Summing two fluxes in magnitude units

This is a basic equation of the study of binary stars.

The apparent magnitude of a star, m, is defined using

$$m - m_{\rm ref} = -2.5 \log_{10} \left(\frac{f}{f_{\rm ref}} \right) \tag{1}$$

where f is the flux of the star and $m_{\rm ref}$ and $f_{\rm ref}$ are the magnitude and flux of a reference star. This can be rearranged to give

$$\frac{f}{f_{\rm ref}} = 10^{-0.4(m - m_{\rm ref})} \tag{2}$$

Therefore the fluxes of two stars, f_1 and f_2, and their total flux, $f_{\rm TOT}$, are given by

$$\frac{f_1}{f_{\rm ref}} = 10^{-0.4(m_1 - m_{\rm ref})} \tag{3}$$

$$\frac{f_2}{f_{\rm ref}} = 10^{-0.4(m_2 - m_{\rm ref})} \tag{4}$$

$$\frac{f_{\rm TOT}}{f_{\rm ref}} = 10^{-0.4(m_{\rm TOT} - m_{\rm ref})} \tag{5}$$

where m_1, m_2 and $m_{\rm TOT}$ are respectively the apparent magnitudes of star 1, star 2 and of both stars combined.

Now $f_{\rm TOT} = f_1 + f_2$ so

$$\begin{aligned} \frac{f_{\rm TOT}}{f_{\rm ref}} &= \frac{f_1 + f_2}{f_{\rm ref}} = \frac{f_1}{f_{\rm ref}} + \frac{f_2}{f_{\rm ref}} \\ &= 10^{-0.4(m_{\rm TOT} - m_{\rm ref})} = 10^{-0.4(m_1 - m_{\rm ref})} + 10^{-0.4(m_2 - m_{\rm ref})} \\ &= 10^{-0.4 m_{\rm TOT}} 10^{0.4 m_{\rm ref}} = \left(10^{-0.4 m_1} + 10^{-0.4 m_2}\right) 10^{0.4 m_{\rm ref}} \end{aligned} \tag{6}$$

Cancelling the common term in the last line of equation 6 and rearranging gives

$$m_{\rm TOT} = -2.5 \log_{10} \left(10^{-0.4 m_1} + 10^{-0.4 m_2}\right) \tag{7}$$