included as free parameters and the gravity darkening exponent values, which were adopted from theoretical analyses, make a negligible difference to the results. The values for the masses and radii of the components of WW Aur can therefore be considered to be entirely empirical.

The T_{eff} s of the components were determined using the Hipparcos parallax of WW Aur and light ratios from our photometric analysis. This method does have a very weak dependence on theoretical calculations (Smalley et al. 2002) but provides T_{eff} values which are almost fundamental in character.

7.7.1 Tidal evolution

The work in this section was undertaken by Dr. A. Claret and is included here for completeness.

WW Aur has a circular orbit and the rotational velocities of both components are synchronous to within the observational uncertainties, so a consideration of theories of tidal evolution is interesting. The timescales for orbital circularisation and rotational synchronisation due to tidal effects have been calculated using the theory of Zahn (1977, 1989) and the hydrodynamical mechanism of Tassoul (1987, 1988). The computations were performed using the same method as in Claret, Giménez & Cunha (1995) and Claret & Cunha (1997). For the theory of Zahn, the critical times of orbital circularization for the system and rotational synchronisation for the primary and secondary components of WW Aur are 1380, 1120 and 1280 Myr, respectively. For the theory of Tassoul, the critical times are 250, 21 and 22 Myr, respectively. The Tassoul timescales are much shorter than the Zahn timescales, as is usually found in similar studies. The physical basis of the Tassoul theory remains controversial (Claret & Cunha 1997).

7.8 Comparison with theoretical models

The masses and radii of the component stars of WW Aur are known to accuracies better than 0.6%. These were initially compared to predictions of the Granada (section 1.3.2.1), Geneva (section 1.3.2.2), Padova (section 1.3.2.3) and Cambridge (section 1.3.2.4) theoretical stellar evolutionary models. It was assumed that the two stars have the same age and chemical composition. The mass–radius relation of WW Aur is very shallow (the mass ratio is 0.92 and the ratio of the radii is 0.95) and could not be matched by predictions for any of the chemical compositions available in the model sets listed above. Predictions from the Grenoble models (Siess, Dufour & Forestini 2000) were then used to investigate whether one or both of the stars of WW Aur is in the PMS phase, but we remained unable to find a match to the observations.

Further investigation using theoretical models calculated by Dr. A. Claret (see Claret 2004 for details) revealed that an acceptable match to the masses and radii of WW Aur could be obtained for a very high metal abundance of $Z = 0.06$ and an age between 77 and 107 Myr (Figure 7.8). Models were calculated for the observed masses

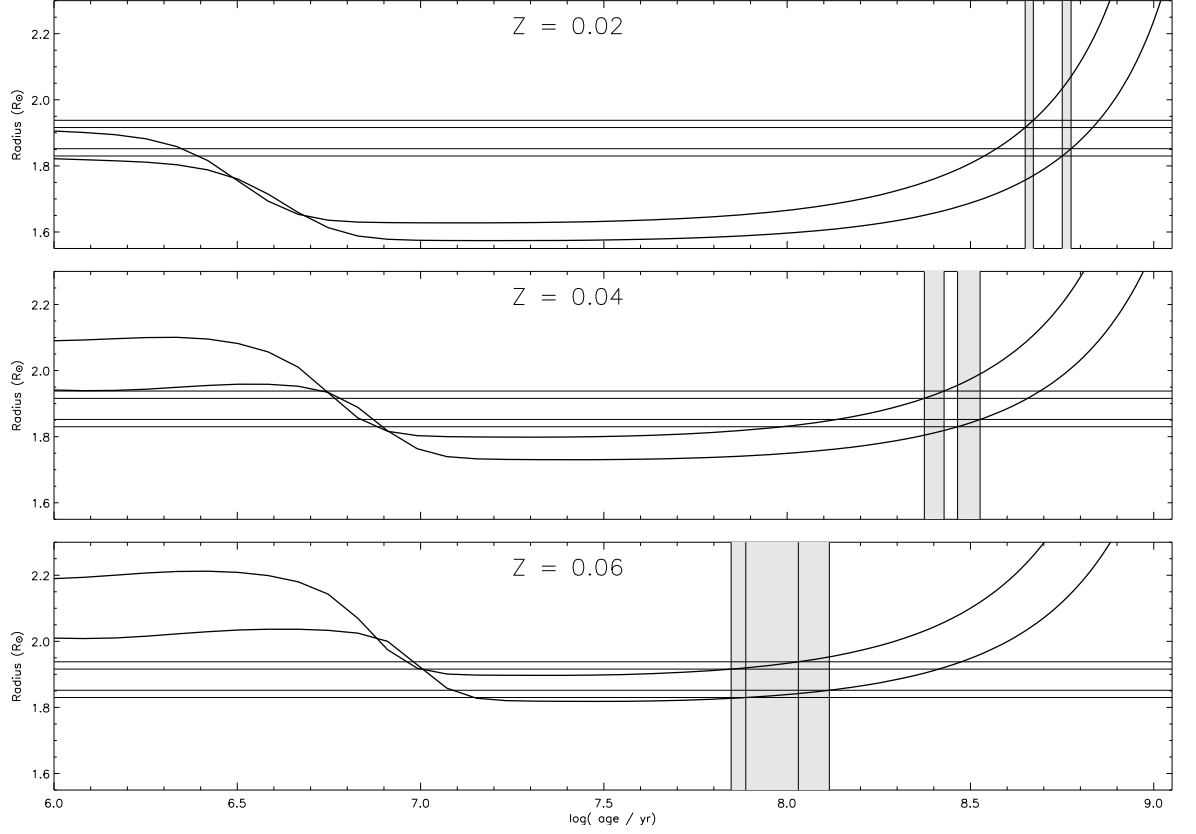


Figure 7.8: Comparison between the radii of the components of WW Aur and the predictions of the Claret (2004) theoretical stellar evolutionary models. Models have been calculated for the observed masses of the components of WW Aur and the evolution of the stellar radii are shown for metal abundances of $Z = 0.02$, 0.04 and 0.06 . The lines of constant radius show the observed radii of WW Aur perturbed by their 1σ uncertainties. Thick lines represent theoretical predictions, and the ages where these match the observed radii are shown by shaded areas.

of the stars, and a helium abundance of $Y = 0.36$ was adopted from the standard chemical enrichment law (section 1.1.1.3) with $\Delta Y/\Delta Z = 2$. The observed T_{eff} s of the components of WW Aur are in agreement with the model predictions for this metal abundance and age, although this is of low significance because the empirical T_{eff} s are quite uncertain. It is possible to match the properties of the stars using models with fractional metal abundances between about 0.055 and 0.065.

An acceptable match to the observed properties of WW Aur can also be found for an age of about 1 Myr and an approximately solar chemical composition, but the age range over which the match is acceptable is extremely small. These results are shown in Figure 7.8.

7.9 Discussion

The component stars of WW Aur are peculiar, both because of their metallic-lined nature and because current theoretical evolutionary models can only match their masses and radii for a high metal abundance of $Z = 0.06$. The strong metallic spectral lines do appear to be caused by the Am phenomenon, and not just by a large overall metal abundance, because the calcium and scandium lines are weak.

It is now important to ask whether the Am phenomenon is just a surface phenomenon, or is it affecting the bulk properties of the two stars and causing us to find a large and spurious metal abundance? There are two ways of answering this question: consideration of other metallic-lined stars in dEBs and investigation into whether the diffusion of chemical species can significantly affect the radii of A-type stars predicted by models.

Several studies of dEBs containing Am stars have recently been undertaken: those of V885 Cyg (Lacy et al. 2004), V459 Cas (Lacy, Claret & Sabby 2004), WW Cam (Lacy et al. 2002) and V364 Lac (Torres et al. 1999). Of these four systems, the components of V885 Cyg and WW Cam are clearly metallic-lined but the presence of chemical peculiarities in the other two systems is rather less certain. The theoretical

models of Claret (2004) can match the properties of V885 Cyg and WW Cam for metal abundances of 0.030 and 0.020, respectively. The same models can also match V459 Cas with $Z = 0.012$ and V364 Lac with $Z = 0.020$. As these dEBs can be matched by models with normal chemical compositions, we have no reason to suppose that the Am phenomenon makes a significant difference to the properties of the component stars of WW Aur. The metallic-lined dEB KW Hya was analysed by Andersen & Vaz (1984, 1987), who found an unusual chemical composition from comparison with the models of Hejlesen (1980). However, the Hejlesen models use opacity data which is significantly different from recent results, so any chemical composition derived using them is in need of revision.

Theoretical models for A-type stars with diffusion have been published by Richer, Michaud & Turcotte (2000). The Am phenomenon is found to be important only near the surfaces of A-type stars, so only affects surface quantities such as T_{eff} (S. Turcotte, 2004, private communication). It will therefore have a negligible effect on the overall radius of a star. In particular, for WW Aur to match theoretical models of a normal chemical composition, the *ratio* of the radii of the two stars must become significantly smaller. Any physical phenomenon must therefore affect one component of WW Aur far more than the other component to change the ratio of the radii significantly.

As we have found no evidence that the Am phenomenon is causing us to find a high metal abundance for WW Aur, we will now consider the existence of stars which are very rich in metals. The highest recent estimation of the metal abundance of a dEB is $Z = 0.042$ for EW Ori (Popper et al. 1986) by Ribas et al. (2000), by comparison with (extrapolated) predictions from the Granada theoretical models, which is still somewhat lower than the $Z = 0.06$ we find for WW Aur.

A metal abundance of $Z = 0.06$, corresponding to $[\frac{\text{Fe}}{\text{H}}] = +0.5$, is at least three times higher than solar (but see Asplund, Grevesse & Sauval 2004 in section 1.1.1.3). However, the metal abundance of the old open cluster NGC 6253 has been found to be between $Z = 0.04$ and 0.06 (Twarog, Anthony-Twarog & De Lee 2003). Metallicities between $[\frac{\text{Fe}}{\text{H}}] = 0.4$ and 0.9 were found from consideration of the Strömgren m_1 and calcium hk photometric indices of its member stars, although the best isochronal match

to the morphology of its colour-magnitude diagram was obtained using the Padova stellar evolutionary models for $Z = 0.04$ and enhanced abundances of the α -elements. Therefore, whilst high metal abundances of $Z = 0.06$ are unusual, there is no reason to assume that they do not exist.

7.10 Conclusion

We have studied the metallic-lined A-type dEB WW Aur in order to determine its physical properties. The masses have been derived to accuracies of 0.4% by cross-correlation against observed spectra of standard stars. The radii have been derived to accuracies of 0.6% by analysing seven good light curves with the JKTEBOP code, without the use of theoretical calculations. The masses and radii of the components of WW Aur therefore are among the most accurately known for any stars. The T_{eff} s of the two stars have been derived from their Hipparcos parallax, using a method which is nearly fundamental.

Attempts to find a good match between the physical properties of WW Aur and predictions from several sets of theoretical stellar evolutionary models were unsuccessful for any of the chemical compositions for which models are available. PMS evolutionary predictions were equally unsuccessful. However, the predictions of the Claret (2004) models match the observed properties of the component stars of WW Aur for a metal abundance of $Z = 0.06$ and for ages between 77 and 107 Myr. An acceptable match can also be obtained for a solar chemical composition and an age around 1 Myr, but the range of possible ages for this match is so short that it is very improbable that WW Aur is currently in that evolutionary stage.

The tidal evolution theory of Zahn predicts timescales for orbital circularisation and rotational synchronisation which are much longer than the age of WW Aur, in disagreement with the observed circular orbit and the synchronous rotation of the stars. The timescales found using the hydrodynamical theory of Tassoul are much shorter and correctly predict that the stars should rotate synchronously, but the Tassoul

timescale for circularisation is inconsistent with the age range suggested by the $Z = 0.06$ theoretical models. However, this could be easily explained by tidal evolution in the PMS phase (Zahn & Bouchet 1989) or the formation of a system with a nearly circular orbit (Tohline 2002).

We have found no evidence to suggest that the Am phenomenon is causing us to derive a spurious metal abundance for WW Aur. In fact, the Claret (2004) theoretical evolutionary models can match the observed properties of other Am dEBs for entirely normal chemical compositions. This is in agreement with the results of stellar models which include diffusion, which suggest that its presence does not significantly affect the radii of A-type stars.

Our conclusion that WW Aur is rich in metals relies on our measurements of the masses and radii of the two stars and the use of theoretical models to determine what initial chemical compositions could produce these properties. The masses and radii of WW Aur have been determined very accurately by our analysis, so the conclusion that the stars are metal rich is robust. However, a separate confirmation of this is important, and could be obtained by observing fundamental T_{eff} s which are more accurate than the values we were able to derive in this work.

A final analysis of the astrometric data from the Hipparcos satellite will soon be available, which is expected to report a significantly more accurate parallax for WW Aur (F. van Leeuwen, 2004, private communication). This will allow more accurate T_{eff} s to be derived. WW Aur will also be a target for the ASTRA robotic spectrophotometer (Adelman et al. 2004), which aims to determine accurate spectrophotometric fluxes for the determination of fundamental T_{eff} s of many types of stars. A rediscussion of the T_{eff} s of WW Aur compared to theoretical predictions may then allow the chemical composition of the system to be studied in more detail.

8 Conclusion

8.1 What this work can tell us

We have studied a total of six dEBs in four stellar open clusters. The first conclusion that can be drawn from this research programme is that one telescope observing run can provide a complete spectroscopic dataset for ten or more dEBs (of which only a minority have been studied in this work). Further conclusions divide easily into three categories: analysis techniques for the study of dEBs, what dEBs can tell us about stellar clusters, and what we can find by comparing the properties of dEBs to theoretical stellar models. I will now summarise these three categories.

8.1.1 The observation and analysis of dEBs

Three conclusions can be drawn concerning the acquisition of data for dEBs:

- Complete spectroscopic observations of many dEBs can be obtained during the same observing run, resulting in a high efficiency in terms of time taken to obtain and reduce data. This is particularly useful for observing a good set of standard stars.
- Photometric data for dEBs requires much more time and effort to obtain, but this can be avoided by using data which has been culled from the literature. These light curves may previously have been analysed using outdated methods (V453 Cyg and WW Aur) or may be previously unpublished (WW Aur).
- dEBs which are located in the same cluster can be studied simultaneously using CCD photometry.

Conclusions concerning the photometric analysis of dEBs are:

- The results from the three different light curve modelling codes, EBOP, WINK and WD98, are generally in excellent agreement (HD 23642 and WW Aur) which confirms that they are reliable tools for the study of dEBs.
- Uncertainties in light curve parameters are reported by EBOP, WINK and WD98, but these formal errors are well known to be significantly too optimistic (Popper 1984; see section 6.4).
- I have implemented a Monte Carlo algorithm to find robust uncertainties in the light curve parameters of dEBs. Its results agree extremely well with the variation in results for different light curves of the same dEB (V453 Cyg, WW Aur) and it is a powerful way of investigating correlations between different light curve parameters (V453 Cyg, HD 23642, WW Aur). I recommend that the Monte Carlo algorithm becomes the standard technique for finding light curve uncertainties.
- Limb darkening and third light must be carefully considered when fitting light curves of dEBs. The value of the limb darkening coefficients can make a significant difference to the result (V621 Per); this can easily be investigated and quantified using the results of the Monte Carlo analysis. Third light cannot be assumed to be zero unless this clearly provides the best fit to the light curves (e.g., WW Aur); if it does not then the uncertainties in the photometric parameters must be increased to reflect this (HD 23642).

Conclusions concerning the spectroscopic analysis of dEBs are:

- The two-dimensional cross-correlation algorithm TODCOR (Zucker & Mazeh 1994) is a reliable tool for extracting RVs from observed composite spectra, performing particularly well compared to other techniques when the data is of a low signal to noise ratio (V618 Per).
- The use of synthetic template spectra with TODCOR can provide precise results, but systematic errors from the mismatch between the template and the

observed spectra must be quantified. One way of estimating these is to run TODCOR using every combination of many template spectra generated for a wide variety of T_{eff} s, surface gravities and rotational and microturbulent velocities (V618 Per; work on NGC 2243 V1 in preparation).

- Template spectra for a TODCOR analysis can be obtained by observing the target dEB at phases where the RV difference between the two stars is minimal, or during total eclipse when the spectrum comes entirely from one star (V453 Cyg). This avoids systematic errors due to mismatch between template and target spectra.
- The use of observed template spectra with TODCOR is an excellent way of deriving accurate spectroscopic orbits as it can avoid the types of systematic errors associated with the use of synthetic templates. If TODCOR is run using every combination of a set of observed templates, systematic errors due to spectral mismatch will average out and the internal errors of the resulting spectroscopic orbit can be found (WW Aur).
- The errors reported by SBOP are an excellent estimate of the actual internal errors of a spectroscopic orbit (WW Aur).

During my analysis of WW Aur I was able to use the techniques covered above to derive accurate masses and radii for both stars using an entirely arithmetical approach. The EBOP code is geometrical by nature and limb darkening coefficients were directly optimised rather than being fixed at theoretical values. A set of nine observed template spectra were used in the TODCOR analysis, avoiding possible systematic errors due to the use of synthetic spectra or to one observed template spectrum which might poorly match the spectra of the target stars. It is clear to me that these methods are a good way with which to analyse observational data on dEBs.

8.1.2 Studying stellar clusters using dEBs

Knowledge of the masses and radii of a dEB allow us to estimate its age and chemical composition from a comparison with the predictions of theoretical stellar evolutionary models. In the case of the h Persei open cluster, a precise metal abundance was derived from the positions of the component stars of V615 Per and V618 Per in the mass–radius plane even though the radii of these stars are known to accuracies of only around 5%. More accurate dimensions of these four stars would enable estimation of a precise age, metal abundance, helium abundance and possibly convective efficiency parameters. We have also provided further evidence that h Persei and χ Persei are physically related because their systemic velocities are the same (V615 Per, V618 Per and V621 Per). The study of dEBs which are near the MS turn-off of their parent open cluster would allow a detailed investigation into the success of convective overshooting approximations in theoretical stellar models.

dEBs are excellent distance indicators, and this was used in the study of HD 23642 to find the distance to the Pleiades open cluster. The resulting distance does not agree with that derived from the parallax observations of the Hipparcos satellite. There are several different ways of finding the distance to a dEB (HD 23642), and the best results are obtained at infrared wavelengths because of the reduced importance of interstellar reddening, stellar metal abundance, uncertainties in the T_{eff} of the stars, and a lower ‘cosmic scatter’. Relations between surface brightness and colour index allow an entirely empirical distance to be found to a dEB, but the results can be inaccurate. The use of methods involving bolometric corrections generally provide more precise results but this comes with either a dependence on theoretical model atmospheres or inaccurate empirical bolometric corrections. To avoid these problems, we introduced a new method to find the distance to a dEB which uses relations between surface brightness and T_{eff} (HD 23642). Whilst this method is not entirely empirical, it provides results which are as precise as methods using theoretical bolometric corrections but are much less dependent on theoretical calculations.

8.1.3 Theoretical stellar evolutionary models and dEBs

dEBs provide excellent tests of theoretical models because it is possible to derive accurate masses, radii and T_{eff} s of two stars which have the same age, distance and chemical composition. V453 Cyg is a particularly rewarding dEB for a comparison with theoretical models because its masses, radii and T_{eff} s are accurately known, as is the central concentration of the mass of the primary star (from analysis of the apsidal motion of the dEB). The theoretical models of the Granada, Padova, Cambridge and Geneva groups were all able to provide a good fit in the mass-radius and T_{eff} - $\log g$ diagrams to the properties of this high-mass slightly-evolved dEB, whilst the Granada models also successfully predicted the central concentration of the primary star. There was a minor indication that models incorporating convective core overshooting provide a better fit to V453 Cyg. It is clear that the current generation of theoretical models are very successful at predicting the properties of main sequence B, A and F stars, and that more evolved, more massive or less massive dEBs must be studied in order to provide useful tests of stellar evolutionary theory.

The formation scenarios of binary stars were investigated for V615 Per and V618 Per. All four stars in these dEBs exhibit slow rotation and circular orbits despite being only 13 Myr old. This is in complete disagreement with theories of MS tidal evolution but may be explained by strong tidal effects during the PMS phase or by formation of binary stars which have these characteristics at birth.

The properties of metallic-lined stars were investigated (WW Aur) and it was found that they have masses and radii characteristic of normal A-type stars, suggesting that the Am phenomenon is a surface characteristic. For WW Aur we were only able to fit the masses and radii using theoretical models with a very high metal and helium abundance ($Z = 0.06$ and $Y = 0.36$). We presented evidence that this was not the case for other metallic-lined dEBs and that indications of such large abundances have been noted elsewhere.

One unique feature of the study of dEBs in stellar clusters is that it is possible to find accurate masses, radii and T_{eff} s for four or more stars with the same age, distance

and chemical composition. This was first noted when studying V615 Per and V618 Per and potentially can provide an extremely detailed test of theoretical models in which values may be found for many different theoretical parameters which would otherwise be left fixed at a reasonable estimate. Another way in which the study of dEBs in clusters will be useful is in forcing theoretical models to fit the masses, radii and T_{eff} of both components of the dEB whilst simultaneously matching the radiative properties of the other member stars in the CMD of the cluster. This requires accurate dimensions for a dEB in a cluster with a well-defined morphology in the cluster CMDs, so we were not able to investigate it further using the dEBs studied in this work.

8.2 Further work

8.2.1 Further study of the dEBs in this work

A definitive study of a dEB is generally expected to provide masses and radii to accuracies of 2% as well as accurate T_{eff} and a reasonable comparison with theoretical models. The studies of WW Aur and V453 Cyg presented in this work can therefore be regarded as definitive, although the characteristics of V453 Cyg are such that further spectroscopy, photometry and times of minimum light would clearly make the dEB even more interesting.

V615 Per and V618 Per are very promising candidates for further study as we have found their masses to within 1.5% but their radii are much more uncertain. As the h Persei open cluster has a well-defined CMD morphology, an improved study of the two dEBs will allow the development of tools for the simultaneous matching of the properties of the dEBs and the cluster to theoretical models.

The dimensions of HD 23642 are also less accurate than they could be and this dEB is also in a nearby and well-known open cluster. It will certainly be the subject of further study in the near future, and I am already aware that new light curves have been obtained by another group.

The study of V621 Per presented in this paper is different to the other work in that we were not able to detect the secondary star spectroscopically and so were not able to measure the masses and radii of either star. This dEB may be difficult to study further but the effort would be very worthwhile because it might provide accurate dimensions of a B-type giant star, a system which would be unique amongst well-studied dEBs (see Andersen 1991). The secondary component in the V621 Per system is known to be unevolved as we were able to calculate its surface gravity to be $\log g = 4.244 \pm 0.054$ from the results of the spectroscopic and photometric analysis. dEBs with a small mass ratio (here expected to be around 0.5) are particularly valuable as they are excellent tests of theoretical stellar models.

8.2.2 Other dEBs in open clusters

We have obtained spectroscopic data for a substantial number of dEBs which were not studied in this work, and hope to be able to publish much of this in the near future. A short list of dEBs in open clusters is presented in Table 8.1; we already have data for some of these systems. I know of two other research groups currently working on dEBs in open clusters.

Many studies have been published on the photometric identification of variable stars by the observation of light curves using telescopes and CCDs. These are often targeted towards open clusters to increase the number of stars in the observed field of view, and because variable stars in open clusters are intrinsically more interesting. In particular, the journal *Acta Astronomica* has published many such studies (Table 8.1). dEBs are usually found towards the MS turn-off as stars increase somewhat in radius during the latter stages of the MS evolution (Kaluźny & Rucinski 1993).

8.2.3 dEBs in globular clusters

The usefulness of studying dEBs in globular clusters was demonstrated by Thompson et al. (2001) by their analysis of OGLE GC 17 in the peculiar Galactic globular cluster

Table 8.1: A list of dEBs in Galactic open clusters and associations for which their study may be very rewarding.

Eclipsing system	α_{2000} (hour, min)	δ_{2000} (degrees)	Cluster or association	Spectral type	Apparent V magnitude	Period (days)	Reference
NGC 188 V12	00 39	+85	NGC 188	F	15.0	2.8	Zhang et al. (2002, 2004)
NGC 581 V1	01 30	+60	NGC 581		11.3	6.0	Wyrzykowski et al. (2002)
NGC 581 V2	01 30	+60	NGC 581		14.5	1.6	Wyrzykowski et al. (2002)
DS And	01 54	+37	NGC 752	F3 + G0	0.5	1.0	Schiller & Milone (1988)
AG Per	04 07	+33	Perseus OB2	B4	6.7	2.0	Giménez & Clausen (1994)
V818 Tau	04 24	+15	Hyades	G8 + K3	8.3	5.6	Schiller & Milone (1987)
EW Ori	05 33	-01	Collinder 70	G0 + G5	9.8	6.9	Popper et al. (1986)
NGC 2099 V1	05 49	+32	Messier 37	G	13.8		Kiss et al. (2001)
NGC 2099 V2	05 49	+32	Messier 37	G	15.0		Kiss et al. (2001)
NGC 2243 V1	06 28	-31	NGC 2243	F	16.3	1.2	Kałużny et al. (1996)
NGC 2243 V5	06 28	-31	NGC 2243	F	16.3	1.2	Kałużny et al. (1996)
V392 Car	07 58	-61	NGC 2516	A2	9.5	3.2	Debernardi & North (2001)
GV Car	11 04	-58	NGC 3532	A0	8.9	4.3	Kraft & Landolt (1959)
QR Cen	13 57	-59	NGC 5381	A	12.5	2.3	Pietrzyński et al. (1997)
V906 Sco	17 51	-34	Messier 7	B9	6.0	2.8	Alencar, Vaz & Helt (1997)
V1481 Cyg	21 42	+53	NGC 7128	B2	12.3	2.8	Jerzykiewicz et al. (1996)

ω Centauri. The faintness of this dEB meant that the masses and radii were found to accuracies of only 7% and 3%, respectively, but the use of IR surface brightness relations meant that a distance of 5360 ± 300 pc (corresponding to a distance modulus of 13.65 ± 0.12 mag) could be derived. The age of the dEB and of ω Cen was also found to be between 13 and 17 Gyr. Further observations of this dEB have been made (Kaluźny et al. 2002) but have not yet been published, and several more dEBs are known in this cluster.

A significant number of dEBs have been discovered in 47 Tucanae (Albrow et al. 2001; Weldrake et al. 2004), but these are quite faint. Additional candidates have been found in NGC 6641 (Pritzl et al. 2001) and in M 22 (Kaluźny & Thompson 2001). A compilation of variable stars which have been detected in the fields of globular clusters has been given by Clement et al. (2001).

8.2.4 dEBs in other galaxies

A large number of dEBs have been found through time-series photometry of the LMC and SMC by the OGLE, EROS, MACHO and MOA groups (see section 1.6.3.4) and several of these have been studied in order to find the distance to the Magellanic Clouds (e.g., Hilditch, Harries & Howarth et al. 2005; section 1.6.3.4). The MOA group have published details of 167 EBs in the SMC (Bayne et al. 2004). The EROS group have published a list of 79 EBs located towards the bar of the LMC (Grison et al. 1995). The MACHO group have published a list of 611 EBs in the LMC (Alcock et al. 1997).

The OGLE group have obtained by far the largest amount of photometry towards both the LMC and the SMC and have found 2580 EBs in the LMC (Wyrzykowski et al. 2003) and 1459 EBs in the SMC (Udalski et al. 1998). Analysis using a difference image analysis algorithm (Żebruń, Soszyński & Wozniak 2001) has allowed the discovery of a further 455 EBs in the SMC (Wyrzykowski et al. 2004). Most interestingly from the point of view of this work are the 127 EBs which have been found to be in optical coincidence with star clusters in the SMC (Pietrzyński & Udalski 1999).

8.2.5 dEBs in clusters containing δ Cephei stars

δ Cephei stars can be used as distance indicators at greater distances than EBs because they are intrinsically brighter objects (with absolute visual magnitudes between about -2 and -6) and can be studied at dimmer apparent magnitudes because spectroscopy is not needed. The distances to Galactic open clusters which contain dEBs and δ Cephei stars can be found from the dEB and used to calibrate the δ Cephei distance scale.

The primary candidate for such an analysis is be QX Cassiopeiae, which is a dEB in the same field as the open cluster NGC 7790, which contains three δ Cephei stars. Sandage (1958) and Sandage & Tammann (1969) find that it is a photometric non-member but E. Guinan finds that it is a radial velocity member (E. Guinan, 2005, private communication). If it is a non-member it cannot be used to find the distance to NGC 7790.

Several δ Cephei stars are known to be members of Galactic open clusters (see Mermilliod, Mayor & Burki 1987) and these clusters should be photometrically surveyed for dEBs which can then be studied in order to find the distance and chemical composition of the clusters and δ Cephei stars.

8.2.6 dEBs which are otherwise interesting

There is a shortage of dEBs which contain well-studied component stars more massive than $10 M_{\odot}$ (Andersen 1991). Such systems are intrinsically rare as massive stars have a low birth rate and very short lifetimes. They can also be very difficult to study photometrically, as long orbital periods are needed for the stars to be well detached, and spectroscopically, due to a shortage of strong metallic spectral lines and often large rotational velocities. Accurate properties of massive dEBs are needed to provide improved constraints on theoretical stellar models and to ensure that we understand such systems well enough to use them as distance indicators in external galaxies.

There is a shortage of dEBs which contain well-studied component stars less massive than $1 M_{\odot}$ (Andersen 1991). Such systems are difficult to find as low-mass stars

are very faint, and the stars are small so are less likely to eclipse. They can also be very difficult to study photometrically, as they often exhibit surface inhomogeneities such as starspots, and spectroscopically as their spectra are complex and relatively poorly understood. Mazeh, Latham & Goldberg (2001) have stated that the best observational data which can be used to improve stellar evolutionary models for low-mass stars is the study of M-type dEBs in open clusters. This will need nearby clusters for the M dwarf stars to be sufficiently bright for study, but may provide accurate masses and radii of low-mass stars with a known metal abundance and age. Several researchers are working on providing accurate astrophysical parameters of low-mass dEBs (Clausen, Helt & Olsen 2001; Oblak et al. 2004; Hebb, Wyse & Gilmore 2004; Pepper, Gould & DePoy 2004).

dEBs which exhibit apsidal motion are intrinsically more valuable because their orbital parameters may be derived more accurately and the central concentration of the masses of the stars can be investigated (section 1.7.2). This allows a more detailed test of theoretical stellar evolutionary models (e.g., V453 Cyg, section 4.1).

Some types of stellar peculiarity can be investigated by studying examples which are in dEBs, for example metallic-lined stars (WW Aur, section 7) and slowly pulsating B stars (Clausen 1996a).

8.2.7 dEBs from large-scale photometric variability studies

Wide-field searches for photometrically variable stars is currently an extremely popular subject in astronomy, mainly due to the possibility of detecting extrasolar planetary candidates which transit their parent stars (so are therefore actually members of eclipsing binary systems). Several of these have targeted nearby open clusters. It is expected that many (possibly thousands of) dEBs will be discovered in the near future, and that the light curves of some of these may be definitive, depending on the observational procedures adopted by the groups involved. A full exposition of the groups pursuing this research is beyond the scope of the current work, but it is relevant to mention some of those groups whose research is either sufficiently advanced to have appeared in pub-

lished journals or is particularly relevant to the study of dEBs in stellar clusters. A full list of groups who are attempting to detect transiting extrasolar planets through wide-field CCD photometry is maintained by K. D. Horne¹.

SuperWasp² is the brainchild of D. Pollacco³ and currently consists of five CCD cameras and telephoto lenses which are mounted on one telescope mount on La Palma. Each camera-lens combination has a field of view of $(7.8^\circ)^2$ and can achieve 1% photometric precision for stars with apparent magnitudes between about 7 and 12 (Christian et al. 2004). This project is the successor to the WASP0 project, which consisted of one CCD camera and telephoto lens piggy-backed onto a commercially available Meade telescope (see Kane et al. 2004).

All Sky Automated Survey (ASAS⁴) is a project to survey the whole Southern sky for photometric variability using four telescopes located at Las Campanas Observatory, Chile (Pojmański 1997). Several thousand variable stars have already been found (Pojmański 2002 and later works) and the project is ongoing.

EXPLORE/OC⁵ is a project to detect planetary transits around stars located towards nearby open clusters. It has obtained substantial photometry of the open clusters NGC 2660, NGC 6208, IC 2742, NGC 5316 and NGC 6235 (Lee et al. 2004; von Braun et al. 2004) using a 1.0 m telescope and large-format CCD camera. Results for NGC 2660 and NGC 6208 will soon be available.

PISCES⁶ (Planets In Stellar Clusters Extensive Search; it is hoped that less attention will be paid to contrived acronyms in the future) is studying open clusters to find variable stars and transiting planets using a 1.2 m telescope and wide-field

¹<http://star-www.st-and.ac.uk/~kdh1/transits/table.html>

³<http://star.pst.qub.ac.uk/~dlp/>

⁴<http://www.astrouw.edu.pl/~gp/asas/asas.html>

camera with a mosaic of four CCDs. Results have been published for NGC 6791 (Mochejska et al. 2002, 2005) and for NGC 2158 (Mochejska et al. 2004).

STEPSS⁷ (Survey for Transiting Extrasolar Planets in Stellar Systems) is studying nearby open clusters using 2.4 m and 1.3 m telescopes equipped with a mosaic of eight CCDs. Results have been published for NGC 1245 (Burke et al. 2004) and are expected soon for NGC 2099 (M 37) and M 67.

I hope that, after a lull during the 1990s, the huge numbers of newly detected dEBs will be used to begin a new golden age of the study of eclipsing binary stars, the properties of which are of fundamental importance to most aspects of astrophysics.

9 Computer codes

Undertaking the research presented in this thesis involved writing a large number of computer codes to perform some of the wide variety of calculations required for the analysis of observations of detached eclipsing binary stars. Two of these codes are of sufficient importance that in themselves they may be regarded as a product of my research. Details on these two codes are given below; their FORTRAN77 source code is available from the author's website, which can be found at <http://www.astro.keele.ac.uk/~jkt/codes.html>.

The JKTEBOP code (section 3.7.1) was written to analyse the light curves of detached eclipsing binaries. Whilst the geometrical model of the binary system is unchanged from the NDE model implemented in EBOP (Etzel 1975), the input and output are unique and the minimisation algorithm MRQMIN (Press et al. 1992) has replaced the differential corrections procedure used by EBOP. The main modifications contained in JKTEBOP are an extensive set of algorithms for robust estimation of the uncertainties in the derived light curve parameters.

The JKTABSDIM code was originally written to combine the results of the photometric and spectroscopic analysis of a binary system for the calculation of quantities including the absolute dimensions, luminosities and tidal timescales. Great care has been taken to properly deal with propagation of uncertainties, and detailed error budgets for every calculated quantity are outputted by JKTABSDIM. This code also calculates many estimates of the distance to the binary system (again with detailed error analysis) using theoretical (two sources) and observational (one source) bolometric corrections. Most importantly, this code contains the implementation of the technique for finding the distance to a binary system through surface brightness calibrations which was introduced in section 6.6.3. The JKTABSDIM code was written in its entirety by the author.

Publications

Refereed publications

I publish under the name of John Southworth for personal reasons.

Southworth J., Maxted P. F. L., Smalley B., 2004, MNRAS, 349, 547–559.

Eclipsing binaries in open clusters. I. V615 Per and V618 Per in h Persei

Southworth J., Maxted P. F. L., Smalley B., 2004, MNRAS, 351, 1277–1289.

Eclipsing binaries in open clusters. II. V453 Cygni in NGC 6871

Southworth J., Zucker S., Maxted P. F. L., Smalley B., 2004, MNRAS, 355, 986–994.

Eclipsing binaries in open clusters. III. V621 Persei in χ Persei

Southworth J., Maxted P. F. L., Smalley B., 2005, A&A, 429, 645–655.

Eclipsing binaries as standard candles: HD 23642 and the distance to the Pleiades

Southworth J., Smalley B., Maxted P. F. L., Claret A., Etzel P. B., 2005, MNRAS, 363, 529–542.

Absolute dimensions of detached eclipsing binaries. I. The metallic-lined system WW Aurigae

Other publications

Southworth J., Maxted P. F. L., Smalley B., 2004, in Spectrally and Spatially Resolving the Components of Close Binary Stars (Astronomical Society of the Pacific Conference Series vol. 318, Dubrovnik, Croatia, October 2003), eds., R. W. Hilditch, H. Hensberge

and K. Pavlovski, pp. 218–221.

Eclipsing binaries in open clusters

(preprint: <http://xxx.lanl.gov/abs/astro-ph/0312506>)

Southworth J., Maxted P. F. L., Smalley B., 2004, in *Transit of Venus: New Views of the Solar System and Galaxy* (IAU Colloquium No. 196, Preston, England, June 2004), eds., D. W. Kurtz and G. E. Bromage, pp. 361–375.

The distance to the Pleiades from the eclipsing binary HD 23642

Southworth J., Maxted P. F. L., Smalley B., Etzel P. B., 2004, in *The A-Star Puzzle* (IAU Symposium No. 224, Poprad, Slovakia, July 2004), eds., J. Zverko, W. W. Weiss, J. Žižňovský and S. J. Adelman, pp. 548–561.

Accurate fundamental parameters of eclipsing binary stars

(preprint: <http://xxx.lanl.gov/abs/astro-ph/0408227>)

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