Distance to an eclipsing binary star using calibrations of surface brightness in terms of effective temperature

This method of determining the distance to a well-studied double-lined detached eclipsing binary was introduced in Southworth et al (2005, A&A, 429, 645) and makes use of the calibrations of surface brightness (expressed as a zeroth-magnitude angular diameter) versus effective temperature constructed by Kervella et al (2004, A&A, 426, 297) for effective temperatures in the range $\left[3600 < \frac{T_{\rm eff}}{k} < 10\,000\right]$.

The surface brightness of a star is a magnitude quantity and is defined to be

$$S_\lambda = m_\lambda + 5\log_{10}\phi \tag{1}$$

where λ indicates an effective wavelength or photometric passband, m_λ is the apparent magnitude of a star at λ and ϕ is the star's angular diameter in milliarcseconds. The zeroth-magnitude angular diameter, $\phi^{m_\lambda=0}$, is defined to be the angular diameter a star would have if its apparent magnitude at λ were zero:

$$\phi^{m_\lambda=0} = \phi\cdot 10^{\frac{m_\lambda}{5}} = 10^{\frac{S_{m_\lambda}}{5}} \tag{2}$$

The angular diameter of a star can be related to its linear radius, R, using the small-angle approximation:

$$\tan\phi = \frac{2R}{d} \approx \phi \tag{3}$$

where d is the distance the star is from the observer. Now rearrange equation 3 and substitute in equation 2 to get

$$d = \frac{2R}{\phi} = 2R\left(\frac{10^{\frac{m_\lambda}{5}}}{\phi^{m_\lambda=0}}\right) \tag{4}$$

$$m_\lambda = 5\log_{10}\left(\frac{d\phi^{m_\lambda=0}}{2R}\right) \tag{5}$$

This method does not assume that the individual magnitudes of the two stars in the eclipsing binary are known. We use the equation which relates the apparent magnitudes of two stars to their combined apparent magnitude. Here $m_\lambda \equiv m_{\rm TOT}$, and m_1 and m_2 are given by equation 5:

$$\begin{aligned}
m_\lambda &= -2.5\log_{10}\left[10^{-0.4m_1} + 10^{-0.4m_2}\right] \\
&= -2.5\log_{10}\left[10^{-0.4\times5\log_{10}\left(\frac{d\phi_1^{m_\lambda=0}}{2R_1}\right)} + 10^{-0.4\times5\log_{10}\left(\frac{d\phi_2^{m_\lambda=0}}{2R_2}\right)}\right] \\
&= -2.5\log_{10}\left[10^{\log_{10}\left(\frac{d\phi_1^{m_\lambda=0}}{2R_1}\right)^{-2}} + 10^{\log_{10}\left(\frac{d\phi_2^{m_\lambda=0}}{2R_2}\right)^{-2}}\right] \\
\frac{m_\lambda}{-2.5} &= \log_{10}\left[\left(\frac{d\phi_1^{m_\lambda=0}}{2R_1}\right)^{-2} + \left(\frac{d\phi_2^{m_\lambda=0}}{2R_2}\right)^{-2}\right] \\
-0.4m_\lambda &= \log_{10}\left[\left(\frac{2R_1}{d\phi_1^{m_\lambda=0}}\right)^{2} + \left(\frac{2R_2}{d\phi_2^{m_\lambda=0}}\right)^{2}\right] \\
10^{-0.4m_\lambda} &= \frac{1}{d^2}\left[\left(\frac{2R_1}{\phi_1^{m_\lambda=0}}\right)^{2} + \left(\frac{2R_2}{\phi_2^{m_\lambda=0}}\right)^{2}\right] \\
d^2 &= 10^{0.4m_\lambda}\left[\left(\frac{2R_1}{\phi_1^{m_\lambda=0}}\right)^{2} + \left(\frac{2R_2}{\phi_2^{m_\lambda=0}}\right)^{2}\right]
\end{aligned} \tag{6}$$

The final equation is obtained by taking the square root of each side:

$$d = 10^{0.2m_\lambda}\sqrt{\left(\frac{2R_1}{\phi_1^{m_\lambda=0}}\right)^{2} + \left(\frac{2R_2}{\phi_2^{m_\lambda=0}}\right)^{2}} \tag{7}$$

This equation will give a distance in parsecs if R_1 and R_2 are given in AU, and $\phi_1^{m_\lambda=0}$ and $\phi_2^{m_\lambda=0}$ are given in arcseconds. The zeroth-magnitude angular diameter for each star is found from its effective temperature and the calibrations of Kervella et al (2004, A&A, 426, 297).