# Surface gravity in eclipsing single-lined spectroscopic binaries

During the analysis of the eclipsing binary star system V621 Persei (Southworth et al, 2004, MNRAS, 355, 986) it was realised that it was possible to accurately calculate the surface gravity of the unseen component of a single-lined spectroscopic eclipsing binary even though its actual mass and radius was not directly observable. This realisation has important implications for the study of transiting extrasolar planets.

The mass function of a spectroscopic binary is given by:

$$f(M) = \frac{(1-e^2)^{\frac{3}{2}} {K_{\rm s}}^3 P}{2\pi G} = \frac{{M_{\rm u}}^3 \sin^3 i}{(M_{\rm u} + M_{\rm s})^2} \tag{1}$$

where $M_{\rm s}$ and $M_{\rm u}$ are the masses of the (spectroscopically) seen and unseen stars in the binary, $P$ is the orbital period, $e$ is the orbital eccentricity, $K_{\rm s}$ is the orbital velocity semiamplitude of the spectroscopically visible star and $G$ is the gravitational constant.

Kepler's third law is

$$P^2 = \frac{(2\pi)^2 a^3}{G(M_{\rm s} + M_{\rm u})} \tag{2}$$

Rearranging equations 1 and 2 to eliminate $(M_{\rm s} + M_{\rm u})^2$ gives

$$(M_{\rm s} + M_{\rm u})^2 = \frac{2\pi G M_{\rm u} \sin^3 i}{(1-e^2)^{\frac{3}{2}} K_{\rm s}^3 P} = \frac{(2\pi)^4 a^6}{G^2 P^4} \tag{3}$$

The fractional radius of a binary star is $r = \frac{R}{a}$ where $R$ is the linear radius and $a$ is the orbital semi-major axis. Therefore the definition of surface gravity becomes

$$g = \frac{GM}{R^2} = \frac{GM}{a^2 r^2} \tag{4}$$

Rearranging equations 3 and 4 to eliminate $a^6$ gives

$$a^6 = \frac{G^3 P^3}{(2\pi)^3} \frac{M_{\rm u} \sin^3 i}{(1-e^2)^{\frac{3}{2}} K_{\rm s}^3} = \frac{G^3 M^3}{g^3 r^6} \tag{5}$$

Rearranging equation 5 for surface gravity gives

$$g = \frac{2\pi}{P} \frac{(1-e^2)^{\frac{1}{2}} K_{\rm s}}{r^2 \sin i} \frac{M}{M_{\rm U}} \tag{6}$$

Converting this into astrophysical quantities ($g$ in cm s$^{-2}$, $K$ in km s$^{-1}$ and $P$ in days) gives

$$\log_{10} g = 0.734694 + \log_{10}\left(\frac{2\pi}{P} \frac{(1-e^2)^{\frac{1}{2}} K_{\rm s}}{r^2 \sin i}\right) + \log_{10}\left(\frac{M}{M_{\rm U}}\right) \tag{7}$$

Equation 7 can now be written specifically for each star, giving

$$\log_{10} g_{\rm s} = 0.734694 + \log_{10}\left(\frac{2\pi}{P} \frac{(1-e^2)^{\frac{1}{2}} K_{\rm s}}{r_{\rm s}^2 \sin i}\right) + \log_{10} q \tag{8}$$

$$\log_{10} g_{\rm u} = 0.734694 + \log_{10}\left(\frac{2\pi}{P} \frac{(1-e^2)^{\frac{1}{2}} K_{\rm s}}{r_{\rm u}^2 \sin i}\right) \tag{9}$$

Note that the mass term becomes the mass ratio, $q = \frac{M_{\rm u}}{M_{\rm s}}$, in equation 8 and disappears entirely in equation 9.

Note that these equations differ slightly from those published in Southworth et al (2004, MNRAS, 355, 986), in which the mass function was retained rather than $K_{\rm s}$. This is also why the constant term given here differs from that in Southworth et al.

It is technically possible to derive the absolute masses and radii of both stars if $\log_{10} g_{\rm s}$ is known independently (e.g., from spectral analysis), but these quantities will be hugely uncertain (compared to double-lined eclipsing binaries) unless $\log_{10} g_{\rm s}$ is known to extreme accuracy (of the order of 0.005 dex).